PERCENTS

Percents are special fractions whose denominators are 100. The number in front of the percent symbol (%) is the numerator. The denominator is not written, but understood to be 100.

Examples $6\% = \frac{6}{100}$ $14\% = \frac{14}{100}$ $87\% = \frac{87}{100}$

Because a percent is a special fraction, then, just like with decimals, all the rules for percents come from the rules for fractions. That should make you feel pretty good. It's not like we are learning brand new stuff you're not familiar with.

Let's take a quick look. To add or subtract percents, you add the numerators and bring down the denominator just like in fractions.

Adding & Subtracting Percents

Example: $34\% + 15\% = 49\%$

Notice I added the numbers in front of the percent symbol, the numerators, then I brought down the common denominator, the percent symbol.

Oh, yes, this is really, really, really good stuff. Don't you wish that - just sometimes - you could make math difficult. As long as you see the patterns develop and you know your definitions and algorithms, math is just plain easy.

Multiplying Percents

If I wanted to multiply percents, again I would go back to my rules for multiplying fractions. To multiply fractions, you multiplied the numerators, then the denominators. To multiply percents, you do the same thing. Multiply the numerators, then the denominators.

Examples $5\% \ x \ 12\%$

Multiplying the numerators, $5x12=60$. Remember, the denominators are not written. They are defined to be 100. Therefore we multiply $100 \ x100,$ that equals 10,000.

$$5\%x12\% \rightarrow \frac{5}{100}x\frac{12}{100}=\frac{60}{10{,}000}$$

Converting Percents to Fractions and Decimals

To convert a percent to a fraction, we just use the definition. The number in front of the percent symbol is the numerator, the denominator is 100, then simplify.

Example Convert 53% to a fraction
53/100 Too easy, right?

What if someone asked you to convert percents to decimals, would you do it the same way? Of course.

Example Convert 53% to a decimal

$\frac{53}{100}$, but that's a fraction.

How do you divide by 100? Move the decimal point 2 places to the left. So, $53\% = .53$.

If we did enough of these, we'd soon realize to convert a percent to a decimal, you move the decimal point 2 places to the left.

Example Convert 3% to a decimal.

Moving the decimal point 2 places to the left, we have .03.

Knowing that you convert a percent to a decimal by moving the decimal point 2 places to the left, how would you convert a decimal to a percent? That's right, you'd do just the opposite, move the decimal 2 places to the right and put the percent symbol at the end.

Example Convert .34 to a percent.

Move the decimal point 2 places to the right and put a percent symbol at the end. The answer is 34%.

That's just too easy.

Now, why are we moving the decimal point 2 places? Because the denominator for a percent is 100, two zeros, and we learned shortcuts for multiplying and dividing by powers of 10.

$$.34 = \frac{34}{100} \rightarrow 34\%$$

When you are first learning these problems and trying to apply shortcuts, remember we call them rules, sometimes we get them confused. So here's a hint that might help you remember.

To convert a decimal, the loop on the "d" in decimal opens to the left, so move the decimal point to the left 2 places.

To convert to a percent, the loop on the "p" in percent opens to the right, so move the decimal point to the 2 places.

Again, those two hints came from patterns we recognized.

Example Convert 63% to a decimal.

The loop on the "d" opens left, move the decimal point 2 places in that direction. The answer is .63.

That's the shortcut, the reason why that works is because 63% means 63/100. Simplifying 63/100 in decimal form is .63

Example Convert .427 to a percent.

The loop on the "p" opens to the right, move the decimal point 2 places in that direction. The answer is 42.7%.

That's the shortcut that allows you to compute the answer quickly. But, shortcuts are soon forgotten, it's important that you understand why the shortcut works.

Let's see what that would look like if we did not use the shortcut.

$$.427 = \frac{427}{1000}$$

To convert that to a percent, I have to rewrite that fraction with a denominator of 100.

$$\frac{427}{1000} = \frac{42.7}{100} \rightarrow 42.7\%$$

Once nice thing about mathematics is the rules don't change. Problems might look a little different, but they are often done the same way. The first example we discussed was converting 6% to a fraction. We said the number in front of the percent symbol was the numerator, the denominator was 100.

$6\% = \frac{6}{100}$ Simplifying, we'd reduce the answer would be $\frac{3}{50}$

What if I asked you to convert $\frac{1}{4}\%$ to a fraction, could you do it? Of course you could. You would do exactly what you did to convert 6% to a fraction. The numerator is the number in front of the percent symbol, the denominator is 100.

By converting $\frac{1}{4}\%$ to a fraction by the definition of percent, we have

$$\frac{1}{4}\% = \frac{1/4}{100}$$

Simplifying that complex fraction, I'd invert and multiply, then reduce.

$$\frac{1/4}{100} \rightarrow \frac{1}{4} \div 100$$

$$= \frac{1}{4} x \frac{1}{100}$$

$$= \frac{1}{400}$$

Notice, the problems looked different, but we used the same strategy, put the numerator over 100 and simplified. Piece of cake! If you simplified a number of fractional percents, you'd probably see a nice pattern develop that would allow you to simplify them in your head.

Let's try a few.

Convert to fractions.

1.	83%	2.	9%	3.	520%	4.	30%
5.	45%	6.	$\frac{2}{3}\%$	7.	.4%	8.	3.5%

Convert to decimals

9.	65%	10.	7%	11.	324%	12.	.43%
13.	$\frac{1}{2}\%$	14.	8.3%	15.	$\frac{2}{5}\%$	16.	$8\frac{1}{4}\%$

Convert to percents

17.	$1/2$	18.	.23	19.	$\frac{3}{4}$	20.	8.6

Percent Proportion

For many of us, a percent is nothing more than a way of interpreting information. We have worked with percents since grade school. In reality, all we are doing is looking at information in terms of a ratio, then rewriting the ratio so the denominator is 100.

For instance, let's say you got 8 correct out of 10 problems on your quiz. To determine your grade, your teacher would typically want to know how well you have performed if there were 100 questions.

In other words, they would set up a proportion like this.

$$\frac{\#\,correct}{total} = \frac{?}{100}$$

Filling in the numbers, I have $\frac{8}{10} = \frac{\quad}{100} \rightarrow \frac{8}{10} = \frac{80}{100}$

Getting 8 out of 10, I'd expect to get 80 out of 100

Notice the right side is a fraction whose denominator is 100. Just as we defined a percent.

Example Let's say you made 23 out of 25 free throws playing basketball. I might wonder how many shots I would expect to make at that rate if I tried 100 shots.

Again, I have a ratio

$$\frac{\text{attempts}}{\text{total}} = \frac{\quad}{100}$$

$$\frac{23}{25} = \frac{\quad}{100} \rightarrow \frac{23}{25} = \frac{92}{100}$$

Now I could solve that by making equivalent fractions or by cross-multiplying. Either way, the missing numerator is 92. I would expect to make 92 free throws out of 100 tries.

These problems are just like the ratio and proportion problems we have done before. The only difference is the denominator on the right side is 100 because we are working with percents.

A proportion that always has the denominator of the right side as 100 is called the **Percent Proportion.**

Percent Proportion

$$\frac{\text{part}}{\text{total}} = \frac{\%}{100}$$

Remembering that you have to describe the ratios the same way on each side of a proportion, we might think this should read.

$$\frac{\text{part}}{\text{total}} = \frac{\text{part}}{\text{total}}$$

Well, the percent ratio actually does compare parts to total on both sides. For a percent, the total is always 100 and the percent is always the part you got.

The point I want to make is we have consistency with the math we have already learned. Now, the real good news. We can use the percent proportion to solve just about any problem involving percents. So, memorize it!

$$\frac{\text{part}}{\text{total}} = \frac{\%}{100}$$

Speaking mathematically, the 100 always goes on the bottom right side. That's a constant. The only things that can change is the part, total or percent. You get that information by reading the problem and placing the numbers in the correct spot, then solve.

There are only 3 different problems, we can look for a part, a total or a percent. Let's go for it.

Example 1 Bob got 17 correct on his history exam that had 20 questions. What percent grade did he receive?

$\frac{\text{part}}{\text{total}} = \frac{\%}{100}$ filling in the numbers, $\frac{17}{20} = \frac{\quad}{100}$

Solving, either by equivalent fractions or by cross-multiplying, we find he made an 85%. In this problem we found a percent.

Example 2 A company bought a used typewriter for $350, which was 80% of the original cost. What was the original cost?

Now does the $350 represent the total or part? $\frac{350}{n} = \frac{80}{100}$

Cross multiplying, we have $80n = 350x100.$ Solving, $\begin{array}{c} 80n = 35{,}000 \\ n = 437.5 \end{array}$

The original cost of the typewriter is $437.50. In this problem we found the total.

Example 3 If a real estate broker receives 4% commission on an $80,000 sale, how much would he receive?

Is the $80,000 represent the part or total?

$$\frac{n}{80,000} = \frac{4}{100}$$

$$100n = 4x80,00$$

Solving, $100n = 320,000$

$$n = 3,200$$

He would receive $3,200 in commission. Here, we found then part.

While the first three examples were all percent problems and we used the percent proportion to solve them, in each case we were looking for something different. That's the beauty of the percent proportion.

In this next example, everything we learned stays the same, but there is a slight variation in how the problem is written. To do this problem, you must understand how proportion problems are set up.

Example 4 Dad purchased a radio that was marked down 20% for $68.00. What was the original cost of the radio!

Now I need you to stay with me. Setting up the proportion, does $68 represent the part or total?

Filling in the proportion,

$$\frac{\text{paid}}{\text{total}} = \frac{\%}{100}$$

This is very, very important, the $68 represents the part you paid, what does the 20% represent? That's the part you got off.

We can not have a proportion with **paid is to total** as **amount off is to total**. If Dad received 20% off, we have to have the same ratio on both sides. That is paid to total as paid to total. If he got 20% off, what percent did he pay? 80%

Now, filling in the numbers, we have $\frac{68}{n} = \frac{80}{100}$

Solving, we have

$$80n = 6,800$$

$$n = 85$$

We were able to solve 3 different type problems using the Percent Proportion. We solved for the part, total, and percent by using what we learned in ratios and proportions earlier.

Now, it's your turn to do some.

1. On last week's math test Carol had 21 correct out of 25 problems. What percent grade did she earn?

2. Bob received an 85% on his history exam. If there were 20 questions, how many did he get correct?
3. Juan receives a 5% commission on his sales. If he received $30, how much merchandise did he sell?

4. Ted got an 84% on his science test, if there were 50 questions, how many did he get wrong?

5. If you receive 30% off on a pair of slacks that cost $25, how much would you change should you receive if you gave the clerk a $20 bill?

6. Jessie earns $250 per week. If 18% is deducted for federal tax and 8% for social security, what is his net income?
7. A radio costs $20, Harold buys it for $16, what percent off the original price did he receive?

8. The Pep Club was decreased from 15 members to 12, what percent decrease was there in the club?

9. Find the 15% tip for a restaurant bill of $42.

10. A store drops the price of a radio 24% to a sales price of $36.48. What was the original price of the radio?

11. After a person receives a 20% raise, his salary is $9,600. What was his old salary?

12. James spends 12 hours per week studying. He spends 3 of those hours studying math. What percent of his study time is spent on math?

13. The Johnson's purchased their home for $160,000. Two years later, the house had increased in value by 15%. How much is the house worth after two years?

14. The membership at a certain church was 450 members five years ago. Today, the membership is 525 members. What percent increase is this?

15. In a recent survey, people were asked if they had a digital camera at home. Six people had digital cameras, this was 30% of the total number of people surveyed. How many people were surveyed?

16. Last week the hotel occupancy rate was 75%. If 600 rooms were rented, how many rooms were still available?

17. Bonnie used 4 gallons of paint-to-paint 80% of her bedroom. How many gallons of paint will it take to paint the entire room?

18. A $30 shirt was on sale for 25% off. What was the sales price of the shirt?

19. On a test with 30 questions, Bill answered 80% correctly. How many answers did he get wrong?

20. Tom bought a radio on sale for 20% off for $68. What was the original cost of the radio?

PERCENT PROBLEMS

1. Find the 7.5% sales tax on the purchase of a sports coat costing $150.

2. Find the 6% down payment on the purchase of a house costing $120,000.

3. 54 is 12% of a certain number. Find the number.

4. A stockbroker sold $350,000 worth of stocks and bonds during the month of October. If her commission rate is 2%, how much money did she earn in commissions?

5. What is the tax rate in Nevada, if a tax of $19.50 is charged on a television set costing $300?

6. Mr. Jones spent $4,800 for a used car. This amount was 20% of his annual income. What was his annual income?

7. What is the sales tax on a dress selling for $70, if the tax rate is 6%. What is the total cost of the dress?

8. Ms. Smith gets a 14% raise in salary, if her original salary is $28,000 per year what is her new salary?

9. A store drops the price of a certain type of radios 24% to a sale price of $36.48. What was the original price of the radios?

10. Anne borrows $1,200 to fix up her home. If the money is borrowed at 7% simple interest, how much does Anne have to pay back at the end of one year?

11. Find the 15% tip for a restaurant bill of $42.

12. The sale price of a used car is $1,837. Find the list price if the discount rate is 16.5%.

13. A pro shop discounted all merchandise in the store by 40%. What are the discount and the sale price on a tennis racket regularly priced at $135.50?

14. Jane was paid a commission of $287 on sales of $5,740. What was her commission rate?

15. The tax rate in Indiana is 5% If the sales tax charged on the purchase of a coat is $13.75, how much does the coat sell for?