SETS

The idea of sets shouldn't be anything new to you. You have probably been classifying things all of your life. We might talk about all redheads with blue eyes or list all the states in the United States. Those are both examples of sets. Another example of a set is the people going to a particular school

Mathematically, we define a set to be a **well-defined collection objects**. The objects are often referred to as members or elements of the set.

Braces are used to enclose the elements of a set and we label the set with a capital letter. The set of single digit numbers can be written as:

$$A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

The order in which the elements are listed does not matter.

We use the symbol $\in$ to show an element belongs to a set. For example $2 \in A$. To show an element does not belong to a set, we use the same symbol with a line through it; $\notin$. Piece of cake, don't you think?

Sometimes making a complete listing of the elements of a set might not be warranted because of the sheer number of elements. In that case, we develop more notation. In those cases we use something called **set builder notation**. Using set builder notation to describe single digit numbers would look like this:

$$A = \{x \,/\, x \text{ is single digit number}\}$$

The way you say that is "A" is the set of all elements *x* such that *x* is a single digit number. The first "*x*" inside the brackets just identifies the variable be used to describe the elements. The "/" is read "such that". Try saying this one.

$$B = \{x \,/\, x \text{ is a prime number less than ten}\}$$

The way you'd say that is "B is the set of all *x*'s **such that** *x* is a prime number less than ten." Or I could have chosen to list them.

$$B = \{2, 3, 5, 7\}$$

As you know, one is not a prime number

A set that has no elements is called the empty set or null set. We use the symbol $\varnothing$ or brackets with nothing in them { } to indicate that.

An example of an empty set would be all the states that begin with the letter z.

The universal set is the set that contains all the elements being considered in a discussion. The universal set is denoted by **U**. It's important that you recognize the universal set in any given problem.

Let's say we are discussing the letters a, b, and c. The universal set could be all the letters in the alphabet So

$$U = \left\{ x \,/\, \text{is a letter in the alphabet} \right\}$$

Now, if the only letters I am actually discussing are a, b, and c, then I could describe those letters in set notation.

$$T = \left\{ a, b, c \right\}$$

I picked "T" because I didn't want coffee. That's a joke. Ok, what happens if I want to discuss the letters that are not elements of T? Well, you know that would be the letters d through z. When you want to talk about members or elements not in a particular set, we call that the complement of the set. There are a number of ways to show this symbolically; $\sim T, T', \text{or } T.$

I'm going to use the $\sim T$ notation. Now back to set T, {a,b,c}. If we want to talk about the letters not in T, we write

$$\sim T = \left\{ x \,/\, x \in U \text{ and } x \notin T \right\}$$

So the **complement** is made up of all the members in the universal set that are not members of T.

I know, you are thinking learning all this notation is pretty cool, so we'll show you some more.

Let's say we are talking about the numbers one through ten. We could call that our universal set $U = \left\{ 1,2,3,4,5,6,7,8,9,10 \right\}$ and let's say we then look at just the odd numbers less than ten, call it **A**.

$A = \{1,3,5,7,9\}$. We might also want to discuss another set, the prime numbers greater than two, but less than ten, call that **B**. $B = \{3,5,7\}$. Notice that all the elements of B are contained in A. Then B is called a subset of A, written $B \subseteq A$. So now we have another definition, a subset, **B is a subset of A if and only if all the elements in B are also elements of A.**

If B is a subset of A and B is not equal to A, then B is a proper subset of A, written $B \subset A$. In other words, B would have to have fewer elements.

Let's say I had another set $G = \{0,5,7\}$, would G be considered a subset of A? Are all the members of G also members of A? We can see that "0" is not in A, therefore it would not be a subset. So we write $G \not\subset A$.

I need you to stay with me, subsets and elements of a set can be confusing for the novice. If we want to talk membership, or an element belonging to a set, we use $\in$.

So we could say that $3 \in A$, 3 is a member of A. Since three is not a set, we can **NOT** substitute $\subset$ for $\in$. However, if we are talking about 3 as a set, we could write $\{3\} \subseteq A$.

In other words, if we are talking sets we have to use either capital letters or $\subset$. If we are just talking membership, we use $\in$ and don't put brackets around the element. For example, $3 \in A$.

Now, the question is, what differentiates the two, membership versus being a set? Membership means you just belong, to be considered a set, it's more than just belonging, if something is to be considered a set, it must be well defined, it must be distinguishable.

That's important, so read it again.

Now that we have the basic definitions and notation out of the way, we can play.

Set Intersection

We'll start with set intersection. There's no need to get worried, you have probably used the concept of set intersection many times in your life, we're just going to describe it mathematically now. Excited?

Let's say a college representative wants to send information to all students that are enrolled in calculus and physics. To do this, they would have to identify students taking both classes. That group of students would be identified as the intersection of the two sets – kids taking calculus, C and the other set, kids taking physics, P.

Mathematically, we'd write the intersection of two sets C and P, written $C \cap P$, is the set of all members common to both C and P.

$$C \cap P = \{x \,/\, x \in C \text{ and } x \in P\}$$

Students not enrolled in both classes would not be in the intersection.

The key in the definition of intersection is the word **and.** In everyday language, as it is in math, and means both conditions must be met.

If sets such as C and P have no members in common, they are called **disjoint** sets. In other words, their intersection is the null set, $C \cap P = \varnothing$

Let's look at an example.

EXAMPLE Let U= {letters in the alphabet}

M = { q, r, s, t, z}, N = { s, p, o, t}

Find M $\cap$ N.

What members are common to both sets? Or another way of asking, which letters are in both set M and set N? The intersection is {s,t}

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Set Union

Other things that happen in our life might be described as the set union. Going back to the two sets, C is the set of kids taking calculus, P is the set taking physics. If we would like to contact students taking calculus or physics, then we would be talking about the set union. That's the kids taking calculus or physics or both calculus and physics..

$$C \cup P = \{x \,/\, x \in C \text{ or } x \in P\}$$

The key word in the definition is *or*. In math, *or* means "one or the other or both". That's called the inclusive *or.* Many of us use the exclusive *or* at home, which means to say one or the other but not both. So *or* in math is used differently than the *or* at home.

Let's look at a couple of examples. The first example will be set intersection.

Example Find $A \cap B$, if $A = \{d, e, f, g\}$ and $B = \{d, o, g\}$

Since we want to find the intersection, we are looking for elements common to both. So looking at the sets, which elements, if any, belong to both sets A and B?

Hopefully, you noticed *d* and *g* are in both sets, therefore

$$A \cap B = \{d, g\}$$

That's just too easy! Let's look at the union of the sets A and B.

Example Find $A \cup B$, if $A = \{d, e, f, g\}$ and $B = \{d, o, g\}$

To find the union we are looking for elements that belong to either set. That would include elements that belong to both sets. What that means is we join the two sets together, therefore

$$A \cup B = \{d, e, f, g, o\}$$

In the set union, any element that belongs to either set gets to join the set union. But in this club, these members (elements) only get one vote. So notice, I did not repeat the elements *d* and *g* in the set union.

Set Difference

Another operation with sets is called the set difference. So let's go back to the calculus and physics students we described earlier by the sets C and P. If we wanted to talk about the set of students taking calculus but not physics, we'd write **C – P**.

Let's see what that would look like if we described these members in set notation.

If $C = \{a, b, c, d, e\}$ and $P = \{b, d, f, g, h\}$, then we'd only want the people taking calculus that are not enrolled in physics.

Let's look at one student at a time, *a* is in calculus, but not physics. So *a* works out.

b is in calculus, but he's in physics also, so *b* won't belong.

c is in calculus, but not in physics, so *c* is OK. How about *d*? *d* is in calculus, but he's also in physics, so *d* won't belong.

e is in calculus, he's not in physics, so e is OK to join the set difference.

Another way to look at that set difference to write the members of the calculus class, and tell anyone in that class if they are also taking physics, they have to leave

C—P = {a,c,e)

I know what you are thinking, you want to try some of these on your own.

If $U = \{a,b,c,d,e,f,g\}$, $A = \{d,e,f\}$, and $B = \{a,b,c,d,e\}$, find each of the following.

1. $A \cup B$
2. $A \cap B$
3. $A - B$
4. $B - A$
5. $\sim A$
6. $.\sim (A \cap B)$

That was fun! Let's see how you did.

$A \cup B = \{a,b,c,d,e,f\}$. The union joins the two sets together.

$A \cap B = \{d,e\}$. The intersection uses only members that belong to both sets.

$A - B = \{f\}$. The set difference takes all the members of A sends members that belong to B home.

$B - A = \{a,b,c\}$. The set difference takes all the members of B and sends members that belong to A home. Just like a subtraction problem, that's probably why they call it the set difference.

$\sim A = \{a,b,c,g\}$ The complement of A means all the members of the universal set except members that belong to A.

$\sim (A \cap B) = \{a,b,c,f,g\}$. The complement of the intersection of A and B means everyone in the universal set, except those that belong to the intersection. We already found $A \cap B = \{d,e\}$, the complement is the set that contains all the members of the universal set, except members *d* and *e*.

Those were just too easy. Would $A \cup B = B \cup A$? How about $A \cap B = B \cap A$?

It turns out the answer to those questions is yes. So it appears we have a communtativity property for set union and set intersection.

Does $A - B = B - A$? Going back to problems we just did and looking at the answers, we can see that is not true. The set difference is not commutative.

Can I make these problems more difficult? Absolutely not, I can make them longer by having more sets, but I can't make them harder.

Let me prove that, I'll make a problem longer – but as you will see, not more difficult!

Example

Let U = $\{m,a,v,e,r,i,c,k\}$, $A=\{r,i,c,k\}$ $B=\{v,e,r,i\}$ and we'll make a third set C. $C=\{m,i,r,e\}$

Find $A \cup B \cap C$

Well, just like in arithmetic, when we don't have parentheses we work from left to right. So, let's find $A \cup B$.

$$A \cup B = \{r,i,c,k,v,e\}$$

Now let's intersect that with C.

$\{r,i,c,k,v,e\} \cap \{m,i,r,e\} = \{r,i,e\}$, therefore

$$A \cup B \cap C = \{r,i,e\}$$

That could have been written with parenthesis; $(A \cup B) \cap C$

Let's change that problem by moving the parenthesis and see if the answer is different..

Example

Fine $A \cup (B \cap C)$.

Just like in arithmetic, we perform the operation in parenthesis first.

$$B \cap C = \{i,r,e\}$$

Now let's join that with A.

$\{r,i,c,k\} \cup \{i,r,e\} = \{r,i,c,k,e\}$, therefore

$$A \cup (B \cap C) = \{r,i,c,k,e\}$$

That wasn't hard. All we had to do is perform the first operation, then perform the second operation using that answer.