Accelerating Mathematics Achievement

A GUIDE FOR PRINCIPALS AND TEACHERS

Not only is what students are being taught important but, how they are taught those concepts and skills is just as important. This book addresses very effective teaching strategies for learning mathematics. The author's knowledge of mathematics combined with his knowledge and insight of working with students of poverty results in recommendations that will result in increased student achievement.

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Bill Hanlon, Director of the Southern Nevada Regional Professional Development Program, has been an educator for over thirty years. His educational experiences include teaching at the junior high, senior high, and college levels. He was the coordinator of Clark County School District's Math/Science Institute and was responsible for K-12 math audits. He served as vice president of the Nevada State Board of Education, Regional Director of the National Association of State Boards of Education (NASBE) and as a member of the National Council for Accreditation of Teacher Education (NCATE) States Partnership Board. Bill was also a member of Nevada's standards writing team in mathematics, and served on the Learning First Alliance Review Team of the NCTM's standards. He hosts a television series, "Algebra, *you can do it!*" and taught mathematics at the University of Nevada, Las Vegas, to prospective K-12 classroom teachers.

Bill's knowledge of mathematics combined with his knowledge and insight of working with students living in poverty brings uniqueness to his style of professional development. Based on the foundation that students should feel comfortable in their knowledge, understanding, and application of mathematics, Bill provides professional development for teachers that will assist them in helping all students succeed in math.

Bill has published two books, Math, *your students can do it!* and Accelerating Mathematics Achievement. He has presented at numerous national conferences including the National Council of Teachers' of Mathematics as well as providing services to local school districts.

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Preface

Nationwide, educators are increasingly committed to helping students succeed on high stakes tests. But to be successful, they need to be just as committed to helping their students to be more comfortable in their knowledge, understanding, and application of mathematics.

Quick fixes seldom work! Too many school district administrators are trying to buy success from companies making extravagant claims about their products and services, that noise tends to drown out common sense. Educators need to remember a simple axiom, *what works is work!*

There have been a number of major initiatives that have come and gone during the last half century. Many have focused on the identification and testing of outcomes without closely examining student work or classroom practice. While the identification of outcomes is an important step in planning effective instruction, there must also be an emphasis on "how" to improve instruction for increased student achievement. To that end, this document offers proven recommendations to improve student achievement in mathematics, recommendations that teachers can incorporate and implement in their classrooms. Importantly, these factors can also be readily identified and monitored by an instructional supervisor. Additionally, more focus has to be placed on acquiring the skills students need to develop to be successful in higher-level coursework and college. There should be no doubt that increasing student achievement depends upon students taking more appropriate, more rigorous coursework. The research suggests students completing a more rigorous high school curriculum tend to be more successful in college.

There are two standards that serve as our foundation for these recommendations. The first is the "Common Sense" standard. Simply stated, it means just what it says, the suggestions should appeal to a person's common sense. The second standard is the "My Kid" standard. The "My Kid" standard simply states that classroom teachers and administrators should treat the students in their classrooms the same way they would like to have other educators treat their own children. As you read this text, challenge the suggestions to determine if they meet those two standards.

There are no shortcuts to increasing student achievement. The underlying belief is *what works is work*. Increasing student achievement does not just happen. Classroom teachers and administrators must work to achieve their goals. That work should be reflected in teacher knowledge, instructional and assessment practices, and how well they work with their students. If instruction is to improve, supervisors must give feedback to classroom teachers.

To address these issues, educators need a plan and a support system, a system that is research based, a system that allows them the opportunity to meet regularly with their colleagues to discuss what they teach, how they teach it, student performance, and instructional strategies that will result in increased student achievement. To develop this plan, professional expectancies should be adopted. Expectancies, protocols, are expected behaviors. We see these in doctors' offices, courtrooms, air traffic controllers, and other professions as a way of setting common standards in practice. We will recommend expectancies that should be adopted in education.

Being an educator is not easy. In business, if you receive raw material that is not up to par, you simply return it. For instance, if the manager of a restaurant received bad meat, he would not cook it and serve it, he'd return it. If a builder received warped wood, he'd return it before he would start building. In education, if students enter kindergarten completely unprepared, public schools have to accept them – as is - there is no expectation of returning them to their parents until they are ready for the experience.

Educators must work hard, but they also have to work smart to be successful. In this era of accountability, high stakes testing is driving instruction. Policymakers often interpret the results of those tests as students' achievement. That's unfortunate because the easiest way to increase those test scores is to teach to those high stakes tests and not the curriculum.

High stakes tests can only measure student responses to 35 to 70 questions, understanding subject matter, the curriculum is hundreds of times larger than what those test items could ever measure. Since so much emphasis has been placed on those high stakes tests and since the public has been led to believe these tests measure student achievement, many educators have abandoned "educating" their students and adopted a "training" model that prepares their students for these tests.

Some might question if there is a difference between educating students and training them. I would suggest that a student who learned to play "chopsticks" on the piano to impress his family went through training. If I asked that student to read music or play other songs, he could not do it. In other words, all he got out of his piano lesson was to learn one song. He received training to play "chopsticks", that was what he was going to be tested on. If another student learned about the keys on the piano, learned how to read and play music, I would suggest he was being educated. There is a difference between training and educating!

In Accelerating Mathematics Achievement, Hanlon recognizes the difference between training students to be successful on a specific high stakes state test versus educating our students that will result in increased performance on all tests. His belief is a good education will result in increased test scores that mirrors increased student performance. But he also believes teachers and school administrators should not expect from their students or subordinates what they are unwilling to inspect. Feedback, follow-up and follow-through are important components to creating and maintaining success.

Chapter 1
Balanced Delivery of Instruction

Some states have experienced math wars between the so-called *traditionalists* and the *constructivists* on the best way to teach math that will result in increased student achievement. They saw it as an either/or proposition. What I profess and recommend is a balance in the delivery of instruction matched by a balance in assessment, a balance defined by what we say we value in math education – what we want students to know, recognize and be able to do.

Teacher-made tests are important, they drive instruction, and they should be balanced. Balance in mathematics is defined as:

1. Concept development & linkage
2. Problem solving
3. Vocabulary and notation
4. Memorization of important facts and procedures
5. Appropriate use of technology

Studying test results and student work would suggest to even the casual observer that students miss far too many questions because they simply did not know what was being asked. For instance, to find a degree of a monomial, all students need to do is add the exponents – not very difficult mathematically speaking. But many students will miss such an easy question. Why? They didn't know or understand the vocabulary. Vocabulary and notation are important and need to be stressed in teaching. Stressing vocabulary and notation, language acquisition, is recognized as being important when addressing the needs of English language learners, it is also important for addressing the needs of all students studying mathematics. There is no more single important factor that leads to comprehension than acquiring the necessary vocabulary.

Remember the Law of Cosines? No, then you illustrate my next point. That is, it is not a matter of if students are going to forget much of the information they learn, it's a matter of when they will forget it. If you were to ask me the sine of thirty degrees right now, the fact is, I can't immediately recall it. But, since I understand how it was defined and developed, give me thirty seconds and I can tell you the answer. Understanding and being able to reconstruct information is important in learning and maintaining knowledge and skills over time.

In the late 80's and early 90's, there were math educators who maintained that any type of drill kills. Others suggested that repetition is the mother of learning. If we want students to be problem solvers, critical thinkers, then they must be able to immediately recall important facts and procedures. Memorization is important and teaching students how to memorize will help them in their learning, thinking, and problem solving.

Now, let's look at these individually and see how nicely they all really fit together so students have a greater appreciation of mathematics.

Concept Development and Linkage

In classrooms lacking sufficient concept development, teachers primarily emphasize memorization of rules and algorithms with little or no attempt made to help students understand the "why" of mathematics processes. Without a thorough understanding of the underlying mathematical concepts, students are not truly learning mathematics. Mathematics becomes an arbitrary set of isolated rules that don't make sense and have a tendency to look like magic. Teachers must do more than just give the rules; they must build student competence through understanding.

Think of the rules we give students. Two negatives are a positive, unless you are adding, then they are negative. Any number to the zero power, except zero, is one. When you add integers, sometimes you add, sometimes you subtract. When you subtract integers, you change the sign and add, but then again, you might subtract. You can't divide by zero. When you divide fractions, you flip and multiply. Who's making up these rules? Were they out of their minds? Standing alone, the rules don't make much sense.

The fact is those rules are just short cuts that allow students to solve problems or compute quickly. Shortcuts in math have fancy names; we call them theorems, corollaries, axioms, postulates, formulas, or rules but- they are nothing more than short cuts. Some of the short cuts we are most comfortable, familiar, and use don't make sense either. For instance, if I asked you to multiply 72 by 10, you'd get 720 very quickly. If I asked how you arrived at the answer, you'd most likely respond by saying you added a zero. That's not true. If you added zero to 72, you'd end up with 72.

As mathematics becomes more abstract, "math anxiety" may develop if short cuts (rules, theorems, and algorithms) have not been developed with an understanding of "why" they work. If this is not addressed, students can become frustrated and eventually quit enrolling in mathematics classes, even though the grade they earned in their last class was average or above. Each lesson should build on and strengthen the students' mathematical foundation. Teachers should not assume students have already seen, let alone remember, an explanation of a particular mathematics concept. Even if they have, a quick refresher may be beneficial.

For example, finding the sum of the interior angles of a triangle might be introduced by having students draw a triangle and cut out the three angles of their triangle and piecing them together to form a straight angle (180˚) – suggesting the sum of the three angles is 180˚ The Pythagorean Theorem might be introduced by examining the areas of the squares formed by the sides of the triangle. Seeing these patterns, students might hypothesize the area of the square formed by the hypotenuse is equal to the area of the squares formed by the other two legs. Teachers should not just draw a right triangle, identify the hypotenuse, give the formula and work out problems. The understanding gained, in combination with sufficient practice and memorization of important information, gives students confidence in their ability to do mathematics. To deliver this balanced approach, time and thoughtful preparation must be given to each and every lesson. This may require teachers to consult a variety of resources, especially professional colleagues, to find applicable concept development activities.

Explaining the "why" will often address different learning modalities. For example, having the students cut out the angles of a triangle to see how the relationship came about would address kinesthetic learners and will help them remember the theorem. Concept development is important because my belief is it is not a matter of if students will forget, but rather, when they will forget. Developing the concepts will allow students an opportunity to reconstruct ideas. For English language learners, conceptual development of ideas comes under the heading of "building background". It's a necessary step in language acquisition and understanding for all students.

Procedural knowledge, while important, is not enough. Understanding is important. The following questions came from high school exit exams. Each question measures students' abilities to find the measures of central tendency – often referred to as averages. If student understanding of the mean was only memorized information, students would have difficulty answering some of the following questions.

Standard: Finding Measures of Central Tendency

If you look at the following problems, they all involve finding the mean. The first question tests students' procedural knowledge. The p-score on that particular question was approximately 0.8. That means about 80% of the students taking the test got the correct answer. They knew procedurally how to find the mean.

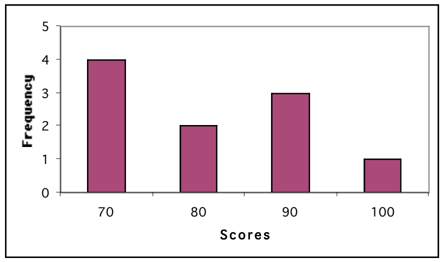
1. Find the mean of the following data: 78, 74, 81, 83, and 82.

2. In Ted's class of forty students, the average on the math exam was 80. Andrew's class of thirty students had an average of 90. What was the mean of the two classes combined?

3. Ted's bowling scores last week were 85, 89, and 101. What score would he have to make on his next game to have a mean of 105?

4. One of your students was absent on the day of the test. The class average for 24 students was 75%. After the other student took the test, the mean increased to 76%, what did the last student make on the test?

5. Use the following graph to find the mean.



On the second question, one of the distracters was 85%. Students who relied only on procedural knowledge, added 80 to 90, divided by two and got the incorrect answer of 85%. The p-score for that question on the exit exam was 0.48. This was a dramatic decrease from finding the mean in the first question. The question was not very difficult, but apparently students need greater understanding to answer the question correctly. Students not understanding they had to re-distribute the total number of points among the seventy students got that question wrong.

Looking at question three, how many times have you been asked what grade a student has to make on their next test to earn a particular letter grade? As you can see from questions two and three, if students do not understand the concept, any variation in problem will cause them difficulty.

Question four caused a tremendous amount of difficulty for students. Many incorrectly think the missing student would have to earn a 77% on the test rather than 100%. Again, not understanding the importance of re-distributing the total number of points led these students to the wrong conclusion. Procedural knowledge is important – but so is understanding. The p-score on this question dropped to .22. What we can quickly see is, without understanding, the slightest variation in a problem will cause students a great deal of difficulty.

Students often interrupt their teachers as they are trying to develop a concept. Why do they interrupt, what's the big deal? The answer is too easy. Typically, they want to know tonight's homework assignment. Students know what their teachers value, even more so than their teachers. Students know if you are not going to ask them a conceptual understanding question on their homework, quiz, or on the test, it must not be important.

Since students value what teachers' grade, concept development and linkage should also be tested. Students should be asked to write a brief explanation of a particular concept as part of the homework assignment, and then be asked an open-ended question on a test where they must explain the origin of a rule or algorithm. If students are tested on the "why" of mathematics, they will be less likely to "tune out" teachers during concept development. Balanced delivery of instruction requires balanced assessment.

Linkage

As teachers teach mathematics, they should remain cognizant of the fact that the concepts and skills they teach will be used later as building blocks to introduce more abstract concepts. Middle-school teachers use concepts and algorithms taught in elementary school, and high-school teachers continue to build on student knowledge gained in middle school. This process is referred to as "linkage" (connections), the introduction of new material through the use of skills and concepts that have previously been taught. The idea of linkage can also be applied to smaller units of time, including material learned yesterday, last week or last month.

Therefore, as lessons are presented, teachers should link the new material to previously learned concepts or outside experiences. By introducing concepts through the utilization of linkages, teachers enable students to place new ideas into a context of past learning. Students are introduced to new or more abstract concepts using familiar language, thereby not being threatened. Teachers, on the other hand, have an opportunity to review and reinforce previously learned topics, topics and skills they often identify as deficiencies and reasons why they are not successful teaching their assigned curriculum. Teachers can then compare and contrast that information, and students see the idea in a different context. Research suggests all the aforementioned leads to increased student achievement. Simply put, students are then more likely to understand and therefore absorb new material when linkage is being used.

The importance of linking concepts and skills to previously learned material and outside experiences can not be overstated. Many of our best students probably don't know the equation of a circle, the distance formula, Pythagorean Theorem, and $\cos^2 x + \sin^2 x = 1$ are all the same formula, just written differently because they are being used in different contexts. By not introducing these concepts through linking, teachers lose valuable instructional time. They also lose opportunities to introduce the new material using language students are most familiar and comfortable, they lose opportunities to address deficiencies by reviewing and reinforcing previously taught material, students are denied opportunities to increase their understanding by comparing and contrasting those ideas, as well as not seeing the math used in different contexts.

Another example, rather than just having students "flip & multiply" when dividing fractions, the division algorithm might be developed through repeated subtraction – just as was done in fourth grade with division of whole numbers. Solving equations should be connected to the Order of Operations. The standard multiplication algorithm that is taught in fourth grade is the same algorithm that is used in algebra to multiply polynomials. Invariably, student memory, over time, will diminish. An understanding of where theorems, formulas and algorithms (short cuts) originated will enable students to reconstruct concepts and solve problems.

Where possible, linkages should also be made between concepts within the course as well as to student experiences in "real life." Buying candy at a store can be linked to such mathematical concepts as ratios, proportions, slope, ordered pairs, graphing, and functions. Students quickly see that if one candy bar costs fifty cents, then two will cost a dollar. The connection is readily translated to the math they learn in the classroom. As a proportion, 1 candy bar is to $.50 **as** 2 candy bars is to $1.00; or written as ordered pairs, (1, .50), (2, 1.00). Linking makes math more relevant and it is very important for students trying to learn the language or students of poverty – by reviewing and reinforcing previously learned concepts and skills in a non-threatening manner. Because I believe introducing new concepts and skills through linking is so important to increasing student achievement, especially for students coming from poverty, I have provided a number of examples that can immediately employed in the classroom.

Example

In middle school, we teach students to add whole numbers by utilizing place value; i.e. tenths to tenths, hundredths to hundredths, etc. In algebra, we teach students addition of polynomials by telling them to combine like terms; i.e., to add constants to constants, x's to x's, x^2's to x^2's, etc. These are not new concepts.

In first, second and third grade, we teach students to add whole numbers by lining up the numerals in columns so that the digits with the same place value are combined; i.e., digits with a place value of one, digits with a place value of ten, etc.

Example

In algebra, we teach students multiplication of binomials by the **FOIL** (**F**irst, **O**uter, **I**nner, **L**ast.) method. In third and fourth grades, students are taught two digit by two digit multiplication by a standard algorithm which is essentially turns into the FOIL method. Therefore, multiplication of binomials should be introduced by using the standard algorithm before the shortcut is introduced.

To multiply 21×32:	To multiply $(x+4)(x+3)$:
2 1	$x + 4$
× 3 2	$x + 3$
4 2	$3x + 12$
6 3	$x^2 + 4x$
6 7 2	$x^2 + 7x + 12$

same algorithm

The next logical step would be to introduce the FOIL Method, linking it to the vertical method of multiplying two polynomials illustrated above.

Besides linking new material to previously learned material, it is helpful to link it to outside experiences as well.

The idea of slope is used quite often in our lives; however outside of school it goes by different names. People involved in home construction might talk about the pitch of a roof. If you were riding in your car, you might have seen a sign on the road indicating a grade of 6% up or down a hill. Both of those cases refer to what we call slope in mathematics.

Kids use slope on a regular basis without realizing it. Let's look at an example, a student buys a cold drink for $0.50, if two cold drinks were purchased, the student would have to pay $1.00.

I could describe that mathematically by using ordered pairs; (1, $0.50), (2, $1.00), (3, $1.50), and so on. The first element in the ordered pair represents the number of cold drinks, the second number represents the cost of those drinks. Easy enough, don't you think?

Now if I asked the student, how much more was charged for each additional cold drink, hopefully the student would answer $0.50. So the difference in cost from one cold drink to adding another is $0.50. The cost would change by $0.50 for each additional cold drink. The change in price for each additional cold drink is $0.50. Another way to say that is the *rate of change* is $.50. In math, we call the rate of change—slope.

In math, the rate of change is called the slope and is often described by the ratio $\frac{rise}{run}$.

The rise represents the change (difference) in the vertical values (the y's), the run represents the change in the horizontal values, (the x's). Mathematically, we write

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Let's look at any two of those ordered pairs from buying cold drinks, (1,$0.50) and (3,$1.50) and find the slope. Substituting in the formula, we have:

$$m = \frac{1.50 - 0.50}{3 - 1} \rightarrow \frac{1.00}{2}$$

Simplifying, we find the slope is $0.50. The rate of change per drink is $0.50

I can not overstate the importance of using linkages to introduce new or more abstract concepts and skills. Linking allows teachers to introduce new ideas in familiar language in a non-threatening manner, to review and reinforce that knowledge, to compare and contrast, and to see mathematics used in different contexts.

Problem Solving

Mathematics is more than just memorizing rules and procedures, it is a discipline. Students must be taught and encouraged to think, to imagine, and to be creative in their approaches to solving problems. The National Council of Teachers of Mathematics materials state, "problem solving is not a mystery." It is also not limited to solving traditional story problems or word problems. It is a way of thinking that can be learned. Teachers need to encourage their students to approach learning/problem-solving activities with an open mind and to realize that this kind of thinking takes time and effort to achieve. Students' answers, whether correct or not, should be viewed as opportunities to explore thinking strategies. Open-ended questions that evoke thoughtful responses and require more than one word answers should be presented. Students should also be encouraged to utilize a variety of problem-solving methods. This process requires that teachers provide students with sufficient thought time. While problem solving is difficult to teach, and requires commitment and patience on the part of both teacher and learner, it is an essential experience.

Requiring teachers to have a balance in their delivery of instruction and assessment should come under the heading of *teacher expectancies*. In math, we'd also expect all teachers to employ recognized problem solving/learning strategies, such as:

Go back to the definition
Look for a pattern
Make a table or list
Draw a picture
Guess and check
Examine a simpler case
Examine a related problem
Identify a sub-goal
Write an equation
Work backward

These problem-solving strategies also help students understand mathematical concepts being taught. Its no mistake that *Go back to the definition* is listed first. Too often it is not listed as a problem solving/learning strategy. Without a good definition, without knowledge of vocabulary, students are bound to encounter difficulty in any subject. I don't know how teachers explain to students not to add the denominators when adding fractions without a good definition of a fraction.

Successful teachers are cognizant of the problem solving/learning strategies in their daily instruction. When students encounter difficulty in understanding a concept or mastering a skill, good teachers encourage students to go back to the definition, draw pictures, look for a pattern, examine a simpler or related problem. Students who have been taught and encouraged to use problem solving/learning strategies don't sit idly when they encounter difficulty, they are better prepared to address the problem at hand.

Some may wonder why some students are successful in one subject, but not in another. Let's look at algebra and geometry as an example. We often see students successful in an algebra class have great difficulty in geometry, why is that?

Algebra teachers tend to use *look for a pattern, make a table, examine a simpler case, and write an equation* as strategies that form the basis for most of their instruction. As a result, students grow comfortable learning math with these strategies. Unfortunately, too many students still try to learn algebra by rote memorization. That results in any variation of a problem causing great difficulty and frustration for students. That can be clearly seen on exit exams in mathematics.

Teachers of geometry tend to use: *go back to the definition, draw a picture, examine a related problem, identify a sub goal, and work backward* as their primary strategies. Students who learned algebra by memorizing often run into difficulty in geometry. Students and teachers that use those same strategies to teach or learn geometry that were successfully used in algebra often run into difficulty too – resulting in higher fail rates.

In geometry, students are typically required to use higher order thinking skills that are not being used in a typical algebra class. In my opinion, too many geometry students do not have a good visualization of the definitions, postulates, and theorems that are being introduced. As we expect students to learn these, they should also be able to draw a picture that reflects the information being taught as well as to measure the drawings and diagrams to explore, discover, and eventually commit to memory important theorems and procedures.

Students should be required to write their definitions, postulates and theorems on their homework, quizzes or tests, they should also be required to draw a corresponding picture. If they can do that, they will be more successful learning geometry.

Geometry is filled with new terminology and notation, teachers need to be mindful that student success in any subject is dependent upon them learning the language. All too often in math, the difficulties experienced by students has more to do with a lack of understanding of vocabulary and notation than the math concept being taught. Classroom teachers should take the time to ensure students are learning and using that vocabulary and notation and they should also be testing students on it.

And while some problem solving / learning strategies are used more routinely in one subject area vs. another, the fact is all problem solving / learning strategies should be used at appropriate times in all of mathematics.

And finally, let's not forget about "linking" geometry to algebra –referred to as coordinate geometry. Linking allows teachers to introduce new concepts in familiar language, to review and reinforce, to compare and contrast, and to teach in a different context – all of which the research suggests leads to increased student achievement. While coordinate geometry is typically a chapter by itself toward the end of a book in geometry, these links can and should be made all year.

Vocabulary and Notation

I encourage teachers to introduce new concepts and skills by linking those concepts to previously learned knowledge using language students are more familiar and comfortable. Having said that, a certain amount of thoroughness, precision, and formality is required in mathematics and specifically in terms of notation and vocabulary; these are the building blocks of concepts and therefore their correct use is vital. So while initially introducing new concepts in familiar language should be encouraged, by the end of the lesson, more formal language should be used to describe the mathematics.

Mathematics notation is a system of shorthand for the language of mathematics. This notation utilizes symbols to denote quantities, relationships, and operations and has evolved over time to enable us to show the manipulation of data and ideas. Notation enables us to designate mathematical concepts and processes with precision and clarity.

All too often student difficulty in mathematics is a direct result of a lack of understanding of the vocabulary and notation. For example, when algebra students are asked to find the degree of a monomial, many are unable to do so. To find the degree of a monomial, you merely add the exponents. It is not that the mathematical concept is difficult, but rather students do not understand what the question is asking. Therefore, the precise use of vocabulary and definition is essential.

As part of developing a new concept, teachers should take great care in developing good definitions. For example, when first learning to add fractions, many students mistakenly add the numerators together and the denominators together ($\frac{1}{3} + \frac{1}{3} = \frac{2}{6}$). The teacher all too often responds by telling the students to add only the numerators and gives no further explanation. (In other words, students are being asked to memorize a procedure without understanding.) In order to develop understanding properly, a fraction needs to be defined as part of a whole – composed of a numerator and denominator. The denominator tells you how many equal parts make one whole unit; the numerator tells you how many equal pieces you have. When students then try to add denominators, the teacher can then have them analyze their work based upon the defined terms and explain that if they added the denominators they would not have one whole unit.

Knowing and understanding vocabulary and notation require teacher modeling, use and reading. There is, and should be, an expectation that students can understand, read, and write mathematics. Students in elementary school should be able to read 16.023 as sixteen and twenty-three thousandths – not sixteen point zero, two three. Similarly, secondary school students should be able to read ${}_nP_r$ as a permutation of n things being taken r at a time – not as “npr”.

Clearly, this falls under the category of language acquisition. Students not acquiring the vocabulary and notation will have great difficulty on high stakes tests. And teachers need to remember, this is not just a problem for non-English speakers, it’s a problem for all students.

Reading and Writing

A common complaint among secondary teachers is their students can’t read. The problem is most of today’s high stakes testing is composed of quite a bit of reading – problem solving. Secondary subject specialist teachers are often in denial with respect to their role in teaching students to read. The vast majority of students have been taught to read by using fictional texts. The way students read fiction and the way technical material is read is different. Math students have to read differently to understand their math.

Who’s going to tell this to the students if it’s not their math teachers? It appears the way most secondary math teachers are handling the reading problem is by ignoring it. In fact, a common strategy seems to stop giving reading assignments because the kids don’t understand them. When you say that out loud, it really sounds stupid. Not assigning reading will surely exacerbate the problem and continue the downward spiral.

Understanding a problem is surely part of mathematics, you can’t understand problems without having acquired vocabulary and notation and that can’t be done without reading. Math teachers have the responsibility to help their students read their texts. Reading should be part of a math student’s homework assignment. As teachers would prepare their students to complete the math problems they will encounter on an assignment, they should also prepare students for the associated reading assignment. Math teachers should, at the very least, introduce vocabulary words, preview the reading assignment, and relate that reading to previous knowledge. Before teachers go over the homework assignment the next day, teachers should have their students retell what they have read checking for understanding, then the teacher can correct and summarize the reading. Students should be told that when they read their math text, they should do so with a pencil and paper at hand so they can write down important information or work out problems they are having difficulty following.

I marvel when I realize how much information is not transferred formally to students. For instance, many teachers in elementary schools teaching subtraction don’t use words like minuend, subtrahend, and difference to describe numbers in that operation. Not only don’t they use that vocabulary, they don’t explicitly teach their students words or phrases that would generally indicate a subtraction problem. Words such as *difference, left, more*, words ending in "*er*", words in the comparative form suggest subtraction. Without repeated exposure to vocabulary and notation, students will not acquire the language of mathematics or any other subject and will experience difficulty in class and on high stakes tests.

To help students clarify their knowledge, teachers should have them write down what they have learned and what they understand in their own words after closing the day's lesson. Toward the end of a unit, teachers might ask students to write definitions, procedures, linkages, applications, compare and contrast the problems they are working, that will help students differentiate between problems that look alike but are handled differently and will help their clarify their own thinking as well as increase their performance on their tests. Teachers might also ask students to write about what they are understanding or not understanding, to summarize or explain. The writing process forces the students to reflect, think and to organize their thoughts.

Writing keeps students on task and reinforces concepts, vocabulary, and notation that teachers say they value. Rather than have students interrupt their instruction, teachers that regularly asked students to summarize the concept being taught would have students who understand that math is more than just memorization of important facts and procedures. Reading, writing, learning notation is extremely important to ELL students as well as students coming from poverty, language acquisition is a necessary component that leads to increased student achievement. That acquisition will not occur without repeated exposure; teachers orally using the language, students orally using the language, and having them read it and write it.

Basic Facts and Algorithms

Mastery of basic facts is an essential part of learning mathematics. When students encounter mathematics concepts they need instant recall of basic facts. Stopping to remember these facts interrupts the flow of thought, which negatively impacts learning.

What constitutes "basic knowledge" depends upon the grade level. Basic facts in elementary school might be arithmetic facts. In middle school, they might be expanded to include the conversions between fractions, decimals, and percents, or the algorithm for adding fractions. In high school, basic facts may also include the Quadratic Formula, the Pythagorean Theorem, knowing what the graph of a 2^{nd} degree polynomial equation looks like or algorithms for solving linear equations.

Since student deficiencies are evident at all levels, teachers should regularly revisit basic facts. Many higher-level thinking processes required for success in high school mathematics courses demand immediate recall of basic facts. The demands of teaching dense curricula and addressing student deficiencies may, at times, overwhelm the teacher. However, if carefully analyzed and incorporated into lesson plans, deficiencies can be addressed successfully.

The most common complaint I hear from teachers is they can't teach their curriculum because their students don't know their basic math facts. The sixth grade teacher will then spend the first three or four weeks of the school year reviewing/re-teaching the arithmetic facts. The students then go on to seventh grade, that teacher has the same complaint, the kids don't know their math facts and they can't teach until they spend three or four weeks having the students memorize them. You guessed it, the kids go into eighth grade, that teacher spends another three or four weeks complaining about how those elementary teachers did not do their job. If you do the math, you can quickly see that approximately nine weeks of middle school is spent addressing a nonexistent problem. Nonexistent?

My experience has been these teachers, well meaning as they are, review all 100 multiplication facts. If they based their instruction upon data, most teachers would have found out the kids knew their 1's, 2's, 3's, 5's,10's, doubles, and 9's. By incorporating the commutative property, teachers would find there are only about 17 facts in which the kids are experiencing difficulty. Rather than pulling out all the flash cards, students might be better served if their teachers would answer the question; "What do your students know and how do you know they know it?" By answering that question, teachers would work on student deficiencies and not waste time re-teaching facts and procedures the students have already mastered.

If students are struggling with their basic addition and multiplication facts, the following strategies for teaching basic arithmetic facts may be of help. What's important to note in these strategies is the facts are not necessarily taught sequentially. These arithmetic facts, like anything else we teach, should be taught in a manner and order that helps students learn.

Strategies for Learning Addition Facts

+	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

There are 100 basic addition facts with sums to 18. By using commutativity (3 + 7 = 7 + 3), we can reduce the total number of addition facts students must learn to 55.

1. **Adding Zero** - Students quickly understand that the sum of zero and any number is that number. For example, 0 + 6 = 6. (This leaves 45 addition facts to have students memorize.)

2. **Counting on by 1 and 2** - Students often find sums with addends of 1 or 2 by simply "counting on." (This leaves only 28 facts left to learn.)

3. **Sums to 10** - Students can readily identify sums to 10 by repeated experiences with ten. (This leaves 25 facts.)

4. **Doubles** - For whatever reason, students seem able to remember doubles more easily than other combinations of numbers. For example, 7 + 7 = 14. (Now, 19 facts are left.)

5. **Doubles plus 1** – Adding consecutive numbers. Knowing that 7 + 8 is equivalent to 7 + (7 +1) helps students remember these sums. (This leaves 13 facts.)

6. **Doubles plus 2 –** Adding consecutive odd or even numbers. Knowing that 5 + 7 is equivalent to 5 + (5 + 2)

7. **Adding 9's** - Students can quickly see that when they are adding the units digit in the sum is one less than the number they are adding to 9. For example, **7** + 9 = 1**6** since the 6 in the units place is one less than 7. (Only 8 facts are left.)

8. **Adding 10's**

Thinking Strategies for Learning the Subtraction Facts

1. **Fact families:** This strategy is the most commonly used and works when students understand the relationship between addition and subtraction. When students see 6 – 2 and think 2 + ? =6. However, if this strategy is used with the following strategies, students will find greater success in a shorter period of time.

Counting backwards: This method is similar to Counting on used in addition. It isn't quite as easy. Some might think if you can count forward, then you can automatically count backward. This is not true –try saying the alphabet backwards. Students should only be allowed to count back **at most** three.

Zeros: The pattern for subtracting zero is readily recognizable. 5 – 0 = 5

Sames: This method is used when a number is subtracted from itself; this is another generalization that students can quickly identify. 7 – 7 = 0**.**

Recognizing Doubles: Recognizing the fact families associated with adding doubles.

Subtracting tens: This is a pattern that students can pick up on very quickly, seeing that the ones digit remains the same.

Subtracting from ten: Recognizing the fact families for Sums to 10.

Subtracting nines: Again, the pattern that develops for subtracting 9 can be easily identified by most students. They can quickly subtract 9 from a minuend by adding 1 to the ones digit in the minuend. 17 – 9 = 8, 16 – 9 = 7.

Subtracting numbers with consecutive ones digits: This pattern will always result in a difference of 9, 16 – 7 = 9, 13 – 4,= 9 15 – 6 = 9 all have ones digits that are consecutive and the result is always 9.

Subtracting numbers with consecutive even or consecutive odd ones digits: This pattern will always result in a difference of 8. 14 – 6 = 8, 13 – 5 = 8, 12 – 4 = 8.

These strategies clearly help students to subtract quickly. How you teach these strategies, allowing the students see the patterns develop, will make students more comfortable using these "shortcuts" and get them off their fingers.

Having said that, as with many of the concepts and skills in math, students need to compare and contrast problems to make them more recognizable to them. Without being able to identify the proper strategy by examining the problem, memorizing these strategies may become more burdensome and cause greater confusion than just rote memorization.

So while you might teach one strategy at a time, as you add to the number of strategies students can use for a specific numbers, you will need to review previous strategies and, this is important, combine strategies on the same work sheets asking students to only identify the strategy they would use for each problem and why they are using it. Being able to compare and contrast will lead to increased student understanding, comfort, and achievement using these strategies..

For example,
16 – 9, students are subtracting 9, they add one to the units digit.
15 – 7, students are subtracting numbers with consecutive odd units digits, the difference is 8.
17 – 8, students are subtracting numbers with consecutive units digits, the difference is 9

Strategies for Learning Multiplication Facts:

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

There are 100 basic multiplication facts with products to 81. The use of commutativity ($8 \times 6 = 6 \times 8$) reduces this to 55 multiplication facts.

1. **Multiplication by Zero** - Teaching students that multiplication by zero results in a product of zero is straightforward.
   Example: $0 \times 3 = 0$. (This leaves 45 facts to be memorized.)

2. **Multiplication by One** - Again, it is easy to see that multiplication by one gives a product which is the number itself.
   Example: $1 \times 3 = 3$. (This leaves 36 facts to be memorized.)

3. **Multiplication by Two** - Students are generally aware that multiplying by two is the same as adding a number to itself and most students have an easy time remembering these.
   Example: $2 \times 3 = 3 + 3$. (This leaves 28 facts to be memorized.)

4. **Multiplication by Five** - The counting chant of 5, 10, 15, 20, can be used to learn these facts. The numbers on a clock are also useful. (This leaves 21 facts to be memorized.)

5. **Numbers Multiplied by Themselves** - Like doubles in addition, many students seem to learn these facts more readily.
   $3 \times 3 = 9$, $4 \times 4 = 16$, $6 \times 6 = 36$, $7 \times 7 = 49$, $8 \times 8 = 64$, $9 \times 9 = 81$.
   (This leaves 15 facts to be memorized.)

6. **Multiplication by Nine -** When multiplying nine by a single digit, the tens digit of the resulting product will be one less than the digit used in the original multiplication problem ($9 \times 7 = 63$). Additionally, the sum of the two digits of the product is always nine ($6 + 3 = 9$).
   (This leaves 10 facts to be memorized.)

There are also other devices that might benefit students. For example:

Finger Multiplication with One Factor 9 - Another device for finding the multiples of 9 is to number your fingers from 1 to 10 from left to right. The product of nine and another single digit number can then be found by bending down the finger which corresponds to the digit being multiplied. The number of fingers to the left of the bent finger gives the tens digit and the number of fingers to the right of the bent finger gives the ones digit.

Finger Multiplication of Two Single Digit Numbers Larger Than 5 - Using both hands let the pinky fingers represent 6, the ring fingers 7, the middle fingers 8, and the index fingers 9. Next, touch the two fingers together that represent the multiplication you are performing. Count the two fingers that are touching AND the fingers that are located "below" these two fingers. This sum gives the tens digit of the product. The ones digit is found by multiplying the number of fingers "above" the touching fingers on the left hand by the number of fingers "above" the touching fingers on the right hand.

I have watched students trying to learn their basic math facts by just memorization, re-reading the tables or using flashcards. It's a painful experience for most students and just as agonizing to watch. Memorization and using flash cards are necessary, but how teachers teach those facts can have a significant impact on students' success.

For instance, many teachers use the "families" for students to memorize the subtraction facts. That is, if 5 + 4 = 9, then 9 – 4 = 5. That's alright, those relationships should be developed.

However, students should be taught when they subtract "10", the units digit remains the same in the difference. They should also be taught to recognize that when subtracting "9", the units digit is always one more than the units digit in the minuend.

There are other patterns that might be developed that would help students learn their subtraction facts. For instance, when you subtract consecutive numbers, the answer is always 9. For example, 16 – 7 = 9; 14 – 5 = 9. Notice the units digit in the subtrahend is one larger than the units digit in the minuend.

Continuing this hunt for patterns, when you subtract consecutive odd or consecutive even numbers, the difference is always 8. For example, 13 – 5 = 8; 14 – 6 = 8. Helping students recognize those patterns will accelerate their learning of the math facts and it will also relieve some of the pain often felt by these students.

Algorithms

The use of algorithms – which are systematic, step-by-step procedures used in computation or problem solving – helps to address the difficulty students often have sequencing complex mathematics problems. The National Council of Teachers of Mathematics' (NCTM) *Curriculum and Evaluation Standards* recommends that students use algorithms to compute and solve problems. However, algorithms should not stand alone and usually need to be preceded by concept development.

By developing an understanding of a concept, students will be better able to understand the objective involved. They will then be more willing and able to identify patterns that lead to the shortcuts we call rules, algorithms, formulas, theorems, or conjectures. These shortcuts were developed in many instances because someone recognized a pattern that would give them the desired result without having to do as much work. Teachers should stress to students that the shortcuts, by themselves, often do not make sense. It is vital that students understand the concepts and how and why the shortcuts work. With this knowledge, students are better able to make sense of mathematics and are more likely to use suitable strategies and algorithms. For instance, students that only memorized an algorithm with no understanding might compute 4x13x25 by multiplying from left to right. That exercise would take a few minutes and need some space on their paper. Students who know the algorithm and understand the concept would probably multiply 25 by 4 first, then multiply that result by 13 – in their head in seconds getting a product of 1300.

While there are many different ways to compute and therefore many different algorithms, the teaching of standard algorithms is important because this ensures that students have common frames of reference. This is significant as the standard algorithms developed in elementary grades become the foundation upon which more abstract material in middle school is introduced. For instance, there are many ways of multiplying. The ancient Egyptians used an algorithm known as Repeated Doubling; the fourteenth century Italians used the Lattice Method. In the United States, to maintain consistency, we have identified a standard multiplication algorithm. A variation of this algorithm is later used in algebra when multiplying two binomials (FOIL Method). In another example, the standard division algorithm taught in fourth and fifth grades is used again in algebra when students divide polynomials. It is also used in synthetic division and synthetic substitution when solving higher degree equations using the Rational Root Theorem. The division algorithm is important. Teachers expect students coming into their classes will have had certain learning experiences. If students lack practice with the standard multiplication and division algorithms, or other standard U.S. algorithms, they will probably experience unnecessary difficulty in future mathematics classes. Here are a few examples to demonstrate how standard algorithms are continually used in mathematics.

Example:

The standard algorithm for multiplying two, 2-digit numbers is:

1. Multiply the multiplicand by the digit in the one's column of the multiplier.

2. Indent a space to account for place value and multiply the multiplicand by the multiplier's ten's digit.
3. Add those partial products.

Illustration:

4th Grade		Algebra I
$\begin{array}{r} 3\,2 \\ \times\, 2\,1 \\ \hline 3\,2 \\ 6\,4 \\ \hline 6\,7\,2 \end{array}$	← SAME ALGORITHM →	$\begin{array}{r} x + 4 \\ x + 5 \\ \hline 5x + 20 \\ x^2 + 4x \\ \hline x^2 + 9x + 20 \end{array}$

Example:

In its simplest form, the standard algorithm for dividing two whole numbers is:

1. Estimate (divide)
2. Multiply
3. Subtract
4. Bring down
5. Repeat procedure

Illustration:

$$\begin{array}{r} 18 \\ 13\overline{)234} \\ -\underline{13} \\ 104 \\ -\underline{104} \\ 0 \end{array}$$

The following illustrates the division algorithm as applied to an algebra problem.

$$
\begin{array}{r}
x - 7 \\
x + 2 \overline{\big) x^2 - 5x - 14} \\
-\left(x^2 + 2x\right) \\
-7x - 14 \\
\underline{-\left(-7x - 14\right)} \\
0
\end{array}
$$

Example:

A general algorithm for adding or subtracting fractions is:

1. Find a common denominator.
2. Make equivalent fractions.
3. Add/subtract the numerators.
4. Bring down the denominator.
5. Simplify.

Illustration:

6th Grade ⟵ SAME ALGORITHM ⟶ Algebra I

$$
\frac{3}{4} = \frac{15}{20} \qquad + \frac{1}{5} = \frac{4}{20} \qquad \frac{19}{20}
\qquad\qquad
\frac{3}{4x} = \frac{15}{20x} \qquad + \frac{1}{5x} = \frac{4}{20x} \qquad \frac{19}{20x}
$$

At some point, students will be expected to identify patterns that will allow them to compute faster.

When young students are asked to model or explain how to determine the number of two's in eight, they might use the repeated subtraction model shown below:

8 - 2 = 6,   6 - 2 = 4,   4 - 2 = 2,   and   2 - 2 = 0

Students can clearly see they subtracted 2 a total of four times. Therefore, there are four 2's in 8.

This same concept applies to division of fractions. If asked to divide 3/4 by 1/8, students should be able to use the same repeated subtraction model.

$$\frac{3}{4}-\frac{1}{8}=\frac{5}{8}, \qquad \frac{5}{8}-\frac{1}{8}=\frac{4}{8}, \qquad \frac{4}{8}-\frac{1}{8}=\frac{3}{8},$$

$$\frac{3}{8}-\frac{1}{8}=\frac{2}{8}, \qquad \frac{2}{8}-\frac{1}{8}=\frac{1}{8}, \quad \text{and} \quad \frac{1}{8}-\frac{1}{8}=0$$

Students can see they subtracted 1/8 a total of six times. Thus, there are six 1/8's in 3/4.

Given opportunities to look for a pattern, for practice and appropriate guidance, students might notice that rather than performing all those repeated subtractions, if they multiplied by the reciprocal of the divisor they would arrive at the desired result. Not only would it give them the correct answer, they would be able to do the problem faster and more efficiently.

$$\frac{3}{4}\div\frac{1}{8}=\frac{3}{4}\times\frac{8}{1}$$

$$=\frac{24}{4}$$

$$= 6$$

When students discover the patterns derived by playing with numbers through teacher guidance, they can be shown that algorithms are nothing more than a faster way to solve problems by applying those patterns. Mathematics then is no longer something magical or mysterious; it becomes a powerful tool to be used in a variety of situations.

By learning an algorithm, students will have a method with which to solve a variety of problems. Likewise, students that know how to solve a problem should be able to verbalize what they have done, verify and defend their solutions, and communicate results. The memorization and utilization of algorithms allows students to do just that.

Technology

"The thoughtful and creative use of technology can greatly improve both the quality of the curriculum and the quality of children's learning. Integrating calculators and computers into school mathematics programs is critical in meeting the goals of a redefined curriculum." (National Council of Teachers of Mathematics. *Curriculum and*

Evaluation Standards. Reston, Virginia: National Council of Teachers of Mathematics, Inc., 1989.)

However, the NCTM also says, "Calculators do not replace the need to learn basic facts, to compute mentally, or to do reasonable paper-and-pencil computation." Therefore, appropriate use of technology is dependent upon the age of a student and his/her ability to demonstrate knowledge of basic facts. It is further dependent on the objective of the activity. If the goal is skill attainment, then calculator use is not appropriate. If the goal is exploration or verification, then calculator use may be appropriate.

Modern technology can free students from tedious computations and allow them to concentrate on problem solving and other important mathematics content. Students should be using calculators to strengthen and extend understanding of concepts, explore mathematical functions, investigate problem-solving activities, employ real world applications, and verify results. (In Algebra I and above, the use of graphing calculators is imperative.) *However, it is essential that all teachers maintain a balance between paper-and-pencil computation/drill and the use of technology to enhance problem solving and conceptual learning.* This requires teachers to make a conscious decision as to the appropriateness of calculator use during each and every lesson. Calculators should not be allowed as a substitute for thinking. To increase the likelihood that calculators will be used appropriately, teachers may need additional training. Total dependence on technology is inappropriate, but when combined with an understanding of the underlying concepts and proficiency with basic skills, it becomes an invaluable tool.

Example:

One method to find the zeros of a quadratic function is through factoring. Students should easily be able to find the zeros of the function $f(x) = x^2 - 4x + 12$ without a graphing calculator.

$$x^2 - 4x + 12 = 0$$
$$(x-6)(x+2) = 0$$
$$x - 6 = 0 \ \ or \ \ x + 2 = 0$$
$$x = 6 \ \ or \ \ x = -2$$

Students should have learned, however, that the graph of the quadratic function $y = x^2 - 4x + 12$ has *x*-intercepts at $x = 6 \ \ and \ \ x = -2$ indicating the function's zeros.

Example:

A quick method for finding the location maximum/minimum of a parabola is to average the zeros. In the above example, the maximum occurs at

$x = \frac{6+(-2)}{2} = 2$. The y-coordinate of the extremum is $f(2) = 2^2 - 4(2) + 12 = 8$. Again, students should have the ability to do this without the graphing calculator.

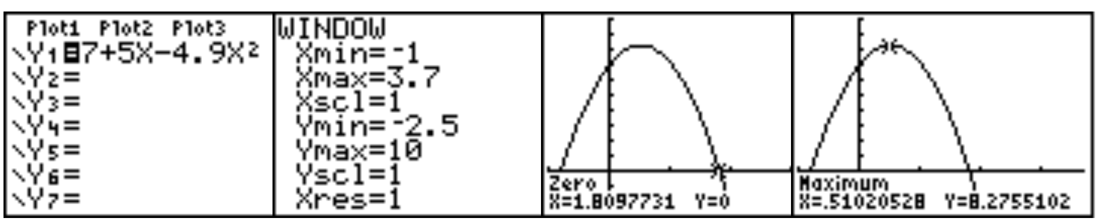
Example:

A graphing calculator is an ideal tool for exploration of the following problem:

A diver stands upon the 7-meter diving platform preparing for a dive. (The 7-meter platform is 7 m above the surface of the water.) The diver jumps vertically upward from the platform with an initial velocity of 5 meters per second. The diver's height above the water can be modeled by the equation $h = 7 + 5t - 4.9t^2$, where t is the elapsed time in seconds since the diver jumped. At what time after the dive does the diver reach the water? At what time does the diver reach a maximum height and how high is it?

The expression $7 + 5t - 4.9t^2$ is not easily factored, nor is completing the square trivial. The quadratic formula leads to solutions, but a graphical exploration yields more information and eliminates tedious calculations.

Students should determine that the diver reaches maximum height of 8.28 m at $t = .51$ s, and enters the water at 1.81 s. Further exploration may prompt discussion on how high the diver jumped above the platform and the meaning of the quadratic's other zero at $t = -0.79$ s.



The Y= Editor Window Screen Finding the Root Finding the Max

To think critically, to problem solve, students need understanding and a body of information to draw from. To do those, they need vocabulary and be able to use technology. Balance in the delivery of instruction and in assessment ensures our students are getting a full, rich curriculum. Balance also ensures students are being taught what the adults say they value in mathematics.

Not only is what we teach important, but how we teach it effects student learning. We need to look at proven instructional strategies that help students learn.

To teach math successfully, students must experience a full, rich, and more in-depth curriculum - a balanced delivery of instruction. That balanced delivery must be accompanied by assessments that reflect that balance.

Chapter 2

Components of an Effective Lesson

Too often in life decisions are made by default – by not making a decision. Many students actually make the decision to not go to college in their freshman year of high school. The decision was made by their class selection – not a conscious decision not to go. The same can be said of some classroom teachers, they do things without consciously making the decision, things that with a little extra thought, they probably would have modified or done differently. Now that we have discussed balance in the delivery of instruction, let's take at how that instruction should be delivered.

The Components of an Effective Lesson is a blueprint for classroom teachers to follow which is easily monitored by school administrators. If fully implemented, the components will result in increased student achievement. My experience is that most teachers already loosely employ many of these components in their current lessons. These components are not controversial, but they are not typically being fully implemented. The following is an overview of the Components of an Effective Lesson.

INTRODUCTION

DAILY REVIEW

DAILY OBJECTIVE

CONCEPT AND SKILL DEVELOPMENT AND APPLICATION

GUIDED / INDEPENDENT / GROUP PRACTICE

HOMEWORK ASSIGNMENTS

CLOSURE

LONG-TERM MEMORY REVIEW

Lesson Objective, Setting the Stage, and Closure

As part of setting the stage for learning, students should be made aware of each day's objectives and how it might be used in everyday activities. Thus, teachers should explain exactly what students are expected to learn, demonstrate the steps needed to accomplish a particular academic task (or let students explore), and then restate what has been taught at the end of the lesson. For additional emphasis, objectives need to be provided both orally and visually. The day's objective should be written on the board and left there for the entire lesson. As an example, an objective might be that all students will be able to find the circumference of a circle. The teacher might set the stage by talking about how tire sizes will affect odometer and speedometer reading in a car. At the close of the lesson, the daily objective should be summarized by the teacher or by the students. Defining daily objectives also helps teachers maintain focus for both themselves and their students. At the close of a lesson, teachers should ask their students to summarize in writing what they learned, how it related to previous knowledge, and how it might be used.

Two Reviews (Oral/Written)

In most cases, teachers are making a good-faith effort to teach the mathematics curriculum. However, there are factors that can hinder their efforts. Virtually all teachers report difficulties stemming from student absenteeism and/or tardies. Also, there seems to be a pattern of students entering middle school and high school with deficiencies in basic skills. To help correct these deficiencies, daily reviews should be employed. Daily review should address both basic skills and recently covered material, including important formulas, facts, algorithms, definitions, and strategies. Daily reviews can also provide an introduction to an upcoming lesson. Reviews can be done as written or oral work, and can be done at the beginning or end of a class or as a transitional activity any time during the period. These reviews are important because they provide an opportunity to reinforce information that all students should know at the completion of the school year. A suggestion to help students remember might be to review current material, such as homework, at the beginning of the class and review previously taught material during the last five or six minutes of class.

Teachers should also teach their students to review using different strategies such as mnemonics, linking, developing relationships, learning in context, and utilizing auditory and visual cues. Knowing "how" to remember is important if we are going to help students learn. Teaching students to recognize that they already use memory skills and helping them to transfer these existing skills will aid them in their efforts to learn. While there is more to learning than just memorization, memorization is an important component of learning.

A second review should be employed to address long-term knowledge. Address deficiencies, mastery, and prepare students for high stakes tests. The reviews should also be based upon student performance – not a whim. Can you imagine sending your son or daughter to a tutor who you are paying, only to find out the tutor is working on material your child knows and understands. My guess is you would not feel that was appropriate. You'd expect the tutor to work in areas where additional assistance is needed. No less should be expected of a classroom teacher, assistance should be provided in areas of deficiency. The long-term review will often address concepts and skills that were learned previously and not part of the current year's curriculum.

High stakes tests require students to maintain knowledge over time, without these long term reviews, there is no doubt students will forget information over time. Forgetting in school is often translated to either not being taught or not having been learned. In either case, it's not only a problem for the students', it's a public relations nightmare for the schools. Schools and math departments must have a plan to address the long-term knowledge of their students, allowing students to forget information over time is not a viable option.

If the second review (long term review) is employed during the last five to seven minutes of a class, students will remain on task till the end of the allocated time. That will maximize time on task as well as having the additional benefit of cutting down on potential discipline problems. Implementing a second review period also has the added benefit of providing teachers opportunities to address mastery, deficiencies, and prepare students for college entrance examinations such as the ACT and SAT.

Presentation Techniques

Nothing ruins a good lesson like a bad example. Teachers must take great care in choosing examples. Teachers need to be careful and pick simple, straightforward examples that clarify what they are teaching that don't bog kids down in arithmetic. Too many teachers think of examples as they are teaching, without much forethought, and wind up picking a variation of the concept or skill which results in confusion for the students. Before variations are discussed, it helps student understanding to first understand the big idea being discussed. Building student confidence and building *success on success* goes a long way to increasing student achievement. Introducing variations of the problems before student understanding is complete often distracts them from learning the objective of the day. Use simple straight forward examples that clarify what is being taught is referred to as "comprehensible input" in the world of teachers working with students whose second language is English

Some teachers have difficulty keeping students focused on the lesson at hand. By "using thorough instructional planning and preparation" and "smooth, rapid transitions between activities," teachers can prevent a great deal of student inattention. Just as important is the recognition of different learning styles. Most secondary school teachers present their lessons visually and verbally while students sit in rows and work independently. This is in sharp contrast to elementary classrooms where students are customarily seated in groups and the majority of instruction is verbal and tactile. Using a variety of teaching techniques should help develop mathematics concepts in a manner that focuses students' attention, increases their level of engagement, and decreases off-task behavior. When teachers use a combination of verbal and visual instruction, hands-on activities, demonstrations, guided practice, and both independent and group work (cooperative learning), they meet the needs of most students.

Visual Component

When learning new material, many students have a need to see it, or at least visualize it, before it becomes meaningful. This implies that teachers need to prepare lessons and materials well before actual instruction. This might include creating models, charts, slides, graphs, videos, manipulatives, overhead transparencies, and handouts.

Many times the visual component of instruction coincides with the development or presentation of the lesson itself, that is, the teacher gives notes on the board or overhead while they verbally explain the material. This is a powerful instructional technique that many teachers use effectively. When teachers shortcut this process by only "talking through" multi-step problems or having students mentally "visualize" complex situations, students may become confused and disinterested. Many students are primarily visual learners and teachers must be concerned about and creative in meeting their needs.

Research suggests keeping material on the board while you are teaching so students can refer back as needed. If teachers are using an overhead projector, this becomes almost impossible for students to do without interrupting the flow of the instruction. More thought needs to be given in using available technology. I recommend limiting the use of an overhead projector when teaching mathematics – it really limits the ability of students to see patterns develop.

Auditory Component

The auditory component of student learning is important and usually not neglected. While it is important for students to visualize what they are learning, additionally they often must hear an explanation. At times, the auditory learner benefits from group work where he can hear the material being verbalized. New terms and material frequently require repeated explanations. This may become cumbersome if concepts are only presented visually. Finally, if teachers give auditory clues to learning, many students will better remember the material

Oral Component (Oral Recitation/Oral Drill)

Oral recitation is the practice of having the entire class recite important facts, identifications, definitions, formulas, algorithms, theorems, and rules during their initial presentation and later when these topics are revisited. To ensure full participation, individuals are also called upon. The number of times an item needs to be repeated depends upon the difficulty of the material and the ability of the class.

In real life, repetition is used frequently to aid memory or to make a point. For example, adults often use it to remember a license plate number or a grocery list. Advertisers use repetition in presenting their message to the public. (How many times have you found yourself repeating a phrase from a commercial or humming a jingle?)

In every course, there are certain items that all students should know at the completion of the school year, information upon which understanding and critical thought can be based. Furthermore, the more sophisticated mental operations of analysis, synthesis, and evaluation are impossible without rapid and accurate recall of bodies of specific information. When these items are first introduced, oral recitation should be utilized to help students memorize the information. For example, after the initial development of the Quadratic Formula, an algebra teacher could provide it visually and then orally read it several times so the students know how to correctly say it. Following this, the class as a whole could recite it several more times. Finally, individual students could be called upon to recite the formula without the use of a visual aide. This process places the information in students' short-term memory with the expectation that it will then, through repeated and extended exposure, be placed in long-term memory. As was stated earlier, oral recitation may need to be used again when important topics are revisited.

Often, teachers report that students have performed poorly on assignments or tests because they did not study. While not a substitute for out-of-class studying, in-class oral drill and recitation provides an opportunity for important repetition, a tool in improving students' achievement. When used correctly, they compel students to become active participants in the lesson and teaches them one method for memorizing new information. These procedures have the additional benefit of meeting the needs of auditory and English language learners.

The use of oral recitation should not lead teachers into the "gotcha" practice. While oral repetition will help most students imbed this information into short-term memory, other students learn differently. Incorporating the idea of building success on success might suggest you don't want to call on students that you think might not have it in short term memory. I used oral recitation from a positive perspective. That is, after the class orally recited for approximately two minutes, I called on students that I thought could give the information back to me. That also created the impression with students that other members of the class were getting the information. As opposed to "nobody is getting it.".

Tactile Component

A National Council of Teacher of Mathematics (NCTM) motto states, "Math is not a spectator sport." The tactile learner learns by doing; he needs direct involvement in the mathematics process. Teachers should provide tactile learners the opportunity to explore and understand mathematics through the use of manipulatives, hands-on activities, labs, group work, projects, and paper-and-pencil problem solving. Additionally technology, including calculators, calculator- based labs, and computers should be used to aid the tactile learner. When using these instructional aids and procedures, teachers may need to do the activity as a learner prior to its presentation in order to fully understand the mathematics involved and anticipate students' difficulties.

The integration of visual, auditory, oral, and tactile styles of teaching and learning ensures students of a better opportunity to learn and feel more confident and competent in their abilities to do and understand mathematics.

Note-Taking

When asked, memory researchers reported the number one "memory aid" which they themselves use is "write it down." Teachers should require students to take notes in all mathematics classes. Notebooks keep students engaged in learning, help them complete their daily homework assignments, enhance their study, and act as a foundation from which to prepare for tests. Also, since students are not allowed to keep their textbooks, the student notebook is usually the only mechanism available for review in later years.

Note taking is a process used by students to record important information that they are trying to understand and need to remember. Because of the importance of a student notebook, teachers need to be prescriptive in how notes are taken and accommodating in their instruction so students can take notes. Notes should typically include a title, the date they were taken, objectives, definitions, identifications, pattern or concept development that leads to some conjecture, a formalized rule or algorithm, and an number of example problems used in guided practice. Teachers should also encourage students to write an explanation of what led to the procedure being used to manipulate or solve problems. Explanations are especially important when a problem-solving method might be construed as a “trick” and whose rationale would not be immediately obvious to the student when reviewed at some future date.

Finally, while note taking is a student responsibility, teachers need to hold students accountable for taking notes. This need not be complicated or time consuming, but it must be done frequently and consistently to further encourage students to take notes.

It’s my opinion that notes are a very important component in increasing student achievement. The notes will help the students complete their homework assignments and should be the primary vehicle used by students to prepare for tests.

Teachers should be very directive, telling the students not only what to write down, but where to write it in their notebook. A student’s notebook sitting in the first row, first seat should be almost identical to the student sitting in the fifth row, fifth seat.

As administrators are monitoring instruction, they should examine student notebooks. If the notes being taken by students will not help them complete their daily homework assignment or prepare for a unit test, that is a subject that should be discussed with the teacher.

Practice (Guided/Independent)

When someone takes up a new sport, they do not expect to be proficient immediately. One expects to practice a new activity to get better at it; long, hard extensive practice is almost always necessary to become proficient. Learning mathematics skills can be equated to learning physical skills. Practice, with frequent reinforcement and evaluation, is essential in order to master abstract concepts. In order to avoid short-changing their students, teachers need to provide practice – both guided and independent.

As part of developing mathematics concepts, teachers need to give students opportunities to practice new skills with immediate feedback. Initially teachers should include several examples as part of the explanation while giving notes.

Guided and independent practice may be more than paper-and -pencil work. It may include labs, projects, and the use of technology.

Guided Practice

Before students are sent home with homework (independent practice), guided practice should be extended to ensure that students are proceeding correctly. Several exercises, similar to the homework assignment, are provided for students to work on in class. Teachers should monitor students carefully, looking for points where they become stuck or confused. If many students stumble on or fail to grasp a given idea or step in an algorithm, the teacher should immediately address the problem on a class-wide basis. If only a few students experience difficulty, these problems can be handled on an individual basis. In either case, a review of the guided practice exercise is recommended before students leave class to begin independent practice.

It must be noted that guided practice should not be "starting homework." Students frequently dawdle during or entirely misuse class time allocated to practice if the time is for "homework". Homework, for the most part, is to be done outside class time; guided practice is done in class with immediate feedback. As an incentive, teachers may add an assessment component to guided practice—essentially a participation grade.

Homework

Student achievement rises significantly when teachers regularly assign, and students regularly complete, homework. The additional study that homework provides benefits students at all ability levels. Furthermore, homework gives students experience in following directions, making judgments and comparisons, raising additional questions for study, and developing responsibility and self-discipline. While homework is not the only or most important ingredient for learning, achievement is often diminished in its absence.

Students are more willing to do homework when they perceive it as an extension of instruction, when it is evaluated, and when it counts as part of their grade. In order to maximize the positive benefits of homework, teachers need to give the same care in preparing homework assignments as they give to classroom instruction. Teachers must carefully prepare the assignment, thoroughly explain it, and give timely comments when the work is completed.

Homework should reflect what teachers' value - balance. Besides assigning a set of problems for homework, teachers should also assign reading, require students to copy definitions, identifications, algorithms, and write brief explanations of the day's work as part of their homework assignments. As students become accustomed to seeing these items as part of notes, homework assignments and tests, they will also begin to understand their value. And don't forget to assign reading as part of the assignment.

Using these strategies in the Components of an Effective Lesson will help students learn. So let's look at them and develop more formal structures to support professional development that results in increased student achievement.

Components of an Effective Lesson
Summary

Before presenting a lesson, refer to the specification sheet and assessment blueprint for the unit.

INTRODUCTION

- Set the stage for today's lesson (students will take notes, participate in a group activity, etc.)

DAILY REVIEWS

- Provide review for short-term memory over recently taught material
- When correcting homework: provide immediate and meaningful feedback and hold students accountable
- Keep reviews and homework checks brief

DAILY OBJECTIVE

- State and write before introducing the day's main lesson and have students record this in their notebooks

CONCEPT AND SKILL DEVELOPMENT AND APPLICATION

- Teach the big concepts
- Provide the "why" for rules and algorithms
- Link concepts to previously learned material and/or real-world experiences
- Utilize a variety of techniques: students need to see it, hear it, say it, and do it
- Hold students accountable for taking notes and keeping mathematics notebooks

GUIDED / INDEPENDENT / GROUP PRACTICE

- Can be done at different times throughout the lesson to help students process information
- Students need time to think, analyze, work on problems, discuss their solutions and become problem solvers instead of watching the teacher do all the work
- Can be done as an entire lesson that enhances conceptual understanding and/or application of concepts through inquiry, investigation, discovery, lab or problem-solving activities

HOMEWORK ASSIGNMENTS

- Assignments should consist of what teachers value and include a variety of assessment items, including, definitions, computations, explanations, applications, etc. (see the assessment blueprint for the unit)

CLOSURE

- Have students explain in writing what they have learned and apply it
- Restate what was taught

LONG-TERM MEMORY REVIEW

Address mastery, student deficiencies, high stakes tests, and stress important ideas - not necessarily part of this year's curriculum, but based on student knowledge

Chapter 3
Structures that Support Increased Student Achievement

BACKWARD ASSESSMENT MODEL
Professional Development Through Sharing

Let's first look at a structure that will support ongoing professional development and changes in instructional practices that will result in increased student achievement.

Educational research strongly suggests that professional interaction—at times informal and unstructured—is often far more influential than formally organized professional development, and is more likely to result in changed behavior on the part of the educator.

The *Backward Assessment Model* (*BAM)* changes the way professional development is delivered. Rather than having an outside expert tell teachers what needs to be done, the assessment model uses the expertise of the school's staff. Research suggests professional development should primarily be on-site, on-going, and regularly scheduled. Professional development should be provided by the people who know best, classroom teachers as active participants, and should focus on the discipline teachers' teach, in both content and pedagogy. The *BAM* places the professional development emphasis on *academic standards, assessment,* and *best practices*.

***BAM* is a communication model.** Its strongest attribute is that it provides teachers an opportunity to share their **knowledge, understanding, skills, experiences, resources, and instructional strategies** with each other**.** Experienced teachers generally know where students traditionally experience difficulty and communicate this to less experienced teachers. Likewise, all teachers can communicate knowledge, model successful strategies, and share accommodations and modifications that help students succeed. *BAM* also provides all teachers, experienced and new, opportunities to reexamine and reflect upon their own practices.

There are two basic premises of *BAM*. The first is data from assessments drive instruction and the second is that teachers make a difference, and teachers working together make a greater difference.

To implement the Backward Assessment Model, teachers need to meet together by grade or subject level to discuss what they teach, how they teach it, their student performance, and changes in instructional strategies that will result in increased student achievement. The following is an overview of what those teachers would do using a typical professional development day.

Before instruction on a major unit of study, grade level or subject area teachers should develop a **SPECIFICATION SHEET**. That is, before they begin teaching, each group (3^{rd} grade teachers, algebra teachers, 6^{th} grade science, etc.) should meet together and identify what they expect students to know, recognize, and be able to do and the timelines to accomplish those goals for a specific unit. This piece fits in very nicely to ensure all teachers are familiar with the state's academic standards, school district curriculum guides, or the already established benchmarks.

The second major component in BAM is the **ASSESSMENT BLUEPRINT.** To ensure students are receiving balanced instruction, teachers should determine how they are going to assess their students. The assessment should include the *Teacher Expectancies* for *balance* and some type of agreement on the types of questions that promote *consistency, fairness and portability* in the grading system. Portability means that a grade of B earned in one class would transfer and be equivalent to a grade of B in another teacher's classroom. The blueprint does not identify specific questions, but rather the approximate number of questions and type that promote a balanced assessment. In math, for instance, 6^{th} grade math teachers might agree to have approximately 20 questions on a test, then determine how many might be computation, vocabulary/identification, concept/linkage, word problems, modeling, etc.

It is important to note that while grade level or same subject teachers agree on the specifications and the assessment blueprint, it is not necessary for all teachers to give the same assessment or have identical benchmark dates. While it is not necessary, I would recommend that there some common assessments.

If one teacher determined it would take six weeks to cover a fraction unit and another indicated he only needed five weeks, that's okay. But, if one teacher indicated he only needed two weeks and another scheduled twelve weeks, then that is a problem that needs to be resolved. Chances are if teachers are that far off on scheduling, then some are not covering the syllabi and others might not be addressing mastery

With respect to the blueprint, one teacher might decide to have four computation problems, while another might choose to have six. That is okay. The goal of the assessment blueprint is to assess students in similar ways and at approximately the same level of difficulty. This approach will help ensure that a grade of "B" earned in one class would transfer to a grade of "B" in the same class or subject, but taught by a different teacher. It would not be fair to students if one teacher had a question like reduce 6/8 while the teacher across the hall had their students reducing 111/213 on the same unit test. It should be noted, the more the tests are identical, the higher the correlation between students' grades between classes.

On the following pages, examples of a Specification Sheet, Assessment Blueprint, and a sample test reflecting the items listed on the specification sheet and the number and types of questions on the assessment blueprint are provided for a sixth grade test on fractions.

Specification Sheet

Fractions

What students should know, recognize and be able to do.

Definitions – fractions, proper, improper, mixed, reciprocal

Identification – numerator and denominator

Equivalent Fractions – converting and simplifying

$+, -, \times$ and $\div$ fractions

Borrowing/Regrouping, whole and mixed numbers

Algorithms for $+, -, \times$ and $\div$

Rules of Divisibility: 2,3,4,5,6,8,9,10

GCF, LCM

Common denominator – methods

Draw models for =, $+, -, \times$ and $\div$

Ordering / comparing

Applications (word problems)

Open-ended concept or linkage

ASSESSMENT BLUEPRINT

Fractions

2 Definitions

1 Identification

2 algorithms / information

1 rules of divisibility

2 concept / linkage problems – open ended

1 draw model

1 ordering

1 simplify

4 computation, $+, -, \times$ and $\div$

1 GCF, LCM

3 word problems (applications)

Cumulative questions

MODEL TEST

Fractions

On questions 1-3, write the definition for each.

1. Fraction

2. Proper fraction

3. Reciprocal

4. In the numeral $\frac{3}{8}$, the 8 is called the ________________.
5. List two methods for finding a common denominator.

6. Write the steps, as discussed in class, for adding fractions.

On question 7-10, evaluate each expression. Simplify your answers.

7. $\frac{5}{7} + \frac{1}{3}$

8. $$\begin{array}{r} 12\frac{1}{2} \\ -7\frac{2}{3} \\ \hline \end{array}$$

9. $5\frac{1}{2} \times \frac{2}{3}$

10. $\frac{3}{4} \div \frac{1}{8}$

11. Find the LCM and GCF of 108 and 72.

12. Simplify the following fractions to lowest terms (simplest form.)

a. $\frac{8}{12}$ b. $\frac{27}{63}$ c. $\frac{111}{207}$

13. Write a five-digit numeral divisible by 2, 3, 4, 5, 6, 8, and 10, but not 9.

14. Order the following fractions from least to greatest. Show your work or explain the strategies that you used.

$$\frac{3}{4}, \frac{7}{10}, \frac{5}{7}$$

15. If the numerator of a fraction remains constant and the denominator increases, what happens to the value of the fraction? (Assume the numerator and denominator are positive.)

16. A student added $\frac{1}{7}+\frac{4}{7}$ with a result of $\frac{5}{14}$. The answer is incorrect. What is his error and how would you explain to him the reason behind the correct answer?

17. Draw a model to show that $\frac{1}{2} = \frac{4}{8}$.

18. Bob owns five-ninths of the stock in the family company. His sister Mary owns half as much stock as Bob. Jill owns the rest of the stock. What **part** of the stock does Jill own?

19. Joel worked $9\frac{1}{2}$ hours one week and 11 hours and 40 minutes the next week. How many more hours did he work the second week than the first?

20. A person has $29\frac{1}{2}$ yards of material available to make uniforms. Each uniform requires $\frac{3}{4}$ yard of material. How many uniforms can be made? How much material will be left over?

With experienced classroom teachers involved in this process, it might take 15 or 20 minutes to create a specification sheet that is based on the school district's curriculum documents and state standards.

It generally takes longer to come to consensus on the assessment blueprint. Teachers need to keep in mind the assessment blueprint is a guide and teachers should work toward building a consensus; **it is not a binding agreement**. Classroom teachers continue to make up their own tests unless they want to create common tests by grade level or subject.

A sample test was provided just as an example. A couple of things should be noted. First, the test did not exactly follow the assessment blueprint. It should also be noted that having teachers identify what they want their students to know, recognize, and be able to do is a straightforward process in which teachers readily agree.

The assessment blueprint gets into testing – teachers tend to argue strenuously over this. What we quickly realize is teachers don't want their students tested on what they don't either stress or teach. For example, teachers who are constructivists may not want students to memorize important facts or procedures. Other teachers might not see the value in students understanding what they are being taught and just want kids to memorize the rules. Believe me, a heated argument will develop in these circumstances and hard feelings will follow unless a balanced curriculum is being followed. To settle such disputes, teachers should refer back to their specification sheet, curriculum documents and state standards to determine what should be tested

One area where teachers should take special precautions is in the writing of test questions. Too often classroom teachers use less formal language on their teacher-made unit tests which may result in students not recognizing the same information that is tested on state or national exams. Care should be taken to write test items so students are exposed to the way in which those questions are phrased or tested on standardized exams. For example, in algebra, a direction on a teacher-made test might be to "solve" an equation. On college entrance exams, the same direction would be "to find the solution set over the real numbers such that…." The way the question is asked might cause some students not to connect what they learned in the classroom to what is being tested on high stakes tests.

To summarize, using the *BAM*, teachers determine the unit of study and how long it should take to teach. Next, teachers create the specification sheet and assessment blueprint based on district curriculum documents and/or state standards. Finally, using the *Teacher Expectancies* to ensure balance, teachers create their own unit tests.

That takes care of the paperwork!

Now on to the most important component of the *Backward Assessment Model*—the sharing of ideas, resources, materials, knowledge, skills and teaching strategies.

As states earlier, educational research suggests that professional interaction—at times informal and unstructured—is often far more influential than organized professional development, and is more likely to result in changed behavior.

After the paperwork is completed, experienced teachers should share their knowledge of where students traditionally experience difficulty on a particular unit. Rather than bemoaning the fact that students have performed poorly on those areas historically, teachers should exchange knowledge, resources, experiences, and successful teaching strategies with each other. Modifying instructional strategies and/or resources can result in greater student understanding and increased student achievement.

Teachers could increase their content knowledge by using this time to share their understanding of conceptual knowledge and application of the knowledge and skills taught in class.

Teachers might also examine areas in which the district has not performed up to expectation on state and national tests and address those areas of concern. Teachers might also study their most recently administered test to determine strengths and weaknesses of their instruction. Once that has been accomplished, decisions might be made on how best to address weaknesses during the current school year and how instructional strategies might be changed in future years.

If specific student populations can be identified as doing poorly, the grade or subject level teachers might want to bring into their meetings ELL, special education, reading or instructional strategists to recommend possible changes in instructional methodologies that would be beneficial to identified students.

A lot of work, time and effort are necessary to effectively use the *Backward Assessment Model*. To assist teachers in working together, a professional development agenda is provided on the next page to guide them through this process. You might notice that the school administrator is first on the agenda to discuss issues they need to communicate to effectively run the school. The rest of the day is devoted to what teachers do, how they do it, student performance, and instructional strategies that will increase student achievement.

Professional Development Day Agenda

I. General meeting – discuss items that site administrators need to address
II. Teachers meet by grade level or subject.
    A. Identify the following and discuss using available data:
        1. The next unit of study
        2. The most difficult unit of study as determined by data and teacher experience
        3. The unit of study causing students the most difficulty as identified by local, state, or national test data
    B. Identify what students should know, recognize, and be able to do on the selected unit (Specification Sheet).
    C. Identify how long it should take to teach the selected unit (Benchmarks).
    D. Determine how and what to assess on the selected unit to help ensure consistency (portability) and fairness between classes of the same grade level or same subject (Test Blueprint).
    E. Using data, identify topics within that selected unit in which students traditionally experience difficulty.
    F. Share with each other successful teaching strategies to overcome those difficulties and/or deficiencies.
    G. Share content knowledge, resources, and expertise to address student success on the identified unit.
    H. Using data, discuss way to involve special education or ELL facilitators if specific student populations are not experiencing the same success as the general population.
    I. Examine the results of the last unit test or other testing data to determine strengths and weaknesses of student's understanding of subject matter.
    J. Identify students not meeting proficiency on standards and a plan to remediate those students.
    K. Identify what instructional practices you will change for next year to correct deficiencies and improve student achievement.

An agenda such as this will focus professional development on teaching and learning. This agenda cannot be discussed in a one or two hour meeting; almost the entire day should be set aside for these discussions.

Secondary teachers are subject specific, so determining what unit(s) they want to discuss might be a little easier than elementary teachers who, before they choose a unit, must select a subject.

Administrators can expect to see classroom teachers gripe as they begin this process; that's natural. In fact, when any administrative group gets together, they spend a few minutes sharing their gripes with anyone who will listen too.

Getting people to change the way they do business is tough. People's first concern is how or if the change will affect them. If it does, then they become concerned about how much additional time will be needed to get the job done. There's always plenty of criticism when change is involved, particularly about how if they don't have adequate time now, then when are they expected to find the time to implement the change.

This is a real concern and it's a real problem that needs to be addressed. But before we get there, teachers need to be reminded to focus on results – increased student achievement.

Yes, change affects them and time is an issue, however if teachers and administrators continue what they are doing, they will continue to get what they have been getting. Chances are that's not acceptable. Part of the answer is to focus on the solution and change what needs to be changed even if it changes the way we have been doing business.

Often, the greatest obstacle to this is the school principal who is unwilling to treat his/her teachers as professionals. These principals want to be in control. Teachers need to be given the opportunity to talk in-depth about their job with colleagues. This means a building principal must give up control on the professional development day. This is very difficult for some. The proposed professional development day agenda, with its required paperwork, will keep classroom teachers on task. *But keep in mind, while the paperwork is important, the most important thing teachers will be doing is sharing their knowledge, resources, and instructional strategies with one another*.

In most schools, there are good teachers and not so good teachers in rooms right next to each other. This professional development agenda will have a tendency to level the playing field with all teachers afforded the opportunity to share their knowledge of teaching with each other.

Most new teachers are brought into the profession using the Pier System. That is, new teachers are given a set a keys, a room assignment, information on where to find books and supplies, a roster, and then sent on to the pier where they are thrown into the water to either sink or swim.

BAM is a Peer System. Can you imagine being a brand new teacher where you get to meet your colleagues and have them share with you what you will be teaching, the order in which you teach it, where students traditionally have difficulty, resources and strategies to overcome those difficulties, what and how they test, how they grade, and they actually give you materials to help you get started and learn your profession? That's *BAM*ing.

While *BAM* is a communications model, it is also a structure that supports professional development focused on increasing student achievement.

BAM and the Components of an Effective Lesson guide teachers' preparation and actual instructional delivery through a typical lesson to ensure efficient and effective use of class time and instructional strategies.

This Components of an Effective Lesson is a guide based upon the instructional strategies we have already discussed. Even though we have discussed them before, I'd like to touch on the two reviews scheduled during the class. More often than not, teachers will say they do not have time for two reviews in a typical class period. The typical instructional day will have one review to go over recently taught material or last night's homework. While that's good, teachers need to remember they live in a world of high-stakes testing. Allowing students to forget translates to students not learning. A second review should be scheduled almost daily to address long-term knowledge, mastery, deficiencies, as well as preparation for high stakes tests. The reviews should be based on student performance. These reviews are particularly helpful to English language learners.

The reviews might also involve concepts and skills not being taught this year. The necessity of these reviews becomes readily apparent when students are required to take promotional or exit exams that might contain material that was taught two or three years before.

Since college entrance exams are reported by state, school district, and school, they determine if a student will gain admittance to a particular college or university, and in some cases used to determine financial aid, some might argue these are important exams. Wouldn't it be reasonable to expect teachers take some time to prepare their students for such high stakes tests?

By doing so, test scores will inevitably go up, which is often perceived by the public as increased student achievement.

As can be seen, there is a great deal expected of classroom teachers. BAM and the components provide a structure that helps the teacher incorporate the teacher expectancies.

Chapter 4

Teacher Expectancies

Teacher expectancies are just the things expected of teachers, they lend themselves to student success as well as address the *common sense* standard. The first expectancy has to do with student-teacher relationships.

Student-Teacher Relationships

One of the greatest concerns expressed by policymakers have to do with closing the achievement gap while increasing student performance. To accomplish this, a number of elements have to be addressed. One that seems to get lost in the fray is the importance of building positive student-teacher relationships.

Many teachers have had the experience of having a student not like them and decide not to work, to flunk, to teach their teacher a lesson. The fact is research suggests that students will work for teachers for no other reason than loyalty. As the professional, educators need to take advantage of that knowledge and talk to their kids. Teachers also need to watch how they talk to them. They need to be positive. Rather than saying things like “If you don’t do your homework, you will fail”, they need to say, “If you do your homework, you will be successful.”

Remember, treat your students the same way you want your own sons and daughters treated by another teacher. So talk to the kids; while you are not their friend, you can be friendly. Talk to them about sports, their social life, the dance, game, or weekend. Form a bond that suggests to the students if they stopped coming to school, someone would miss them – that you care about them.

If you have ever been in a long-term relationship, your partner may have expressed their frustration about you not expressing your feelings in words. They want you to say you love them even though you think it should be just understood. The same is true in your classroom, you need to tell your students you like them, that you want them to be successful. Reading my evaluations while teaching at the university, many of my students would fill out their anonymous evaluations and comment about how much I liked them, how I liked teaching, and how I wanted them to succeed. A colleague once asked me why my students felt that way about me. The answer was simple, I did my best to tell my students at least twice during the semester that I liked them, I liked teaching, and I wanted them to succeed. They reciprocated by liking me! Remember this Law of Reciprocity, people you like generally like you, people you don’t like generally don’t like you either.

If you want your students to be successful, teach them how to learn. Many students think the reason some kids are successful in math is because they are smart. They don't equate studying with being successful or being smart. Talk to your students about how they learn; are they visual, audio, kinesthetic learners, or a combination?

Most students don't know how they learn, students who really don't know what it means to study use strategies such as the "Stare & Glare" of method of study. Others use the "Pray to God" method, and others, because of their after school job tend to grasp on to the "Osmosis" method. They placed the open text on their chests, took a nap, thinking when they woke that somehow that print would work its way to their brain. While these methods are not very effective, students continue to use them because they don't have other strategies that work any better for them. Some students need to be taught how to study effectively and efficiently. They tend to think methods saying, writing it and having someone ask questions – like they did in primary school still work. Poor students tend to study until they "almost" have the information, then quit. Students need to told that after they study, they should be able to discuss what they studied without having their notes as a prompt – or they need more study. Sometimes teachers inadvertently fail their students by not requiring them to verbalize their knowledge.

Bottom line, teachers need to reinforce to their students that accomplishment is more dependent upon hard work and self-discipline than on innate ability. School administrators also need to be told the same thing; adoption of the latest new program will not lead to success, *work does*. In fact, the only place *success* comes before *work* is in the dictionary.

We also need to talk to students about their concentration times. How long can they study before they start looking around? I know I'm good for about forty-five minutes. While I can talk for days, after forty-five minutes of listening to someone else, I begin to notice how many lights are in the ceiling, a bird flying by the window, and a whole host of other items. Students need to know not only how they study, but how long they can study effectively and efficiently. Students need to be taught to extend their concentration time by studying a few more minutes before they take a short break from studying.

How many times have you heard a frustrated parent tell someone they sent their children to their room for two hours to study to improve their grades? Bottom line, if a student whose concentration time is only thirty minutes is sent to his room for two hours to study, chances are great that an hour and a half will be wasted. Students need to be taught how to study effectively and efficiently. If teachers don't tell them, who will?

Teachers and administrators must explain expectations explicitly and give examples. Don't have a 12 to 15 year old interpreting those expectations. If you want the students to class on time, does that mean running in the door as the bell rings, at his seat, or in the seat with his book or notebook open and ready for instruction? You need to tell them or you will be frustrated the rest of the school year.

Build trust with your students. Make sure they know you are there for them. Grading papers is not about taking points away from students. It should be about finding out how much they learned and helping them become more successful. Don't get caught up in arguing about points deducted in a test. If a student deserves the points, give them.

And while we are talking about testing, teachers should make testing as much a reflection of their own instruction as student preparation. If students are failing, the first place a teacher should look is in their bathroom mirror at home to find the problem. Remarkably, there is a relationship between not only what students are taught, but how they are taught and what they learn.

Belief Systems

Student-teacher relationships is an important component in increasing student achievement, but so are belief systems. Most of us have experienced a college professor that we might have thought was brilliant, but could not teach worth a hoot. Or we may have had the one that may have thought the way to increase student achievement was to separate the men from the boys by being overly demanding. In either situation, that class was not a good place to be. The first example just further illustrates that while teachers' content knowledge is extremely important, there's more to teaching than just knowing information. The second case does certainly does not seem like a very good method of inviting students into more rigorous classes. In fact, it seems like they use this as a method to decrease class size.

As K-12 educators, we don't have the option of trimming the herd like our colleagues in college; we teach the students in our classes. Or, do we? We hear the occasional teacher talk about getting students more appropriately placed – out of his classroom. This trick, often referred to as dumping, is accomplished by giving some sort of pretest. Then, after the test is graded, not so successful students are transferred to another, supposedly more appropriate, class.

Listening to some teachers, one might conclude they don't believe their students can learn. If that's the case, they need to resign and allow someone else to get the job done. Administrators and teachers must believe their students can succeed, that if they do a better job teaching and their students work harder, success will follow.

If you are really interested in your students succeeding, then you should *build success on success*. I have always used the first unit of the year as the unit I shape beliefs, teach students to study effectively and efficiently, as well as teach mathematics.

To *build success on success*, students must first experience success. So, over-teach the first unit, the students over-learn it, all the while teaching them what kind of learners they are, their concentration times, how to take notes, how to study effectively and efficiently. Provide examples of how you remembered important information, allow time at the end of the class for note reviewing, ensuring they have the information they need to successfully complete their homework or prepare for a test.

My belief is that I can successfully teach math to anyone willing to learn. If I can get the students to be successful on the first test and I can show them that success was based on what they did to prepare – not just being smart, I will be on my way to a great school year. Preparing them to learn will help them succeed and make you feel better about your students' accomplishments

If you hear yourself or others talking about "those" students, not *my* or *our* students, then chances are you are not taking ownership in their success. There is a disconnect, a disassociation that acts as a disclaimer to your part in your students' learning.

Elementary administrators have an advantage over secondary principals in that they have typically taught all of the subjects the teachers they supervise teach. They know the subject matter, they are familiar with both the sequencing and benchmarks, and instructional strategies to help students learn. Secondary principals normally come from subject specific areas. They have backgrounds in math, social studies, physical education, or science. All too often, people who don't have a background in the natural sciences feel threatened by their secondary math teachers. Some will acknowledge they didn't understand math, they didn't get the math gene and that's why they were not successful in math. So when they evaluate their math teachers, they are looking at classroom environment, instructional strategies, classroom management, and not really paying close attention to the math content being delivered to the students. That has to change. A lesson's worth should be determined by what students learned – not how well the class seemed to go.

Administrators must also change their belief systems. Many administrators will sit in a math class, evaluate the instruction, knowing full well they did not understand the day's lesson. My guess is if the administrator did not understand the lesson, and they probably took the class in high school, probably graduated high school, earned a bachelors and masters degree, and are mature, then how would they expect the 13, 14, or 15 year old to understand the lesson. If administrators are not understanding the lesson, they need to address that with their teachers, because it is doubtful that the students are getting it.

Make a good faith effort to teach the curriculum

Most school districts have established policies similar to the following:

"Guides or course syllabi are established for all areas of the curriculum and are to serve as the basis for instruction in district. Members of a professional staff shall utilize these guides as a means of meeting the needs of individual students. Making a good faith effort to teach the curriculum means that teachers plan to cover all the material in the appropriate syllabus."

The development of specific teaching techniques is the responsibility of the individual teacher. It is suggested that these teacher expectancies be incorporated into daily plans.

These should be consistent with the district's objectives and proven principles of learning.

In addition, many school districts have also established position statements or guidelines as follows:

STANDARD FOR QUALITY: Adopted secondary course syllabi serve as the basis for classroom instruction.

I. Instructional activities are correlated with stated objectives in adopted secondary course syllabi.

II. Resources are selected to support objectives in course syllabi.

III. Daily, unit, and semester planning includes goals and objectives contained in course syllabi.

IV. Appropriate accommodations and/or modifications are made in alignment with goals and objectives in adopted course syllabi to meet the instructional needs of all students.

Imagine you are the parent of twins, they are enrolled in the same school, but don't have the same teachers. Would you expect them to be learning the same material at about the same time? Covering the established curriculum with the appropriate benchmarking will create the consistency needed to develop and maintain credibility in the community.

There is no greater factor that affects student achievement than the content of the classes they take. If teachers do not cover the curriculum assigned to them, students will end up with gaping holes in their knowledge. All too often you hear teachers talking about how much time they must spend to review topics that students were supposed to learn the year before. While I sympathize with those teachers and the work they think they must do to remediate those students, the fact is by not covering their assigned curriculum, they will contributing to the problem as well. To address these deficiencies, it is my belief teachers could better utilize the idea of linking new concepts and skills to those areas of concern and reinforce those basic skills.

The simple fact of the matter is teachers who spend too much time remediating students who have already been remediated will not be able to cover their own curriculum assignment and that will result in the school not making adequate yearly progress. One of the first things I would check if a school is not making AYP is if they were teaching the curriculum assigned to them. How teachers' teach is important, what they teach is of greater importance! Teachers must teach the curriculum assigned to them.

Benchmarks

Is there a need for a consistent, standards-based curriculum? That question may best be answered by asking another question: Have you ever had students transfer into your class who have not acquired the necessary prerequisite skills and knowledge to be successful? If you answered "Yes," then you see the need for a curriculum that is consistent, not only between classrooms within a school, but between schools as well.

Although curriculum guides and syllabi provide classroom teachers with clear goals and expectations, they are often not accompanied by explicit timeframes. Therefore, in order to maintain a consistent mathematics curriculum, benchmarks should be established. (Benchmarks are approximate time lines by which particular concepts and skills are to be taught.) It is suggested that teachers within a school teaching a particular class, work jointly to develop benchmarks. Once developed, benchmarks must be revisited on occasion to allow for necessary revision. Setting and following these benchmarks should ensure adequate coverage of essential course objectives which lead to mastery. By adopting a professional development model such as the Backward Assessment Model, these timeframes would be established for each unit of study.

To further ensure students are meeting academic expectations, common periodic testing during the school year should be scheduled to determine the level at which students are achieving mastery on specific topics.

Frontloading

We discussed the importance of teaching the curriculum and using the benchmarks to assure students are spending the appropriate amount of time on the concepts and skills being taught to reach mastery. Having said that, there are times when it appears the system is setting students up to fail. While teachers clearly want to teach to mastery, a dense curriculum can get in the way of achieving that goal. Teachers often talk to each other about the issue of *coverage versus mastery* and how time effects student performance. Another factor that impacts student achievement is the sequencing of the material to be taught during the year. If important concepts and skills are left to the end of the school year, teachers might not get to them or if they do, they might have to rush to cover the topic. Not having the time to address mastery will have a negative impact on student performance on high stakes tests.

Frontloading suggests that teachers examine the curriculum assigned to them and determine the most important topics to be taught during the year. Once that determination is made, teachers should ensure they teach that material early enough in the year so they know they will get to it, teach to mastery, and that they will have opportunities to review and reinforce those concepts and skills regularly during the school year.

Naturally, material should not be just arbitrarily moved around, sub-skills still need to be taught as does developing the foundation for what is to be taught. Having said that, let's look at an example.

Fractions are often taught in sixth grade. I would guess there is a consensus that students should be able to compute with fractions. If fractions were introduced late in the year, some teachers might not get to them, others might feel rushed trying to cover them. The seventh grade teachers will be upset if their students come into their classes without the prerequisite knowledge and skills to be successful for them to teach the curriculum assigned to them. That would result in them trying to re-teach the material, giving up the valuable time they need to cover the material he's assigned to teach.

Another concern I would have if teachers felt rushed to cover concepts is they may decide to teach by procedure only – with little or no understanding of the "why" behind the procedure. For instance, the procedure for multiplying fractions is easy for students to memorize and multiplying fractions is relatively easy. Mission accomplished! While students might be able to multiply fractions without that concept being developed, their lack of understanding of why might make math look like a magical mystery tour rather than a system of logical reasoning.

If, in fact, there is a consensus that students entering seventh grade should be able to work with fractions, then the sixth grade teachers should teach fractions early enough in the year to assure they get to them, that they are able to teach to mastery, students understand the math, and have plenty of opportunities to review and reinforce that knowledge so it is embedded in the long term memory of their students.

Frontloading allows teachers to address mastery of important concepts and skills early in the year and allows the teachers to continually review and reinforce that knowledge. Frontloading will also result in increased student performance because students are not given the opportunity to forget important ideas.

Adopted Text/Programs

The adoption and use of approved textbooks and programs in the classroom by teachers should clearly be defined by district policy.

While textbooks are an important resource, the course syllabi drive the curriculum. Most adopted textbooks will not provide all of the resources necessary to satisfy all aspects of the individual course syllabi and, in many cases, include topics not addressed/covered in the syllabi.

The Third International Math and Science Study (TIMSS) research clearly recognizes the importance of textbooks. The study suggests teachers pay more attention to material in their textbooks and the order it is presented than to curriculum guides or state standards. Knowing this, careful attention must be given when schools are selecting textbooks to try and find a close match to the district's adopted curriculum.

Teacher expectancies require teachers to teach the curriculum that is assigned to them, to use common benchmarks to assure an adequate amount of time is set aside so teachers can more fully and appropriately develop concepts, make their students feel more comfortable in their knowledge, understanding, and application of the mathematical sciences.

Classroom teachers should be expected to, especially on the first test, to over-teach so students over-learn and are successful and while they are teaching content, they should teach students how to learn. Most students don't not know if they are visual, audio, or kinesthetic learners, how long they can concentrate, or how to study. Classroom teachers should use the first unit of study to teach content and teach students how to learn.

As discussed earlier, teachers should be expected to use linkage to introduce new or more abstract concepts and skills. Teachers need to develop concepts. For English language learners, that's referred to as "building background" and it helps all students acquire language skills.

Continuing with these expectancies, teachers need to use simple straight-forward examples to clarify the concept or skill being taught when introducing new material. Bogging students down in arithmetic is counter productive. For English language learners, that's referred to as "comprehensible input."

Balance - Teachers should be expected to have balance in their delivery of instruction and assessment as well as full implementing the Components of an Effective Lesson.

Homework -Homework should include what teachers say they value – not just problem sets. Guided practice should be used to monitor student learning.

Reading and writing – Teachers should have students write about what they learned at the end of instruction. And while reading ability is an issue for teachers, they must assign reading as part of the homework. But there's more to it than just assigning it, teachers need to introduce new vocabulary words, preview the reading, connect the reading to previously learned concepts and skills, check their understanding of the reading, and correct that understanding when appropriate.

Testing – Teachers need to test what they say they value (vocabulary, notation, conceptual understanding, procedures, modeling, etc.). Teachers should also provide practice tests halfway through the unit to prepare students for the real test.

Memory aids – teachers need to be more directive and prescriptive in how their students take notes. Teachers need to check the notebooks to ensure it can be used to complete the daily homework assignments and prepare for tests. Oral recitation should be used to embed information in short-term memory. These two memory aids benefit all students, especially English language learners.

Reviews – a second review should be scheduled at the end of each class period to address student mastery, deficiencies, and prepare for high stakes tests. While these reviews benefit all students, the review is of particular help to English language learners. These expectancies will be addressed in more detail in the next chapter.

Since the research suggests a great dependence upon textbooks, the use of a common text further leads to increased consistency. Books are written by different authors with different emphasis, textbooks should be chosen carefully to ensure they reflect the expectations of the community and school system.

As you can see, we expect a lot of our teachers. These expectancies should be discussed and their meaning agreed upon by the school administrator and classroom teachers, then put into practice in the math classroom.

Some textbooks lean toward a very traditional curriculum, memorization of rules, do the problems. Other textbooks are more constructivist in nature, having the students discover everything that is taught with very little or no memorization. My belief, expectation, is there needs to be a balance in the delivery of instruction and in assessing student performance. That balance should be defined by what we say we value in math education – not by a textbook or program.

Use of Instructional Time

State and local school districts usually determine the classroom time available to teachers and students. Regardless of the quantity of time allocated to classroom instruction, it is the classroom teacher and school administrator who determine the effectiveness of the time allotted.

According to a survey conducted by the American Association of School Administrators, teachers identify student discipline as the single greatest factor that decreases time on task in the classroom. Generally, teachers with well-managed classrooms, have fewer disciplinary problems. These classrooms typically have teachers who have established rules and procedures are in the classroom when the students arrive, and begin class promptly. They reduce the "wear and tear" on both themselves and students by establishing procedures for make-up work, they arrange their room to accommodate their teaching philosophy and style, and they develop routines that increase overall efficiency. The benefits of establishing these classroom procedures and routines become apparent as the total time on task approaches the allocated time.

The research says: "When teachers begin class immediately, students view them as better prepared, more organized and systematic in instruction, and better able to explain the material. Students also see these teachers as better classroom managers, friendlier, less punitive, more consistent and predictable, and as teachers who value student learning."

That's an awful lot coming from just starting class on time. But, when you think of it and understand our culture, it does shed light on those conclusions. In countries such as Italy or Greece, it is alright for boys who are friends to hug or even kiss each other on the cheek. In America, hugging and kissing is not well received. The way American boys show affection is by hitting. So when two boys like each other, they show it by hitting their friend on the arm. If there was too much "love" being delivered in the punch, the other boy might respond with a little extra affection of his own. Too much of this affection might lead to a fight.

Routines, like beginning class immediately, reviewing recently taught material, orally reciting new material, having students take notes, and ending the class by reviewing important definitions, formulas, algorithms, and the daily objective keep students engaged and on task. Quality time on task is not a "silver bullet" that can cure all the problems facing education; however, it can play an important role in increasing student achievement. Teachers must ensure that the entire class period is used to its full potential. That is, that "An academic focus and on-task behavior are [to be] maintained by the effective use of allocated instructional time." Teachers should begin class as soon as the bell rings with an activity such as a review or a quiz displayed on the overhead. This encourages students to arrive to class on time and it also allows the teacher a few moments to take care of attendance, tardies, new students, etc. This immediate focus on academics sets the tone for the entire class period. Carrying this idea to the end of the period, students should not be allowed to pack up their books and supplies early. (The bell should not be viewed as a signal to students to leave, but instead as a reminder to the teachers that they may dismiss their students.) When students complete their assignment/activity early, they should be encouraged to work on other material or should be provided with an enrichment activity. Students should not be allowed to socialize with those around them as it disturbs the class as a whole. The last few minutes of class should be used to bring closure to the lesson, to review important skills, to work on problem solving, or to further check the level of student understanding of the day's lesson.

Assessment (including test preparation and content)

Before a test is administered, teachers should be able to answer the question; "What do my students know and how do I know they know it?" All too often I hear teachers sounding surprised about the results of a test they administered in their own classrooms. If teachers were monitoring student learning along the way, that should not happen. It would seem counter productive to test students over material they don't know or understand. It certainly does not follow our belief in building success on success. Clearly teachers need to balance covering the curriculum with taking too much time addressing mastery, but if the Components on an Effective Lesson and teacher expectancies were utilized, some of those issues might dissipate. Teachers failing 30%, 40% or 50% of their students is not acceptable. Principals could hire people right off the street, without preparation, to get those results.

Teachers should prepare students to succeed on tests. As part of this, it is important that students know what is to be tested in time to correct any deficiencies. Students also need verification they are understanding and following correct procedures. Sufficient class time should be allocated to give students feedback. Students need to recognize the commonalities among and differences between problems that appear to be similar. To accomplish this, chapter tests and reviews from the text or other sources can be assigned as practice. These assessments/reviews should also contain the types of questions students can expect to see on tests. Additionally, in-class study groups can help students resolve their questions in preparation for tests. Finally, students should use their notebooks to review concepts and problems.

Tests should include definitions, algorithms, identifications, strategies for solving problems, and questions that require students to describe linkages between concepts. This will ensure that all levels of understanding are evaluated. To encourage students to review previously taught material, teachers might consider having cumulative tests. This will further serve to emphasize the importance of student notebooks.

In addition, tests should include a variety of question styles: multiple choice, true/false, true/false with correction, quantitative comparison, matching, and fill-in-the-blank. These types of questions are found on many forms of assessment including standardized tests. Each test should also include short answer and essay questions, as communication of mathematical ideas is an important component of understanding.

Discriminating between problems that appear to be similar, but requiring different solutions, needs to be practiced. Students often do not have difficulties learning rules or algorithms in their daily assignments for specific problems; however, when those problems are mixed with similar problems some students experience difficulty telling them apart. Some clear examples of this occur when working with integers.

Example:

Simplify.

1. 4 + (-3)
2. 6 (5)
3. -5 - 6 - 4
4. 6 - (-5)
5. (+6)(-4)
6. -24/8

Many students might have difficulty recognizing problem 3 as an addition problem. Again, students need to be taught how to distinguish between problems that look very similar. In summary, if teachers do not teach this differentiation, if students are not taught to compare and contrast and given time to see how problems that look alike are different, they probably will not learn it. Research suggests comparing and contrasting helps student understanding and will result in increased student achievement.

Teachers should prepare students to succeed. In preparing students for tests, teachers should provide tips on how to study. For instance, students sometimes confuse the definitions of complementary and supplementary angles. Teachers might suggest the "c" in complementary comes before the "s" in supplementary as 90^o comes before 180^o. Teachers should also take the time to help students differentiate between problems that look alike. For example, while students might learn several different methods of factoring, they may not be able to determine an appropriate method of factoring when a mixture of factoring problems is presented. Students have to be taught how to recognize differences and when to apply the appropriate method.

And, as mentioned before, how students perform on a test should be a reflection of what was taught and how well it was taught. If teachers teach the concepts and skills in a unit or chapter, but don't take the time at the end of that unit to give students an opportunity to compare and contrast those concepts and skills, to many students they will look alike. That inability to distinguish between problems and apply appropriate strategies will result in poor performance.

Some teachers may see that poor performance as an indication that the students did not study or learn what was presented. The fact is, the students probably did learn what they teacher taught. What they didn't learn was what the teacher did not teach – to differentiate between the problems.

For instance, I'm pretty sure most students can learn how to factor polynomials using the Distributive Property. I'm also pretty sure most students have little problem factoring using the Difference Between Two Squares. Teaching these factoring methods on different days – the teacher would see students successfully learning how to factor polynomials. As the days continued and students were taught to factor trinomials using Linear Combination, my guess is again, most students would be successful. The next method introduce would be factoring by Trial & Error. Many students experience some difficulty factoring by Trial & Error.

However, my experience tells me that while students can factor these polynomials as they are being taught, they have a great deal of factoring those same problems when the are mixed together. Leading teachers to believe the students did not learn how to factor. The real problem is students just could not tell them apart when they were all presented on one sheet or one test.

$4x^2 - 25$ looks an awful lot like $4x^2 – 12$. The first polynomial would be factored by the Difference of Two Squares, the second by the Distributive Property. How would students recognize that if they were not taught to compare and contrast?

Looking at $2x^2 + 11x + 12$ and $x^2 + 7x +12$. They certainly look similar to students learning about polynomials for the first time! Why would one be factored by Linear Combination, the other by Trial and Error?

From a student standpoint, they are just learning how to factor, these problems look alike, but the strategies used to factor them are different. Time needs to be taken to ensure students can tell these problems apart.

It probably goes without saying that feedback is needed for student improvement. The same is true when you ask students to summarize a lesson, tell what they have learned , or compare and contrast information. In order for students to improve their writing, their math teachers must give them feedback.

Chapter 5

Unsuccessful Learners

Student expectations impact learning

People enjoy participating in activities when they are able to participate successfully. Some of us who may be monotonic may be reluctant to sing with a group of friends because we don't feel like our voice is that good. Some people really enjoy a game of chess or putting puzzles together, typically they enjoy those activities because they have experienced a certain amount of success or feel, with additional time, they can be successful. The bottom line is most people will spend time on activities in which they have experienced success or have an expectation of success. We tend to shy away from activities that we don't do well or don't have an expectation of doing well.

The same can be true of students, they shy away from activities in which they have not been successful and feel like they will not succeed. Students who have not experienced success in mathematics might be viewed as reluctant learners. They are typically forced to enroll in a class, a class in a subject in which they have not been successful, and are expected to perform with a certain amount of interest and/or enthusiasm. Their reluctance to participate is the same reluctance adults have when asked to sing, speak before a group, or otherwise participate in an activity they don't enjoy or don't visualize themselves as being successful.

We spoke earlier about the importance of building success on success, students who experience success in math will generally spend more time on math than students who can not see the light at the end of the tunnel.

Teacher expectations impact student learning

Many students sit in the back of the room for this very reason, they want to be left alone. These unsuccessful learners could be classified because of their behavior as reluctant learners. Teachers know them and are often thankful when those students sit in the back - quietly.

And while teachers do feel thankful, we need to refer back to the "My Kid" standard discussed earlier and put ourselves in their parent's position. When children are four and five years old, they see their parents as the smartest and strongest people in the world. By the time these same kids reach sixth grade, they begin to view their parents as not very hip, up to date, or sometimes just not smart. As those youngsters reach twenty-five, it seems the parents start to regain some of the intelligence their offspring thought they lost when they were in their teens.

Students in middle school and high school tend to rebel against authority which more often than naught includes their parents. They would rather listen to the advice of friends, than their own parents. Teachers need to help these parents with their children just the way they would like some other teacher guide their own kids.

Would a classroom teacher want their child to sit in back another teacher's classroom napping during instruction? If they would not want that for their own child, using the "my kid" standard, they should not let it happen for someone else's children.

As stated earlier, student-teacher relationships are important. Teachers need to earn the trust, confidence and respect of their students by constantly communicating with them, encouraging them to be successful, and showing them how they can be successful.

Students who sit in the back of a room napping or seemingly ignoring instruction appear to be defiant however, upon closer examination, teachers might just find out that behavior is a defense mechanism. A mechanism that allows them to escape ridicule, ridicule for not being successful, from being seen as stupid.

Many of these students have not experienced success in the math classroom and have lost hope they ever will succeed in mathematics. Without teacher intervention, these students will be doomed and underachievement in math may become a way of life. Too many students equate success in mathematics with being smart, not with hard work. That is sometimes unconsciously reinforced by classroom teachers who recognize students that learn quickly as having higher ability than those students that have to work harder or longer to complete a task or assignment. Speed at completing tasks appears to have replaced effort as a sign of ability; high levels of effort might even carry the stigma of low ability – as not being seen as smart by their peers. That results in students not expending the effort needed to achieve to their full potential. Once students begin believing they have failed or are failing because they do not have the ability, they lose hope for future success, they stop trying. Hard work and effort have to be recognized by teachers as critical to student learning.

Teacher expectations can often be interpreted by listening to their own commentary. As we stated earlier, when teachers talk about "those" students, not "my" students, there appears to be a disconnect between what they teachers do in the classroom and what students learn. You hear these teachers lamenting that these students are absent often, won't come to class on time, won't do homework, won't take notes, etc. What's interesting about those commentaries is that some of those same students can be observed going to other teachers' classes on time, doing their homework, and taking notes. That suggests that teachers do make a difference, teacher expectations and their relationships with students have an affect on what students are willing to do in a classroom.

Classroom teachers must make an effort to build a positive relationship with all students and especially with unsuccessful learners, to build confidence and trust so the students won't feel threatened if they try. To build a relationship by communicating with those students daily that results in those students feeling they can be successful and they would actually be missed if they did not come to school.

To develop such a relationship, teachers need to talk to their students outside the classroom, in the halls, lunchroom, at ball games, dances, and at the store. Since many of these students have confidence problems, teachers might have to schedule a one-on-one conference with individual students during the school day - during another teacher's class. Teachers would be wise to conference with these students in a non-threatening atmosphere. For example, teachers should sit with the student at a table, not sitting behind a desk. Teachers should discuss what they are able do to help the student succeed in their class. In other words, the teacher must be part of the plan too. They also need to elicit from the student what they are currently doing and offer constructive suggestions on how to more efficiently and effectively use their time. Once it is determined how students are using their time, the teacher should encourage them to do more. The teachers need to think about what they would like another teacher to say to motivate their own child and make the same kinds of comments in a friendly, helpful manner. Teachers must teach students how to be successful.

Classroom teachers must build success on success. To do that, they must teach students how to be successful. Here are some suggestions to help students succeed in mathematics classroom.

Student Suggestions

1. Neatly copy in your notebook any problems that are put on board. Be sure you understand each step of the problem as it is being explained. Ask questions to clarify any step that you do not understand. Do not wait to have the point explained at a later time.

2. Always try to do as much of the assignment as possible without help. To a great extent, the amount you learn is dependent upon how well you have worked independently. When you practice a skill, it is more likely to become part of your long-term memory. Relying excessively on the teacher, or anyone else, to answer questions and to solve all the problems could result in a lack of understanding. If you are still confused after making your best effort, consider discussing the problem with a classmate.

3. It is necessary to spend time studying at home in order to reinforce what you have learned in class. Do not think that once you have obtained all the answers on an assignment, you are through with the material. After completing an assignment, review the concepts with the idea that you will be expected to know the material on a test. By studying at home, you will discover what you do not understand and will be ready to ask questions in class the next day. A student who has done little studying on his own frequently knows so little that he is embarrassed to reveal his ignorance. He is often afraid to ask a question that he feels everyone else in class can already answer.

4. When material you have already learned is being discussed, use the opportunity for "over-learning." Try to work a step ahead of the person presenting the problem.

5. DO NOT WASTE TIME IN CLASS! Most of your learning will occur during class time; it is foolish to waste this time. Each class period gives you an opportunity to concentrate on learning a specific concept, to correct your mistakes, and to direct your learning efforts.

6. ALWAYS COME TO CLASS IF AT ALL POSSIBLE! When you are absent, there is no way to fully make up for the class instruction you miss.

7. Always seek to understand rather than simply to "squeak by." The grade you receive is important, but not nearly as important as the mental growth you gain from the process of learning the subject.

8. Memorization will help you absorb and retain factual information upon which understanding and critical thought is based. Knowing and using mathematical vocabulary and notation is key to the understanding of the mathematical sciences.

9. PREPARE FOR TESTS! Your tests are often made up of questions that come directly from homework exercises, class notes, the chapter test or the chapter review. Meet in study groups to discuss items that you think will be on the test. Use the study group for remediation and peer tutoring. Individuals who help others learn, gain a better understanding themselves.

10. If you are making a serious effort and still not doing well, come in after school and talk with the teacher. He/She can probably help you overcome your difficulties.

Unsuccessful students must be slowly integrated into the mainstream classroom to avoid possible embarrassment from their friends. The first time the student that plays the role of "sleeper" in the back of the classroom answers a question correctly, well meaning friends of that student might laugh or make some sounds of surprise or astonishment that result in that student feeling ridiculed. That embarrassment could cause that student to revert to the defense mechanism that has served him so well – napping in the back row.

Since reluctant learners often sit in the back of the class, teachers might try moving them toward the front of the class in a very inconspicuous way – like making a new seating chart for the entire class. It would be wise to let that student know what you are doing and why you are doing it so they are not caught off guard making an inappropriate remark. All students should be encouraged to take notes, thereby participating in class. The teacher should initially check for the reluctant student's understanding by simply asking if the lesson is understood – not looking for verification by answering an academic question. As time progresses, the teacher might ask if the reluctant learner agrees with another student's understanding of the lesson. And finally, as the reluctant learner becomes more comfortable and immersed in the class, the classroom teacher should begin to ask the student questions that the student is likely to know and answer correctly – building success on success and self confidence.

Building confidence and trust takes time, unsuccessful learners have developed defense mechanisms over time to protect themselves from embarrassment and ridicule. Change is not usually embraced quickly or wholeheartedly by everyone – not even by classroom teachers or administrators. Realizing this, teachers need to work patiently with students. If teachers move too quickly, they may inadvertently embarrass the student they are trying to help. That embarrassment could result in a student's trust and confidence being shaken or broken in that teacher.

The reason students generally become reluctant learners is because they were unsuccessful learners. The reason they are unsuccessful is because they probably don't understand what exactly is expected of them.

In the student suggestions for instance, it was suggested the students neatly copy any problems on the board into their notebook. While that is great, what if the student does not know how to take notes, does not title what is in the notes, doesn't date it, doesn't draw pictures, write definitions or procedures? Chances are there are many students in a classroom who could use guidance in the configuration of their notebook. Something simple like leaving white space so there is no visual overload might help students study more effectively and efficiently. Chances are your most unsuccessful students really have nothing to take home to help them with their homework or prepare for tests. Students need to be taught how to take notes.

Another suggestion requires students to memorize information. Is it possible that unsuccessful learners don't know how to memorize material effectively or efficiently? Earlier, we discussed student learning, to tell a student to go home and study does not help that student if they really don't grasp what they need to do at home to study.

So the suggestions are meaningless unless students know what they really mean and know how to apply them. Explaining those suggestions is part of teaching.

Many unsuccessful learners come from homes where the parents might not be able to help their children with schoolwork because they were unsuccessful learners as well. Additionally, a disproportionate number of unsuccessful learners come from poverty, their parents are more likely to work in the evening and not be available to assist their children with homework or make suggestions on how to study effectively.

All to often these students are left to make decisions at very early ages, decisions about what to make for dinner, to watch television, study, or interact with their friends. While many of these students might have good intentions going out the schoolhouse door, when they get home, those intentions are not carried through. It is a lot easier to watch their favorite television show or talk to their friends than it is to stop having fun and study.

Because of this, it is important that teachers implement the *teacher expectancies* and the *components of an effective lesson*. The components and expectancies provide structure to daily instruction that is helpful to students who have not experienced a great deal of success in a math class because of the long and short term reviews, oral recitation which embeds information into short term memory, note taking that helps students complete their daily homework assignments and prepare for tests, as well as the guided practice to monitor student learning.

While homework and home study are important, teachers need to use their class time effectively so students learn as much as possible in class. Learning difficulties among special populations stem largely from instructional practices that: do not build upon informal knowledge, does not foster learning, or teachers that do not monitor student learning. Special populations will experience difficulty if the instruction begins with the abstract and moves too quickly or if the instruction relies on memorizing mathematics by rote.

Thinking about what causes learning difficulties for special populations, one would realize those are the same factors that cause difficulty for the general student population as well. Good teaching matters!

EMPHASIZE THESE STRATEGIES:

- State the day's objective, teach it, and then tell them what you taught them when you close the lesson – closure.

- Develop concepts. Teach the big ideas.
- Link concepts to previously learned material and/or real-world experiences.

- Use simple, straightforward examples that clarify what is being taught.
- Use numbers in examples that allow students to focus on the concept that don't bog students down in arithmetic.
- Have students discuss among themselves and write about what they are learning.

- Incorporate guided practice to monitor student learning before assigning homework.
- Use practice tests to prepare students for unit tests. In first year algebra, use multiple test versions.
- Tell students how you remembered (learned) important information.

- Use choral recitation to imbed information in short-term memory.
- Require students to take notes and keep notebooks.

- Use the second review period to reinforce long-term knowledge and address student deficiencies.

Contradictory rules – Misunderstood behaviors

As classroom teachers, there are times we just scratch our heads in bewilderment because of student behavior. It just seems that some students find trouble, then continue to dig deeper when confronted about their behavior. That additional digging is often interpreted as a sign of disrespect by many teachers, but it might not be.

For instance, as a sign of respect, many cultures require eye contact when correction in behavior is being discussed. Other cultures, as a sign of respect, discourage eye contact and, in fact, the person on the receiving end of the message is taught to look downward. If classroom teachers are not sensitive to their students' backgrounds, they may interpret a student looking down as a sign of disrespect, which in turn, causes the student an additional problem to deal with.

In more affluent areas, when adults have disagreements, frequently they are handled through litigation. This method to resolve problems has been adopted in the schools using peer mediation. Since a form of peer mediation is used at home to resolve conflicts, some students view this as very consistent to home life. In less affluent areas, disagreements are often handled by fighting. In fact, the people who win those fights are often held in high regard. Students coming to school from those communities are taught at home to take care of themselves by fighting, peer mediation may be construed as a coward's way to resolve a problem. The end result of this might be a student beating up another student to address a grievance – which is seen by educators as bad behavior.

These same types of misunderstandings happen quite often in other circumstances, again the behaviors can be traced back to what is learned at home. For instance, when parents argue at home, children from more affluent homes might be more inclined to listen quietly or take leave. In less affluent areas, children try to diffuse volatile situations at home by using humor to decrease the tension. Learning from that home experience, students might try to apply it at school. For instance, teachers often have to correct the behavior of students. In some cases, rather than being a quick direction, the teacher might spend some time discussing the seriousness of the situation. If students coming from less affluent neighborhoods see this discussion escalating into a more volatile situation, they may say something that to them seems to be funny - not as a sign of disrespect, but a way to calm the situation.

Again, if classroom teachers and administrators are not sensitive to their students' backgrounds, those student might find their way to the office for being disrespectful and possibly being punished for using a strategy they learned at home to successfully diffuse an uncomfortable situation.

The reason that students are not held to adult standards is because they are not adults. They make mistakes in judgment because of their own experiences and/or lack of experiences. As the adult role model, classroom teachers must make every effort to ensure their students know how to act in different situations. Without explicit guidance from teachers and school administrators, students may apply the rules and behaviors they learned at home to school – not realizing those rules contradict standards of behavior at school.

I can not overstate the importance of developing positive student-teacher relationships, especially with students who don't care for school, students who have not experienced much success in the classroom. The research strongly suggests that students will work and work harder for teachers out of loyalty.

Research from the U.S. Department of Education suggests these factors also impact student achievement. They include:

- teacher preparation
- course content
- student-teacher relationships
- time on task
- verbalization
- repetition
- outside study groups
- hands-on activities
- teacher attitude / personality
- humor
- enthusiasm, energy and interest of teacher
- loyalty, reciprocity
- positive reinforcement

Over half those factors are based on teachers!

Chapter 6

Testing

Assessment drives instruction

Monitoring student progress frequently and systematically helps teachers identify strengths and weaknesses in student learning as well as in instruction. Assessing student work comes in many forms, but teachers need to know the answer to this question to improve their instruction and address student needs, "What do my students know and how do I know they know it?"

There is often a disconnect between what teachers say they value in mathematics and what they test. When talking with classroom teachers, they will indicate the importance of vocabulary and notation to be successful in math. But when you look at their own classroom tests, typically there are no questions on vocabulary or notation. Teachers will tell you how important it is to have student understanding, but again, when you look at their unit tests, there won't be questions that require open ended answers that measure student understanding of the concepts being taught.

In fact, many teachers will readily admit that students have raised their hands in the middle of them developing or explaining an important concept only to ask the question; "What's the homework assignment?" Now, why would a student interrupt instruction to ask such a question? The answer is the students know what the teachers' value better than the teachers themselves. Students know teachers' value what's being graded – that is, what's on the homework, quiz, or test. If teachers don't ask questions dealing with conceptual development, linkage, vocabulary, and notation on tests, then chances are students won't spend time studying it.

Teacher-made tests should reflect what is taught and valued in mathematics education. For example, while many teachers say mathematics is a language, this may not be reflected on their tests. If we value students' ability to verbalize their knowledge, then definitions, identifications, and procedures should be part of tests. Many of the rules in math don't make a lot of sense standing alone. For example, when students add fractions, it seems natural to add both the numerators and denominators, however they are told not to add the denominators. If students learned that a fraction is a part of a unit, made up of a numerator and denominator and the denominators told them how many equal parts makes one whole unit, it would make sense not to add the denominators so they know how many equal pieces makes one unit. Another example, while it is important for students to know they can not divide by zero, they should also be able to give some rationale behind the rule besides saying that's what my teacher said. Having the students write a brief explanation using the definition of division will clarify their understanding. Manipulation of data, vocabulary and notation, open-ended questions, problem solving and appropriate use of technology should be included on tests. Also, to encourage students to review and reinforce previously learned material, teachers should make their tests cumulative.

Teachers using the specification sheet, assessment blueprint, and benchmarks discussed in the Backward Assessment Model would be more likely to have balanced assessments that measure what they say they value in math education. Teachers should not expect of their students what they are not will to inspect.

Teachers should teach students to successfully prepare for tests. For instance, students sometimes confuse newly introduced terms such as domain and range. It might be helpful if teachers wrote an ordered pair (x, y) and suggested those are in alphabetical order as is the (domain, range), (abscissa, ordinate) and (horizontal axis, vertical axis). Those connections might help students remember. Teachers should also take the time to help students differentiate between problems that look alike. For example, while students might learn several different methods of factoring polynomials in algebra, they may not be able to determine an appropriate method of factoring when a mixture of problems is presented on factoring. Students have to be taught how to recognize differences and when to apply the appropriate method. Comparing and contrasting leads to increased student achievement. All too often, teachers successfully teach each method to their students on a section-by-section basis, but don't take the time to teach them to compare and contrast these problems so they know which method to use. When the students perform poorly on the test, the conclusion reached by many teachers is the students did not learn how to factor. But the reality is the students did learn what the teacher taught - to factor, but they did not learn what they were not taught – to differentiate between problems that looked similar and factor using the appropriate method.

Tests are formalized vehicles to not only evaluate student learning, but should also act as an assessment tool. As such, tests provide students a blueprint to increase their knowledge. Teachers should use assessment information, particularly questions answered incorrectly, as one way of increasing student performance. Addressing these deficiencies will result in increased student achievement. As will addressing their own instructional practices.

Assessing student work

In order to address student deficiencies, teachers need to know what students know. On many state mandated tests, it would be pretty difficult, if not impossible, to determine deficiencies based on the make up of the test. Often, one test question is meant to measure multiple state standards, including a student's ability to compute. If test items are all made up of word problems, how could a teacher determine if a student missed a specific question because:

a) the student does not speak English,
b) the student has a reading comprehension problem,
c) the student did not understand how to solve the problem,
d) the student did not know how to compute using that number set, or
e) the student made a simple computational mistake.

Without knowing what students know, teachers will frustrate themselves and their students by having to re-teach what the students already know. That boredom often results in students getting off task resulting in classroom management problems. That re-teaching also takes time, time that teachers say they don't have.

Identifying deficiencies and addressing them provide teachers more time to teach the curriculum assigned to them to mastery. In the factoring example given above, if the teacher re-taught the five methods of factoring taught in a first year algebra class, they would spend a lot more time than if they identified the deficiency as being able to distinguish between polynomials.

Teachers complain all the time about their students not knowing their basic arithmetic facts. If teachers looked at each operation, they would find that students actually know most of the facts. For instance, if you asked teachers, can your students multiply by one, by 2, 3, 5, 10, 9, and doubles, the answer to each of those questions is generally yes. That means the students know most of their facts. That information would allow teachers to concentrate on student deficiencies. Knowing that, teachers should spend their time reviewing and reinforcing multiplying by 4, 6, and 8. But not even all those need to be addressed, because the students, using the commutative property, could multiply four by 1, 2, 3, 4, 5, 9, and 10. That means the teacher needs to concentrate on 4x6, 4x7, and 4x8. Continuing, multiplying by 6, students could multiply by 1, 2, 3, 4, 5, 6, 9, and 10 leaving only 6x7 and 6x8 as the facts that need to be addressed. By identifying what the kids don't know, the teacher is able to cut down quite a bit of unnecessary re-teaching. That knowledge of what students know takes a lot of frustration out of teaching and does not bore the students to death.

Another example of assessing what students know might be with adding fractions. On a test, if a student could successfully add 1/4 and 1/3, but could not combine 5/18 and 7/24, some teachers might deduce the student does not know how to add fractions. Another teacher, upon closer examination, might conclude the student has procedural knowledge because of successfully adding the fractions on the first problem, but is getting the second problem wrong because of difficulty finding a common denominator. Rather than re-teaching the procedure for adding fractions, good teachers will know what their students know and concentrate on finding common denominators. Teachers find out what students already know and what they still need to learn by monitoring and assessing student work – not by mere perception.

Introducing new concepts by linking them to previously learned concepts and outside experiences was emphasized for a number of reasons. One of those was familiarity with language, making the students more comfortable in the concept being introduced. Having said that, the importance of the formality of language was also discussed. On teacher made tests, teachers must be on guard to use the more formal language that students will find on standardized tests on their own classroom tests. Otherwise, students might not recognize the question being asked as one that was taught in school. For example, on an algebra test, a teacher might ask students to "solve" an equation. On a college entrance test, students would be asked to solve an equation by "finding the solution set …". This difference in language might result in a disconnect in what was taught and learned in class and what is being asked on a test. Students not recognizing these different directions mean the same thing might miss a problem they really knew how to do. Unfortunately, this kind of mistake might result in people in the community believing students are not being taught these concepts and skills in school.

If teachers truly understand the importance of vocabulary and notation and its relationship to increasing student achievement, then they would ask students to translate English to math and math to English on their unit tests. For instance, a student might be asked to translate ${}_5P_2$ to English. Students not understanding math or not paying attention to the detail and rigor required in mathematics might say five P two. All students should be taught to read ${}_5P_2$ as a permutation of 5 things taken 2 at a time.

Another example, a word problem might contain the expression, four less than a number, some novice students might translate that very literally and write $4 - x$ or $4 < X$, because the four was written first. However, if English to math translations were taught and tested explicitly, more students would recognize that as $x - 4$.

If teachers tested what was taught, the tests could be used to explicitly identify deficiencies. For instance, students asked to solve counting problems should be encouraged to use a calculator. A correct answer would suggest the student knew how to solve the word problem. Students not using a calculator who answered incorrectly may not have understood the problem, not have known a particular formula, may not have known how to use the formula, or could have made a simple arithmetic mistake. If teaching students to solve counting problems is important, then test the students on that and allow them to use a calculator. If it is important for students to know a formula, such as ${}_5C_3$, I think it is, then have them write the formula and evaluate it. And make sure they know how to say it – a combination of 5 things being taken 3 at a time.

Experienced teachers can predict common errors students make. All algebra students are taught to solve quadratic equations by the Quadratic Formula. Most students will memorize the formula, find the values of a, b, and c, plug them in the formula correctly, and evaluate the resulting algebraic expression, then simplify the last step incorrectly. The most common mistake is reducing the fraction without factoring first. When this occurs, some teachers might conclude their students did not learn what they were taught. The fact is, if they memorized the formula, could identify the values for a, b, and c, were able to plug them into the formula correctly, and successfully evaluated the expression, but made a reducing error, then the students did learn what was taught. The teacher needs to address reducing or modify their instruction so students are more successful. For instance, the quadratic formula could be written as a sum of two fractions.

To determine if teacher-made tests are fair, balanced, consistent, cover the curriculum and the grades earned in different classes are portable, site administrators need to compare the content and achievement on grade level or unit tests given by different teachers testing the same topics. This should be a common practice, a practice that almost always necessitates a follow-up discussion by the teacher(s) and administrator.

If the site administrator did not see the balance in the assessment to match the balance on the specification sheet discussed in BAM, consistency should be a point of discussion. If one teacher asked students to reduce a fraction like 4/6 and another asked students to reduce 111/123, a supervisor might ask about the fairness of these questions.

Setting a date for a test

As teachers do their long range planning or use BAM to prepare instruction on a unit, benchmarks are identified. Setting benchmarks is important so teachers can plan to teach their assigned curriculum to mastery. As part of that planning, testing dates are typically identified. Since teachers are in the planning stages, the dates should be flexible. If a test is scheduled for a specific Friday and on the preceding Wednesday the teacher determines the students are not ready for the test, the test should be postponed. There is no sense administering a test to students that are not prepared for the test. Remember, build success on success.

Postponing a test requires the teacher to move forward with new instruction, while giving the students a few more days to successfully prepare for the test. It does not mean stop, re-teach the entire lesson, then give the test. Teachers incorporating the *Components of an Effective Lesson* in BAM could use the last five to seven minutes of each class to address the deficiencies that led to postponing the test. Teachers that stopped all new instruction would run the risk of not adequately covering the curriculum assigned to them.

Teachers would be more inclined to give students additional time, when warranted, if they viewed the test results as a reflection of their own instruction. As we have discussed, teaching is a whole lot more than just presenting material, teachers that do not monitor student learning as they are teaching might not know their students are not understanding the material being taught. A teacher that is surprised by test results probably needs to pay closer attention to questions being asked in class, performance on homework and quizzes, guided practice assignments, as well as discussions taking place in class.

Practice tests

Classroom tests, unit tests are criterion referenced tests. They are tests based upon what is taught and learned in the class. As such, the contents of a teacher-made test should not be a secret. As part of the Components of an Effective Lesson, we ask teachers to not only state, but write the daily objective on the board – to be explicit so students know what they are learning.

Testing should not be any different. Students should know exactly what is expected of them. After all, as teachers, many of us had to take exams, teachers prepared for them by studying older forms of the ACT, SAT, PPST, or PRAXIS. Not only did teachers study for these exams using practice tests, chances are great those teachers wanted to know the rubric used in grading the tests.

Providing students with practice tests is another way of ensuring students know what to expect on their tests and it also provides the teacher another opportunity to monitor student learning.

Rather than scheduling a day for students to take a practice test in class, the recommendation might be to provide a practice test to all students approximately half way through the unit to be tested. Identify the questions students should be able to answer based upon instruction and the questions that have yet to be covered. At the end of each successive class, be willing to answer questions from the practice test as well as identifying additional questions the students should be able to answer based on the new instruction. This activity keeps the students engaged in test preparation over a period of time and results in students having a clearer picture of what is expected – it also leads to increased student achievement.

Some might argue providing a practice test is teaching to the test. I don't agree. Using the assessment blueprint in BAM, the types of questions asked are based on the specification sheet. That is, what do we expect students to know, recognize and be able to do after instruction. On a practice test, students might be asked to add 5/18 and 7/24, on the real test, different numbers would be used. On the practice test, students might be asked to write the procedure for adding/subtracting fractions, on the real test, the procedure asked for might be for multiplying fractions. Practice tests don't have to give the test questions away, but the types of questions, the assessment blueprint, should be known by all – especially the students.

Chapter 7
Department Improvement Plan

Adequate Yearly Progress

The No Child Left Behind Act of 2001 expands the rules and regulations for school accountability to ensure all students meet state standards. NCLB also changes how states determine whether schools make adequate yearly progress and delineates the school improvement procedures and consequences when schools do not make that progress.

Each state defines what constitutes adequate yearly progress for all elementary and secondary schools, as well as districts within a state. Adequate yearly progress must contain the following components:

1. Applies the same standards of academic achievement to all students
2. Is statistically reliable and valid
3. Results in continuous and substantial improvement for all students
4. Measures the progress of schools, school districts, and state on primarily state assessments
5. Includes separate measurable annual objectives for continuous academic growth for the following sub populations:
    a. economically disadvantaged students
    b. students from major racial and ethnic groups
    c. students with disabilities
    d. students with limited English proficiency

Each state is required to establish a timeline for adequate yearly progress beginning in 2001-02. The timeline ensures all students in each subgroup will met or exceed the state's proficient level of academic achievement on state assessments no later than 12 years after the beginning date.

To make adequate yearly progress each year:

1. each group of students must meet or exceed the objectives determined by the state
2. 95% of the enrolled students in each subgroup must participate in the testing

School Improvement

If schools fail to meet adequate yearly progress in two consecutive years, the school district must identify those schools. Each of those schools must, with in three months of being identified, develop or revise a school plan in consultation with school staff, the local school district, and outside experts. The school improvement plan must:

1. include a review of the school's accountability report and other data
2. include an identification of problems and/or factors causing the schools to be in improvement section
3. use scientifically based research strategies to strengthen the core academics
4. adopt policies and practices concerning the school's core academic subjects that have the greatest probability of ensuring all groups of students will meet state standards
5. establish specific annual, measurable objectives for continuous and substantial progress by each subgroup to make adequate yearly progress
6. include strategies to promote effective parental involvement
7. incorporate remedial activities
8. determine strategies to improve achievement
9. specify the responsible entities identified in the plan
10. establish a timeline
11. determine measurable criteria for evaluating the effectiveness of each provision in the plan including achievement, attendance, and decreasing dropouts
12. describe resources available to the school
13. provide a summary of effectiveness of Legislative appropriations to improve school achievement and of programs approved by the Legislature

Data

There is little doubt educators are hardworking, dedicated and caring people. But to be successful, not only do they have to work hard, they have to work smart. That means they have to know what's going on and have a plan to address concerns with their students' current status.

Before developing a plan, one needs to know the strengths and weaknesses in their program. Collecting data is the first step on the journey to increase student achievement. To get a sense of the department's knowledge of their own data, one might ask individual department members to answer the following questions:

Why does our data look like it does?
What are the root causes and contributing factors of the data results?
Do all the department members know what and how material is assessed and what a good response looks like?
Do all department members teach and assess the standards being taught on high stakes tests?
How does the department monitor individual student progress on the standards?
How does the staff intervene with students not demonstrating proficiency?

It's very likely that department members don't know what the data looks like, they depend upon perceptions or generalizations made by others. Those perceptions might not be on target.

Hitting the target

Let's look at five processes to hit instructional targets and ask some questions to help guide teachers in developing a plan.

1. Understanding the target

Do all department members understand what students are asked to know and do on state assessments?

Do all department members understand how student performance is scored and what a satisfactory and excellent student response looks like?

The fact is many teachers might not be able to answer these questions with any degree of specificity. That lack of specificity might be a contributing factor leading to the data looking the way it does.

2. Teaching the indicators

Do all department members know the goals, expectations, and standards they are responsible for teaching?

Do all department members teach them?

Do all department members review them?

Anecdotal evidence suggests that teachers do teach the standards, but to often they are only teaching procedural knowledge. When students do not have conceptual knowledge, any variation in the way a standard is tested tends to cause them great difficulty. Additionally, over time, students forget information. Classroom teachers need to review standards over time to ensure their students remember – to sustain that knowledge.

3. Assessing the indicators

Does each department member know how to assess the content and performance outcomes?
How are the content and performance outcomes being assessed by the department?
What do the results indicate?

4. Monitoring individual student progress

Is each staff member monitoring progress of individual students on these performance outcomes?
How is staff using data to address instruction?

How does the department share results?

5. Intervening with students not succeeding

Does each department member provide interventions for students not demonstrating attainment of an outcome?

What are the department's most commonly used interventions for students not achieving?

How successful are those interventions?

What percent of the students need interventions?

Teachers' ability to answer these questions might suggest areas in which professional development is required and processes that need to be put in place to ensure student needs are being addressed.

Now it's time to look more closely at the data. That data might include criterion and norm referenced tests, college entrance tests such as the ACT and SAT, NAEP, end of subject tests, and proficiency exams, observations, surveys, interviews, as well as demographic information. That can be looked at in conjunction with student achievement, curriculum and instruction, school culture, parent community involvement, school community characteristics, and professional development.

The Department Improvement Plan

As teachers and administrators develop a plan, this should not be perceived as a 5-year plan, the department plan must have immediate and long term impact. That plan should have the My Kid and Common Sense standards built in as well as implementing the building *success on success* model as a cornerstone.

Additionally, the following topics should be identified as *teacher expectancies* adopted and reinforced by each member of the department and included in the department improvement plan.

a. First test: over teach – over learn; teach content while teaching students how they learn, concentration times, and how to study.
b. Improve student-teacher relationships – talk to students, be positive!
c. Use linkage to introduce new or more abstract concepts and skills – develop concepts, teach the big idea.
d. Use simple straightforward examples to clarify concepts being taught when introducing new material. Don't bog students down in arithmetic.
e. Adopt a balanced approach to instruction, emphasize vocabulary & notation, concept development & linkage, memorization of important facts and procedures, appropriate use of technology, and problem solving.
f. Fully implement the Components of an Effective Lesson.

g. Adopt a homework format to include what teachers' value – not just problem sets.
h. Include reading and writing in the instructional plans.
i. Test what you value, provide practice tests halfway through the unit to help students prepare for the real test. Use the more formal language students will see on high stakes tests.
j. Require students to take notes, use oral recitation to embed information in short-term memory.
k. Use reviews at the end of each class to address mastery, deficiencies, and prepare for high stakes tests.
l. Use time effectively, start the class on time, end the class at the end of the period.

As preliminary work, the department members need to develop a Department Mission or Vision Statement, then identify school highlights; list any successes, honors, or unique features.

Once the mission statement has been developed, let's get to the plan.

Section A

1. Perform a Comprehensive Needs Assessment
   A. Review and analysis of data
      a. Identify areas of strength
      b. Identify areas of concern
   B. Identify Goals
      Goal 1
      Goal 2
      Goal 3 (optional)

2. Inquiry Process
   For each goal, identify causes/factors and solutions and strategies.

3. Plan Design
   For each goal, state objectives.
   For each objective create action steps that include timelines, resources, and responsible entity.

4. Monitoring Plan Implementation
   For each goal, create action steps to monitor goals, include data to collect, timelines, and responsible entity.

5. Evaluation of Plan Implementation
   For each goal, identify the outcome indicators, when to collect it and the entity responsible for collecting it.

Section B

1. What are the policies and practices in place that ensure proficiency of each subgroup in the standards

2. List and briefly describe, as appropriate, how the department has incorporated activities or remedial instruction, or tutoring to help students achieve the standards

3. Describe the resources available to the department to carry out the plan.

Summarize the effectiveness of any appropriations for the department to improve student achievement.

Effective Schools

School administrators can not be left out of any school improvement effort. While we are describing a department improvement plan, educational research suggests effective schools generally have strong instructional leaders, a safe and orderly climate, a school-wide emphasis on basic skills, high teacher expectations, and regular assessment of student progress. For that reason, the school administration has to be included in any plan.

According to the U.S. Department of Education, schools with high student achievement and morale show the following traits:

Vigorous instructional leadership
A principal who makes clear, consistent, and fair decisions
An emphasis on discipline and a safe orderly environment
Instructional practices that focus on basic skills and academic achievement
Collegiality among teachers in support of student achievement
Teachers with high expectations of their students
Teachers that believe their students, through hard work, can and will learn
Frequent review of student progress

Effective schools have effective principals, principals willing to observe, supervise, and evaluate their classroom teachers.

Teacher Supervision

Teachers welcome instructional suggestions that result in increased student achievement, but they rarely receive them. According to the U.S. Department of Education, supervision that strengthens instruction and teachers' morale has the following elements:

- agreement between supervisor and teacher on specific skills and practices that characterize effective teaching,
- frequent observation by the supervisor to see if the teacher is using these skills and practices,

- a meeting between the supervisor and teacher to discuss the supervisor's impressions,
- agreement between the supervisor and teacher on areas for improvement, and
- a specific plan for improvement, jointly constructed by the teacher and supervisor.

Many secondary teachers report principal involvement in their classroom does not occur often. Those teachers often indicate when principals do observe classroom instruction, rarely do they receive recommendations that are specific enough to implement in their classrooms.

For schools to improve performance, building level administrators must be willing to inspect what they expect of their teachers.

The recommendations made in this text are very easy to observe and monitor. Using the Professional Development Day Agenda in the Backward Assessment Model, principals can be involved and monitor the self identified strengths and weaknesses within a department and the changes in instructional practices identified to address student deficiencies. Additionally, the Components of an Effective Lesson can be easily observed and monitored by the principal. Stating and writing the day's objective on the board, closing the lesson by restating the objective and providing a brief over view, having students write about what they learned at the close of the lesson. By providing two review periods, one in the beginning of the period to go over recently taught material and the second to review long-term knowledge and prepare for high stakes tests. Principals can easily determine if homework is more than just a problem set out of a textbook and monitor whether students had an opportunity to practice with guidance from their teachers.

If the components were adopted within a department improvement plan, principals could focus their recommendations on their implementation. While a checklist could be developed for the components, teachers would be much better served if the principal sat down with the teacher to discuss their observations in greater detail.

While we said this before, it is worth repeating. With respect to school administrators, elementary administrators have typically taught all of the subjects their teachers teach. They know the subject matter, they are familiar with both the sequencing and benchmarks, and instructional strategies to help students learn. Secondary principals normally come from subject specific areas. They have backgrounds in math, social studies, physical education, or science. All too often, people who don't have a background in the natural sciences feel threatened by their secondary math teachers. Some administrators will acknowledge they didn't understand math, they didn't get the math gene and that's why they were not successful in math. So when they evaluate their math teachers, they tend to look more at classroom environment, instructional strategies, a checklist for the components, and classroom management, and not really paying close attention to the math content being delivered to the students. That has to change. A lesson's worth should be determined by what students learned – not how well the class seemed to go.

Administrators must also change their belief systems. Many administrators will sit in a math class, evaluate the instruction, knowing full well they did not understand the day's lesson. If administrators are not understanding the lesson, they need to address that with their teachers, because it is doubtful that the students are getting it.

Improvement plans rarely work unless the school's administration is an integral part of the plan and is actively participating in the process. On the following pages, I have provided an observation sheet that teachers and administrators might discuss and come to an agreement on items that should be observable on most days during a regular class period.

SECONDARY MATH OBSERVATION SHEET

This observation sheet will use a four-point scale. The observer will provide a rating on what was "seen" or "heard" in the classroom. If no evidence was provided to give a rating, then that component or expectancy will be left blank. To give a Level '4' or '1' rating, the observer must give documented evidence of what was "seen" or "heard" in the classroom.

Total Number of Students		# of students	Minutes after the start of class period					
			Start	11	22	33	44	End
Male	Female	On-Task						
		Task						

Level 4	Distinguished	Level 2	Basic
Level 3	Approaching Distinguished	Level 1	Approaching Basic

MATH OBSERVATION NOTES

COMPONENTS OF AN EFFECTIVE LESSON	4	3	2	1	TEACHER EXPECTANCIES	4	3	2	1
1. Introduction					13. Reinforce Study Skills				
2. Daily Review					14. Student/Teacher Relationships				
3. Daily Objective					15. Use Simple Examples				
4. Concept/Skill Development					16. Assessment				
5. Concept Linkage within Discipline					17. Student Note-taking				
6. Concept Linkage outside Discipline					18. Vocabulary is Stressed				
7. Guided Practice					19. Reading and Writing				
8. Group Practice					20. Facts and Procedures				
9. Independent Practice					21. Technology Implementation				
10. Long-Term Memory Review					22. Problem Solving Process				
11. Closure					23. Memory Aids				

12. Homework Assignment					24. Questioning Strategies				

MATH OBSERVATION NOTES

COMPONENTS OF AN EFFECTIVE LESSON	TEACHER EXPECTANCIES
1. Introduction: What will be learned and why it is useful.	13. Reinforc Study Skills: Teach students how they learn (visual, audio, kinesthetic), concentration times, and how to study and how they learned (remembered) information.
2. Daily Review: Provide review for short-term memory over recently taught material. When correcting homework: provide immediate and meaningful feedback and hold students accountable. Keep reviews and homework checks brief.	14. Student/Teacher Relationships: Show your students mutual respect. Talk to your kids, be positive, and use humor to engage them into the lesson or activity!
3. Daily Objective (Specify skills/information that will be learned.) Write this information on the board and have the students record it in their notebooks.	15. Use Simple Examples: The teacher makes a conscience decision to start the development of skills using simple, straight-forward examples that clarify the concept or skill being taught which do not bog students down in arithmetic.
4. Concept/Skill Development: (Give and/or demonstrate necessary information) Teacher focuses on the big concepts. Utilize a variety of techniques: students need to see it, hear it, say it and do it.	16. Assessment: Continually assesses the progress of students through the lesson and adapts the lesson according to students' successes and difficulties.
5. Concept Linkage within Discipline: Link concepts and skills to previously learned material and outside experiences.	17. Student Note-taking: Teachers are very directive and prescriptive in how students take notes. Notes should be used to complete homework assignments and prepare for tests. There is accountability for taking notes and keeping mathematics notebooks.
6. Concept Linkage outside Discipline: Link concepts to real-work experiences.	18. Vocabulary is Stressed: Teachers are using direct instruction to teach vocabulary and there is evidence that the teacher is using and requiring the students to use appropriate vocabulary to describe the mathematics.
7. Guided Practice: Partially completed problems are given to students to check for student understanding. Can be done at different times throughout the lesson to help students' process information.	19. Reading and Writing: Teachers introduce new vocabulary, previews reading, connect reading to previous reading and checks student understanding of reading. Teachers also give students time to write about what they have learned at the end of the lesson. Reading and writing (including note-taking) are used during daily lessons.
8. Group Practice: Students need time to think, analyze, work on problems, discuss their solutions and become problem solvers instead of watching the teacher do all the work. Can be done as an entire lesson that enhances conceptual understanding and/or application of concepts through inquiry, investigation, discovery, lab or problem-solving activities.	20. Facts and Procedures: Provide the "why" for rules and algorithms.
9. Independent Practice: The majority of students who are given time to start their homework, worksheet, or seatwork can do so with out disturbing the teacher for clarification.	21. Technology Implementation: The teacher builds on the concept using technology through modeling or presentation.
10. Long-Term Memory Review:	22. Problem Solving Process:

Maintain skills, address deficiencies, and stress important ideas for the year. During this review teachers should be review typical questions that are used on the proficiency exam or other high stakes tests.	Students are lead to be problem solvers by the teacher modeling or prompting for a variety of ways to solve problems that do not have answers that are automatic
11. Closure: The new material from the lesson is summarized or wrapped-up for the students. An example is having students explain what they have learned and apply it.	23. Memory Aids: Teacher is directive and prescriptive in how students take notes in the class. The use of oral recitation was used to embed information in short-term memory.
12. Homework Assignment: The homework should include vocabulary and notation, procedures, and open-ended questions to check their understanding. The assignment sent home is clear and is manageable for a student to complete in a 20 – 30 minute timeframe.	24. Questioning Strategies: Several different types of questioning strategies were used including, but not limited to front loaded, rear loaded, higher level thinking, and effective use of "wait-time"

Chapter 8
Summing it up
There is no silver bullet

There's a lot more to education than meets the eye. For students to succeed, schools need to have very good instructional leaders as well as teachers knowledgeable in their content area who possess the skills to present that material in understandable terms to their students, teachers that can motivate their students while maintaining a positive learning environment.

There is no silver bullet, there is no program that can be purchased that will automatically result in increased student achievement. A good program put in the hands a poor administrator and less than average teachers will be doomed to fail. A poor program put in the hands of a good administrator and good, hardworking, dedicated teachers will be successful. What teachers need is a plan to address student achievement, a plan whose foundation is based on data and an understanding that *what works is work*.

Two standards were introduced in this discussion, the *Common Sense* standard and the *My Kid* standard. Along with those standards, there are two basic premises, the first is *testing drives instruction*. The testing meant to drive instruction is teacher made tests – not state or national testing. The belief is if teachers' tests are balanced and cover the curriculum set forth in the state's standards, then students should be able to perform well on any test administered by the state or federal government. The second premise is that *teachers make a difference, teachers working together will make a greater difference*. Teachers need an opportunity, as do site administrators, to talk to their colleagues about what they do, how they do it, their student performance, and the strategies, skills, and knowledge that need to be addressed based upon student performance - data.

Outside the parents, there is no one more important to students' education than their classroom teachers. Therefore training in content, pedagogy, and strategies to address all student populations becomes paramount in raising student achievement. It should go without saying that student achievement increases when students are enrolled in appropriate and challenging coursework, and also, when the course is more rigorous and requires higher order thinking skills. That, however, does not mean that math has to be hard!

A great deal of emphasis was placed on using linkage to introduce new topics to demonstrate that math does not have to be hard. How you teach math, how you introduce more abstract concepts and skills makes a difference in student motivation, comfort, understanding, and application. Linking allows teachers to introduce topics using familiar experiences and language which puts the students at ease, it provides teachers an opportunity to review and reinforce those concepts and skills – not affording the students an opportunity to forget, permits the students to see the material in different contexts, as well providing opportunities to compare and contrast that information, all of which, according to educational research, will result in increased student achievement. Linking is a very powerful tool in teaching mathematics that results in increased student understanding, comfort as well as increased student achievement in mathematics. Because of my very strong belief in the power of linking, I have added an appendix to this book with full examples of linkages.

To be successful in math, students need to learn the language of mathematics. Language acquisition is too often thought of in terms of non-English speaking students, it should be about all students. Classroom teachers need to explicitly teach vocabulary, provide regular opportunities for students to read, write and speak in mathematics.

Teaching is more than presenting material, then leaving. Building relationships with students is an important component in successful teaching. The research suggests that if students perceive their teachers know them and care for them, they will work harder for that teacher for no other reason than loyalty. That sense of caring translates to a sense of belonging that ultimately encourages students not only stay in school, but to perform to higher expectations and academic standards.

A great concern for educators is closing the achievement gap. A number of items were stressed in the teacher expectancies. One of the first was introducing new concepts or skills using simple straightforward examples that clarify what is being taught, that does not bog students down in arithmetic. In second language circles, that's referred to as "comprehensible input". We discussed "building background" by reviewing and linking new material to old. We discussed the importance of reading, writing, note taking, as well as oral recitation in terms of language acquisition. While some might believe these strategies should be used only for special populations, we need to remember, good teaching is good teaching. If those instructional strategies can be used successfully with one population, they can be used successfully for the general population as well.

Educational research has strongly indicated that one-shot professional development does not support changes in instructional behaviors that result in increased student achievement. For that reason we discussed structures that support professional development over time and focuses on what teachers do, how they do it, student performance, and changing strategies to increase student achievement based upon their performance. Those structures were the *Teacher Expectancies*, *Components of an Effective Lesson*, and the Backward Assessment Model (BAM).

The Components of an Effective Lesson incorporate the *teacher expectancies* with instructional strategies to facilitate student learning in the classroom. The components are a template that guide teachers in the delivery of daily instruction.

Success in mathematics is very dependent upon the acquisition of the language of mathematics. To acquire any language, repeated exposure to vocabulary and notation is a very necessary component for success. Teaching vocabulary explicitly, modeling it in class, giving students time to discuss the math they are learning using that vocabulary as well as having students read and write about the mathematics they are learning will greatly increase the probability that all students are successful in mathematics.

The Backward Assessment Model is another structure that supports ongoing professional development. The research indicates professional development should be primarily on-site, ongoing, and regularly scheduled. That it should be focused on the discipline teachers' teach, in content and pedagogy, and with teachers as active participants. BAM places the professional development emphasis on academic standards, student performance, and best practices. One of the most important attributes of BAM is it provides teachers an opportunity to share their knowledge, understanding, skills, experiences and resources with other teachers. It's clearly a communications model.

While educators face many obstacles in their endeavor to increase student achievement such as large classes, not enough books, the primary question that most parents really care to have answered is "*What are you doing to help my child learn?*" Your problems are *your* problems, not the parents, not the students', parents want you to help their child be successful.

Using data to develop and implement a department improvement plan can lead to teachers and administrators working together to attain the goal of increasing student achievement.

The pieces of this puzzle to increase student performance are on the table. Those pieces include addressing teacher content knowledge, instructional practices, classroom management, monitoring student learning, sequencing, benchmarking, frontloading, linking concepts and skills, utilizing practice tests, administering balanced tests using language used on standardized tests, be able to diagnose deficiencies, student teacher relationships, working with special populations, providing long and short term reviews, teaching the students how to learn effectively and efficiently just to name a few. Implementing some of these is not the solution. For instance, while using linkage will help introducing new concepts and skills, standing alone it won't do as much to increase student achievement without implementing other factors that address student achievement. If teachers and administrators choose not to use *all* the pieces of the puzzle available to them, then the picture simply can not be completed. These pieces should be part of the departmental improvement plan.

There are many people critical of public education. They propose simple solutions to a complex problem. Those simple solutions are a testament to their lack of understanding of the magnitude of the problems facing public education. While this text has provided a number of suggestions to increase student achievement, they are not meant to be picked from – but to be implemented collectively. As this text would suggest, there is no one thing that teachers can do that solves the student achievement problems in mathematics.

Goal Setting

The goals you set for yourself, your school will have an effect on how you and your teams work. Sports franchises have produced outstanding, average, and teams in need of improvement over long periods of time. Some teams' goal is to have a winning season, other teams might set their goal at making the playoffs, and there are teams whose goal is to be the best by winning the "Super Bowl".

My experience suggests that many educators go to work with the goal of just getting through the day or week. They work hard, but without a real goal. Schools that have been categorized by the state as low performing have probably set a goal of getting off that list and their school improvement plan mirrors and supports that goal.

I believe schools must have goals, while working hard is admirable, it often not enough. School administrators should work with their teachers to set goals – to set high expectations not only for their students, but for themselves as well. Educators in schools should strive to be the best! Not just the best in their district, not just the best in their county, not just the best in their state, but the best school in the nation. Once the goal is identified, a lot of discussion will result. Some of it from people who believe the goal is not attainable. The only thing we know for sure is if you don't strive for it, it will not be attainable. Just having the discussion will change the focus of the school. A plan put in place to be the best will cause a school to look a lot different than a school where teachers and administrators just go to work and work hard.

Appendix
Introducing Math by Linking

As we have discussed before, linking can be a very powerful tool in teaching mathematics. Unfortunately, its not used enough. In fact, because linking is not used, for many students math looks like a bunch of disconnected rules that don't make a lot of sense.

As I watch math being taught, I can almost predict that the Pythagorean Theorem will not be connected to finding areas of squares formed by the legs of a right triangle. What's really troubling is later in the year or the next year, the distance formula will be introduced and never linked to the Pythagorean Theorem. Students will not be told that the distance formula is the Pythagorean Theorem, just written differently because it is being used in a different context.

Later, as the students are studying the conic sections in algebra, they will be introduced to the equation of a circle. In all probability, the algebra teacher will not link that equation with the distance formula or the Pythagorean Theorem. And as the students progress into Trigonometry, will the $\cos^2 x + \sin^2 x = 1$. Most students will never know the four formulas are the same, just written differently to describe different situations. This really becomes a problem when teachers are expected to teach a very dense curriculum. If they introduce each new topic like it has no relationship to what has been taught before, then teachers spend more time teaching the new concept. From my perspective, that teacher is wasting valuable time.

By introducing concepts in familiar language, students are more comfortable and confident. Teachers have the opportunity to review and reinforce important concepts and skills, address long-term knowledge, compare and contrast that information, and see how the same math is used in different contexts.

In this chapter we will look at how concepts and skills can be introduced by linking them to previously learned material or outside experiences. It is important because the linking builds background for concepts and skills learned in a math class. It's important because it places the new topics in a context they have experienced that increases the probability the material will be comprehensible. This is important for lower performing students, it is very important for English language learners. It's important for all students to be successful.

While many algebra teachers might complain they can not teach algebra because of student deficiencies in arithmetic, the following lessons suggest these deficiencies can be addressed successfully while teaching algebra and making students more comfortable in their knowledge and understanding of basic mathematics.

I have watched many algebra teachers introduce functions by stating a definition, then doing problems. They'll write on the board; A function is a special relation in which no two ordered pairs have the same first element. What could be more clear to a student!? Rather than using the abstract, it might be better to try an approach that students might be more comfortable and familiar.

For instance, students typically have read a menu. Menus are typically written with a food item on the left side of the menu, the cost of the item on the other side as shown:

Hamburger ………….$3.50
Pizza..………………..2.00
Sandwich……………..4.00

Menus could have just as well been written horizontally;
Hamburger, $3.50, Pizza, 2.00, Sandwich, 4.00.

But that format (notation) is not as easy to read and could cause confusion. Someone might look at that and think a sandwich costs $2.00 rather then $4.00. To clarify that so no one gets confused, I might group the food item and its cost by placing parentheses around them:

(Hamburger, $3.50), (Pizza, 2.00), (Sandwich, 4.00)

Those groupings would be called ordered pairs, pairs because there are two items in each parenthesis. Ordered because food is listed first, cost is second.

We now have a relation. By definition; a relation is any set of ordered pairs.

Another example of a set of ordered pairs might describe buying cold drinks. If one cold drink costs $0.50, two drinks would be $1.00, three drinks would be $1.50. I could write those as ordered pairs – as a relation:
(1, .50), (2, 1.00), (3, 1.50), and so on

For most students, the idea of a relation is something they have experienced and are now just learning more formally in their algebra class. If I continued talking about buying soft drinks, students would expect the cost to increase by $0.50 for each additional drink.

Students might be asked to determine how much ten drinks would cost. Some students might just keeping adding fifty cents for each additional drink until they reached ten. Other students might quickly realize ten drinks cost $5.00. The teacher then might ask the students how they got that answer. After some prodding, students would come up with a rule cost = .50 x number or y = .50x) that would generate more ordered pairs. Those ordered pairs could also be graphed so students might be introduced to the "vertical line" test.

A teacher might then pose another question; What do you think might happen if one student went to the store and bought 4 drinks for $2.00 and his friend who was right behind him at the counter bought 4 drinks and only paid $1.75?

My guess is the students would think the first guy got cheated, that it was not right, or that this was not working. The teacher might conclude this was *not functioning*. That the first guy would expect anyone buying four drinks would pay $2.00 - just like he did!

Let's look at the ordered pairs that caused this problem.

(1, .50), (2, 1.00), (3, 1.50), (4, 2.00), (4, 1.75)

The last two ordered pairs in that relation highlight the malfunction, one person buying 4 drinks for $2.00, the next person buying the same 4 drinks for a $1.75.

For this to be fair or functioning correctly, we would expect that anyone buying four drinks would be charged $2.00. Or more generally, we would expect every person who bought the same number of drinks to be charged the same price. When that occurs, we'd think this is functioning correctly. So based on this situation and the ordered pairs, the teacher could define a function.

A function is a special relation (ordered pairs) in which no two different ordered pairs have the same first element.

Since the last set of ordered pairs have the same first elements (4,2.00) and (4,1.75), those ordered pairs would not be classified as a function because it resulted in somebody being cheated. One person paying $2.00 for four dinks, the other paying only $1.75.

The idea of relations and functions if taught abstractly can cause students to have high anxiety in mathematics. But, by introducing those ideas in a context they have used in their daily experiences and probably not thought of as mathematics, students can readily see the connection between math and the real world. Not to mention, making math a lot easier along the way.

Most of us would see the glaze coming over students' faces if a function was first introduced by the abstract definition.

From this example, it can be clearly seen teachers make a difference in student achievement. Algebra does not have to be hard, if connected to what students already know and understand, algebra can often further clarify student understanding of mathematics. Teachers' content knowledge, instructional strategies, and decision-making effects student achievement. Connecting algebra to outside experiences also can make math look a lot more interesting. Too often students ask when will they ever use the math being taught in school. And too often, teachers respond by saying "on the test on Friday."

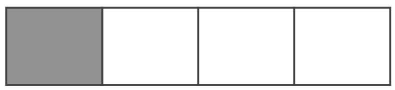
Let’s look at an another example of introducing a skill by linking. This time, rather than linking algebra to an outside experience, we’ll link it to a previously taught math skill.

Over the years, I have heard many algebra teachers comment, how can I teach these students to add or subtract rational expressions if they can not add or subtract fractions? Well, let’s see if we can find a solution – that’s what mathematicians do.

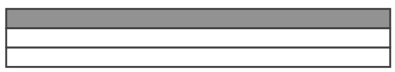
The teacher might want to review and reinforce student understanding of combining fractions by drawing a model.

Let’s draw a picture to represent this:

$\frac{1}{4}$



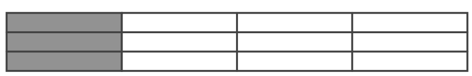
$+\frac{1}{3}$



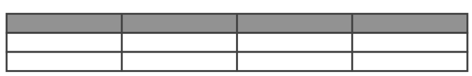
Notice the pieces are not the same size. So saying you have two pieces doesn’t describe very much.

Making the same cuts in each cake will result in equally sized pieces. That will allow me to add the pieces together. Each cake now has 12 equally sized pieces. Mathematically, we say that 12 is the common denominator. Now let’s add the numerators.

$\frac{1}{4} = \frac{3}{12}$



$+\frac{1}{3} = \frac{4}{12}$



$\frac{7}{12}$

From the picture we can see that 1/3 is the same as 4/12 and $1/4$ has the same value as 3/12. Adding the numerators, a total of 7 equally sized pieces are shaded and 12 pieces make one unit.

By doing a few more examples and examining the corresponding pictures, we might be able to come up with a procedure for adding fractions without drawing the picture.

The procedure to add and subtract fractions would look like this:

1. Find a common denominator
2. Make equivalent fractions

3. Add/subtract the numerators
4. Bring down the denominator
5. Reduce

That algorithm allows students to compute very quickly.

Now, if I wrote a number of problems with just their answers, we might be able to see a pattern develop that will allow us to add fractions mentally. Look at the following addition problems and their respective answers, see if you can identify a pattern.

$$\frac{1}{5}+\frac{1}{2}=\frac{7}{10} \qquad \frac{1}{2}+\frac{1}{3}=\frac{5}{6} \qquad \frac{1}{3}+\frac{1}{5}=\frac{8}{15}$$

Look at the numbers in the problems, look at the answers. See anything interesting? Use that pattern to add $\frac{1}{3}+\frac{1}{4}$ in your head. It appears that you are adding the numerators, then adding the denominators so the answer is 7/12.

Notice, I used a pattern that was fairly easy to see, a pattern that clarified what I wanted students to see that did not bog them down in arithmetic. A pattern in which most students would experience success. Remember the success on success model.

All of those fractions had a numerator of 1, students might ask what happens if the numerators are not 1. Have the students look for a pattern in these problems.

$$\frac{1}{5}+\frac{2}{7}=\frac{17}{35} \qquad \frac{2}{9}+\frac{1}{10}=\frac{29}{90} \qquad \frac{2}{5}+\frac{4}{7}=\frac{34}{35}$$

From these examples, its easy to see where the denominator comes from, a teacher would need to ask their students to manipulate the numbers in the problem that would suggest where the numerator is coming from? It won't just jump out at them, they have to be given some time to play with those numbers. The numbers I have chosen for these examples are important! They are common enough that students will concentrate on finding the relationship (pattern) that I am looking for – not bogging them down in arithmetic.

After they played long enough some students could tell you the common denominator was found by just multiplying the denominators. On the other hand, finding the numerators was not as obvious. The numerator was obtained by multiplying the addends diagonally, then adding those products.

$$\frac{2}{9}+\frac{1}{10}=\frac{29}{90}$$

Another example $\frac{3}{5}+\frac{2}{7} \rightarrow \frac{3\times 7+5\times 2}{5\times 7}=\frac{31}{35}$

After doing a few of those problems, students might be asked to generalize that pattern.

Generalizing that, we have $\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$

That pattern or formula would allow me to add or subtract fractions very quickly, they would be able to add and subtract rational expressions using the same generalization.

$$\frac{2}{x}+\frac{3}{y}=\frac{2y+3x}{xy} \qquad \text{or} \qquad \frac{2}{x-1}+\frac{3}{x+2}=\frac{2(x-2)+3(x-1)}{(x-1)(x+2)}$$

By reviewing and reinforcing computation with fractions, students see the pattern develop. Seeing that link, they would be very happy to learn that formula because it would allow them to add fractions in their head with a little practice and add rational expressions in algebra. Unfortunately, many students never see these patterns develop, never told of the connection between algebra and anything else they have learned in mathematics.

For those that are only told the shortcut, they view them as tricks. Those shortcuts, those tricks are what mathematicians call theorems, corollaries, rules, and axioms, if students see them develop, it won't seem like math is magical.

Let's look at a link in basic mathematics. Addition and subtraction of decimals is often taught by algorithm. Students are told to line up the decimal points, fill in zeros, add the numbers, and bring the decimal point straight down. Teachers do this because students tend to have a lot of success adding and subtracting decimals by algorithm. Teachers see little advantage in taking time to explain the concept when the kids have attained the desired skill. While I have no problem having students memorize that procedure, if that procedure was not linked to addition and subtraction of fractions, then teachers lost an opportunity to review, reinforce, compare, contrast, and teach fractions in a different context – all of which leads to increased student understanding and achievement.

Teachers should take time to explain to students that when they lined up the decimal points and filled in zeros, they were finding a common denominator and making equivalent fractions. That when they added the numbers, they were adding the numerators in fractions, and when they brought down the decimal point, that was synonymous to bringing down the denominator when adding fractions.

Creating that link does not take a lot of time, but it does increase the students' awareness of the connections in mathematics, it does increase their understanding, and will result in increased student achievement.

It’s not just the content that teachers teach, how they teach that content affects student understanding and achievement.