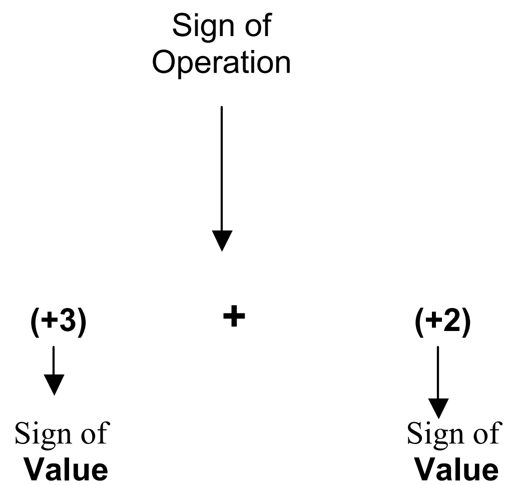
INTEGERS

Integers are positive and negative whole numbers, that is they are;
$\{\ldots -3,-2,-1,0,1,2,3\ldots\}$. The dots mean they continue in that pattern.

Like all number sets, integers were invented to describe things that happen in our environment.

When we work with signed numbers, we are often working with two different signs that look exactly alike. They are signs of value and signs of operation. A sign of value tells you if the number you are working with is greater than zero (positive) or less than zero (negative). Signs of operation tell you to add, subtract, multiply or divide.

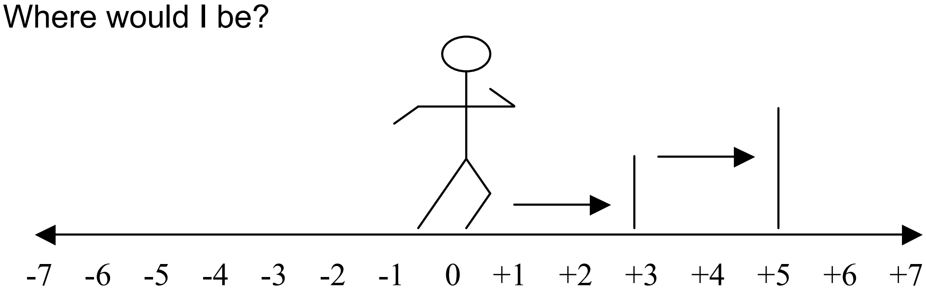
Example



Notice that the signs of value and the sign of operation are identical.

Adding Integers

One way of explaining integers is with a number line. Let's say I was standing on zero and I walked three spaces to the right, then walked two more spaces to the right. Where would I be?



You've got it. You'd be 5 spaces to the right. Piece of cake, you're thinking.

Now let's see that same example incorporating mathematical notation. Let's agree that walking to the right is positive, walking to the left will be negative. Easy enough.

So 3 spaces to the right could be labeled, 3R or +3
2 spaces to the right could be labeled, 2 R or + 2.
Now we said we'd end up 5 spaces to the right, 5R, or +5.

Now let's define walking mathematically, we'll agree to define that as addition.

Translating that problem of walking to the right, then walking further to the right mathematically, we have

$$3R \ + \ 2R \ = \ 5R$$
$$(+3) + (+2) = +5$$

The sign of operation tells you to walk, the sign of value tells you which direction.

Guess what happens next, that's right we make a rule that will allow us to do problems like this without drawing a picture.

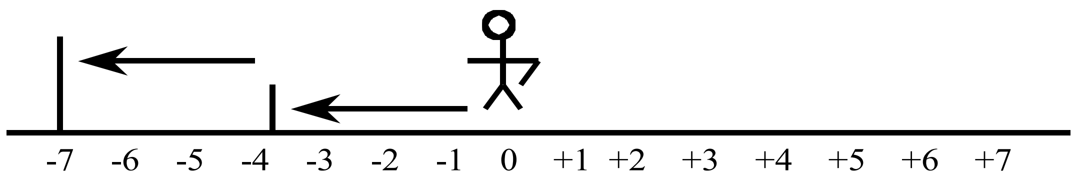
Rule 1. ***When adding two positive numbers, find the sum of their absolute values, the answer is positive.***

Example $8 \ + \ (+9) = +17$

From the above example, notice that the 8 does not have a sign of value. We will now agree that when a number does not have a sign of value, it is understood to be positive.

Using those same agreements we just made, walking is still defined by addition, going right is positive, going left is negative.

Let's see what happens when we walk 4 steps to the left from zero, then 3 steps to the left. Where will we end up? If you said we'd up 7 spaces to the left, we're in good shape.



Mathematically, we'd express that like this

$$4L + 3L = 7L$$
$$(-4) + (-3) = -7$$

This idea of walking around the number line is pretty cooool! Of course, now what we do is generalize this and make that into a rule so we don't have to always draw a picture.

Rule 2. ***When adding two negative numbers, find the sum of their absolute values, the answer is negative.***

Example $(-5) + (-6) = -11$

Try a couple for yourself

1. $(-12) + (-8)$

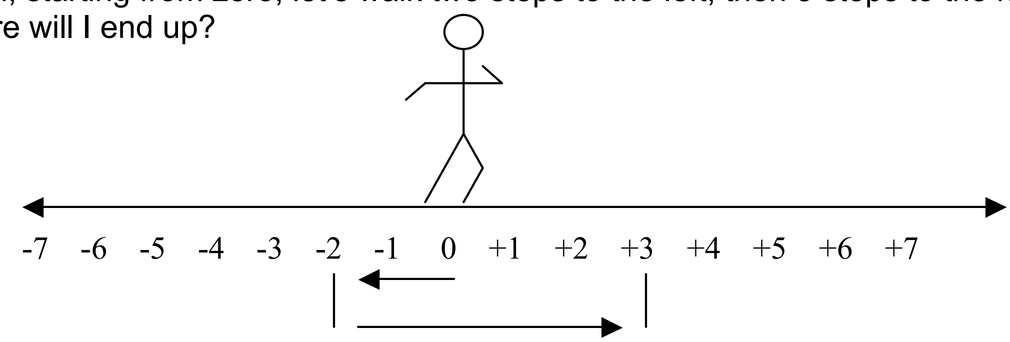
2. $(-9) + (-6)$

Some real good news, the rules we are developing also work for other number sets like fractions or decimals.

3. $\left(-\frac{1}{4}\right) + \left(-\frac{3}{5}\right)$

Yes, I know you love to walk, so let's take another walk on the number line. This time we are going to walk in different directions.

Again, starting from zero, let's walk two steps to the left, then 5 steps to the right. Where will I end up?



Hopefully, by using the number line, you see that we'll end up 3 spaces to the right.

Mathematically, changing $2L + 5R = 3R$

to $(-2) + (+5) = +3$

Let's do another one, this time starting out walking 4 to the right, then going 9 to the left.

Again , using the number line, where should we end up? If you said 5 to the left, you are making my life too easy.

Mathematically, we'll change $4R + 9L = 5L$

to $(+4) + (-9) = -5$

Now if we play with the two preceding examples long enough, we'll come up with a rule (shortcut) that will allow us to do these problems without drawing a number line.

Rule 3. ***When adding one positive and negative number, find the difference between their absolute values and use the sign of the Integer with the greater absolute value***

***Examples*:** $(-12) + (+8) = -4$

$7 + (-5) = +2$

Simplify

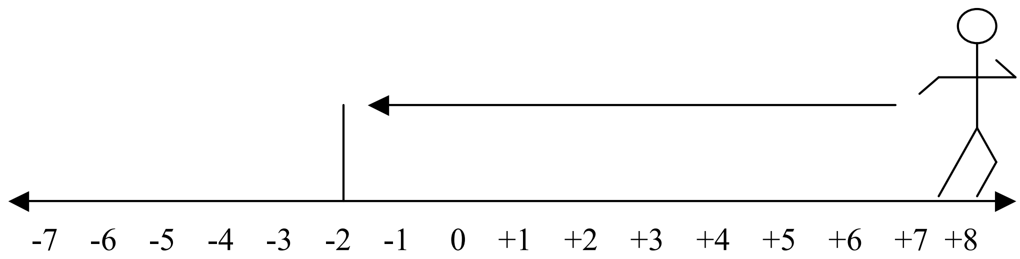
$(+7)+(+3)$	$(+8)+(+5)$	$(+2)+(+4)$
$(-7)+(-3)$	$(-8)+(-5)$	$(-2)+(-4)$
$(-7)+(+9)$	$(+8)+(-11)$	$(-5)+(+2)$
$+8+6$	$+7-5$	$-6-4$
$+8+(-2)+(-7)$	$-7-2-5$	$6-4-5$
$-8+3-5+4$	$5-3+6-7$	$7-8+5-4$

Subtracting Integers

Now that we have learned to add positive and negative numbers, I'll bet you know what's coming next. Yes indeed, it's subtraction.

Remember, we defined addition as walking the number line from zero. Well, we are going to define subtraction as finding the distance between two locations on the number line and the way you have to travel to get to the first address. Going right is still positive, going left is negative.

EXAMPLE Let's say I want to know how far you must travel if you were standing on $(+8)$ and you wanted to go to the location marked as (-2).



Looking at the number line, you are standing on +8. Which direction will you have to go to get to –2? If you said left, that's good news and mathematically it translates to a negative number. Now, how far away from –2 are we? Using the number line we see we would have to walk 10 spaces to the left or –10.

Mathematically, that would look like this

$(-2)-(+8)$ _ walking 10 spaces to the left, (-10)

Another example. This time you are standing on –5 and want to go to –1. How far and what direction would you have to move? 4 spaces to the right would be the correct answer.

Mathematically, we have $(-1)-(-5)=+4$

Rule 4. When subtracting signed numbers, change the sign of the subtrahend (second number) and add using rule 1, 2 or 3, whichever applies.

Example $6-(+13)$

$=6+(-13)$ change sign & add

$=-7$

Simplify

$(+4)-(+6)$	$(-5)-(-7)$
$8-(-3)$	$8-(+3)$
$-4-(-6)$	$-8-(+7)$
$-10-(-3)$	$10-(-3)$
$6-(+2)$	$(+7)-(+11)$
$(-2)-(-12)$	$(+2)-(-12)$

Up to this point, we have developed rules for adding and subtracting signed numbers. Those rules came from observations that we made that allowed us to do problems without drawing pictures or using manipulatives. Using pictures and/or manipulatives is important so the kids have an understanding of the concepts being introduced.

Multiplying/Dividing Integers

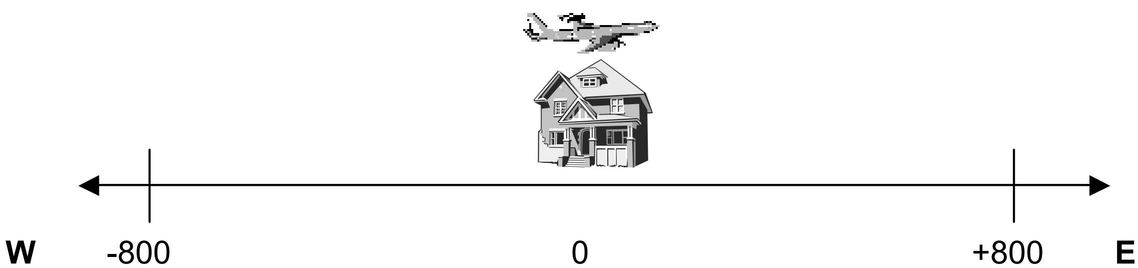
Here's a new agreement for multiplication and division. Traveling east (right) is positive, traveling west (left) is negative. Sounds familiar, doesn't it?

Now, future time will be defined as positive, past time as a negative number. And you'll be at your lovely home which will be designated as zero.

ILLUSTRATION 1

If you were at home (at zero) and a plane heading east at 400 mph passed directly overhead, where will it be in 2 hours?
If you don't know what distance equals rate x time, now you do.

Translating English to math, going 400 mph East is +400, and since we are looking at future time, 2 hours will be +2.



Now, standing at zero and the plane heading east for 2 hours at 400 mph, it will be 800 miles east in 2 hours.

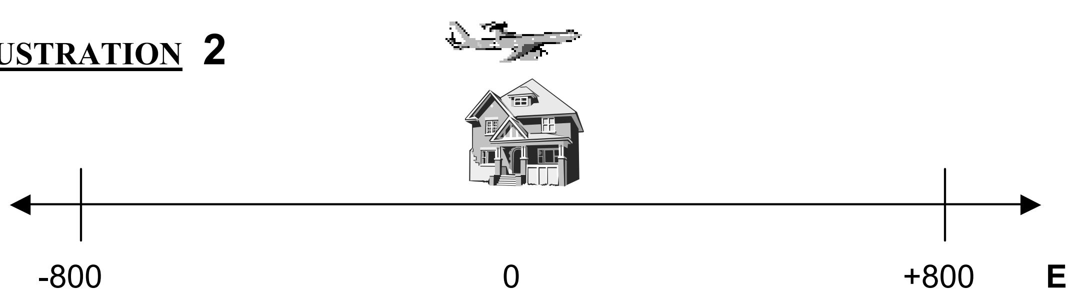
Mathematically, we have:

400 mph east x 2 hrs future = 80 miles east

$(+400) \times (+2) = +800$

Makes sense.

ILLUSTRATION 2



W

The plane is directly over your house heading east at 400 mph. Where was it 2 hours ago? Going east at 400 mph is written as +400, we are using past time, so that's –2.

He’d be 800 miles west.

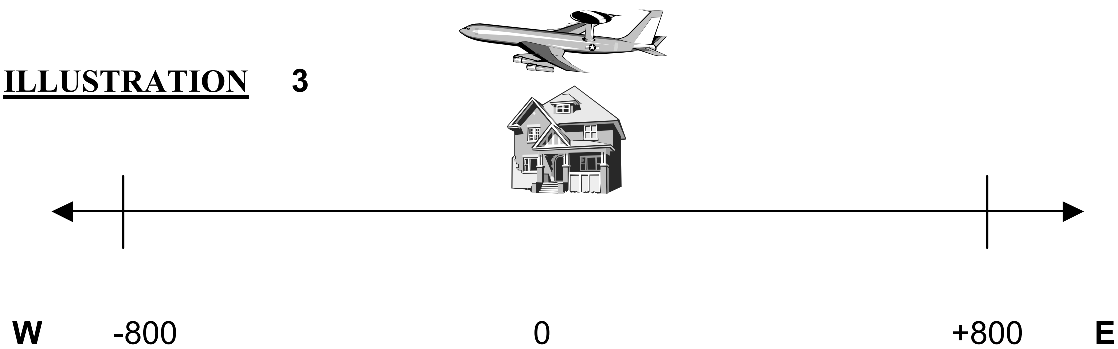
Translating English to math we have

400 mph east x 2 hrs past = 800 miles west

$(+400) \times (-2) = -800$

Oh, yes, this is a piece of cake. Don’t you just love math?

<u>ILLUSTRATION</u> 3



The plane is heading west at 400 mph, where will it be in 2 hours if it is directly over your head now?

He’d be 800 miles west.

Translating English to math we have

400 mph west x 2 hours future = 800 miles west

$(-400) \times (+2) = -800$

<u>ILLUSTRATION</u> 4 Using the last illustration

The plane is heading west at 400 mph, where was it 2 hours ago if it is directly over your home now?

Translating English to math we have

400 mph west x 2 hours past = 800 miles east

$(-400) \times (-2) = +800$

Now, if we looked at those 4 illustrations and checked out the math, this is what we would see.

From the illustrations, we have

1. $(+400)x(+2)=+800$

2. $(+400)x(-2)=-800$

3. $(-400)x(+2)=-800$

4. $(-400)x(-2)=+800$

That might lead us to believe multiplying numbers with like signs results in a positive answer, while a negative answer appears when you multiply numbers with unlike signs.

Those observations leads us to a couple more rules.

Rule 5. ***When multiplying or dividing numbers with the same sign, the answer is positive.***

Example: $(+5)x(+4)=+20$ $(-6)x(-7)=+42$

Rule 6. ***When multiplying or dividing numbers with different signs, the answer is negative.***

Example: $(-5)x(+8)=-40$ $(+9)x(-3)=-27$

Don't you just love it when things work out?

Multiply or Divide

1)

$-7(+6)$ $\frac{-48}{8}$ $\frac{+54}{-9}$

2)

$-6(+10)$ $-8(3)$ $\frac{32}{-4}$

3)

$\frac{-8}{+2}$ $-6(3)$ $6(-12)$

4)

$8(-7)$ $-6(3)$ $8(-5)$

5)

$\frac{-40}{4}$ $\frac{+40}{-8}$ $-5(9)$

6)

$\frac{-60}{+5}$ $4(-9)$ $-8(+6)$

7)

$-7(+9)$ $\frac{-63}{+7}$ $-3(12)$

Let's look at the rules we developed by looking at those patterns on the number line. We have three rules for addition, one for subtraction and two for multiplication/division.

Addition

Rule 1: Two positive numbers, take the sum of their absolute values, the answer is positive.

Rule 2: Two negative numbers, take the sum of their absolute values, the answer is negative.

Rule 3: One positive, one negative, take the difference between their absolute values, use the sign of the number with the greater absolute value.

Subtraction

Rule 4: Change the sign of the subtrahend and add using rule 1, 2, or 3,. whichever applies.

Multiplication/Division

Rule 5: Two numbers with the same sign are positive.
Rule 6: Two numbers with different signs are negative.

When working with these rules, we must understand the rules work for only two numbers at a time. In other words, if I asked you to simplify $(-3)(-4)(-5)$, the answer would be –60.

The reason is $(-3)(-4)=+12$, then a $(+12)(-5)=-60$

In math, when we have two parentheses coming together without a sign of operation, it is understood to be a multiplication problem. We leave out the "X" sign because in algebra it might be confused with the variable x.

Stay with me on this, often times, for the sake of convenience, we also leave out the "+" sign when adding integers.

Example: $(+8)+(+5)$ can be written without the sign of operation _ $+8+5,$ it still equals $+13$ or $8+5=13$.

Example: $\left(-8\right)+\left(-5\right)$ can be written without the sign of operation _ $8-5,$ it still equals -13 or $-8 \;\; -5=-13$

Example: $\left(-8\right)+\left(+5\right)$, can be written without the sign of operation _ $8+5$, it still equals -3 or $-8 \;\; +5 \;\; =-3$.

For ease, we have eliminated the "X" sign for multiplication and the "+" sign for addition. That can be confusing.

Now the question is: How do I know what operation to use if we eliminate the signs of operation?

The answer:: If you have two parentheses coming together as we do here, $\left(-5\right)\left(+3\right)$, you need to recognize that as a multiplication problem.

A subtraction problem will always have an additional sign, the sign of operation. For example, $12-\left(-5\right)$, you need to recognize the negative sign inside the parentheses is a sign of value, the extra sign outside the parentheses is a sign of operation. It tells you to subtract.

Now, if a problem does not have two parentheses coming together and it does not have an extra sign of operation, then it's an addition problem. For example, $8-4,-12+5$ and 9 - 12 are all samples of addition problems. Naturally, you would have to use the rule that applies.

Simplify and name the appropriate operation

1. $\left(-4\right)+\left(-9\right)$
2. $\left(-5\right)\left(6\right)$
3. $-7-\left(+3\right)$
4. $-10 \;\; -4$

Answers: 1. add, -13, 2. mult, - -30 3. sub, -10

4. add -14.

Simplify. First determine if the problem is $+, -, X,$ or $\div$, then write the rule that applies to the problem.

1. $(+8)+(-3)$

2. $(-5)+(-4)$

3. $(-5)(+6)$

4. $-8-5$

5. $24 \div (-6)$

6. $5-(-3)$

7. $(-5)-(+8)$

8. $-3+9$

9. $(-6)(-5)$

10. $(-5)(-4)(-2)$

11. $-5-4-2$

12. $(-4)^3$

13. $-2+8-10+4$

14. -4^2

Problem Solving

1) In a certain game one couple made a score of 320, while another couple made a score of –30, what was the difference in scores?

2) At noon the thermometer stood at $+12^0$, at 5 pm it was –8. How many degrees had the temperature fallen?

3) The height of Mt. Everest is 29,000, the greatest known depth of the ocean is 32,000 ft. Find their difference.

4) On 6 examination questions, Bob received the follow deductions for errors $-4, -2, 0, -5, -0, -8$. What was his mark based on 100 pts?

5) A team lost 4 yards on the 1st play and gained 12 yards on the 2nd play. What was the net result?

6) The average temperature of Mars is –60. The average temperature of Venus is 68^0. What is the difference in temperature?

7) How long did a man live who was born in 73 B.C. and died in 25 B.C.?

8) Roberto traveled from an altitude of 113 ft. below sea level to an altitude of 200 ft below sea level. What was the change in altitude?