

PROBABILITY

If someone told you the odds of an event **A** occurring are 3 to 5 and the probability of another event **B** occurring was $\frac{3}{5}$, which do you think is a better bet?

Most of us would probably believe they are the same, it would not make a difference. But, in fact, they are different. Let's see how.

Probability is defined as the ratio; success to total. To write that mathematically, we have;

$$\text{Probability} = \frac{\text{success}}{\text{success} + \text{failure}}$$

Success plus failure is equal to the total number of things that could happen.

Odds, on the other hand, is defined as the ratio; success to failure. Writing that mathematically, we have;

$$\text{Odds} = \frac{\text{success}}{\text{failure}}$$

Notice, odds and probability are defined differently. Now, going back to the original question, which is a better bet, we can substitute numbers and compare.

If the odds are 3 to 5, $\frac{3}{5}$, by using the definition, that means you can succeed 3 ways and fail 5 ways. In other words $s = 3$ and $f = 5$. How would that be translated to probability?

By definition, probability is $\frac{\text{success}}{\text{success} + \text{failure}}$. By plugging in the numbers, we have

$$\frac{3}{3 + 5} = \frac{3}{8}$$

So if the odds are 3 to 5 of an event occurring, then the probability of that event occurring is $\frac{3}{8}$

Now, which is the better bet, **A** or **B**? Hopefully, you would now say **B**. In **A** you have three chances out of eight to win, in **B** you have three chances out of five.

If I know the probability of an event, I can determine the odds associated with the event.

If the probability is $\frac{3}{5}$, by using the definition $\frac{\text{success}}{\text{success} + \text{failure}}$, that means you can succeed 3 ways out of a total $(s + f)$ 5 ways. We already said $s = 3$, so if $3 + f = 5$,

what's the value of f . You've got it, 2. So $f = 2$. Now writing the odds we have

$$\frac{\text{success}}{\text{failure}} = \frac{3}{2} \quad \text{The odds are three to two.}$$

Piece of cake, don't you think?

Converting Probability – Odds

To convert probability to odds and odds to probability, we can formalize what we just did using this procedure.

1. Write the information given with its respective ratio
2. Determine the value of s and f
3. Write the ratio for the information you are looking for
4. Plug in the values of s and f

Let's try one.

Example Find the odds of an event if the probability is given by $\frac{3}{4}$

1. $P = \frac{s}{s+f} \rightarrow \frac{3}{4}$.
2. In our case $s=3$. The denominator is 4 and that is equal to $s+f$
If $s = 3$, then f would have to equal 1. Knowing that s is 3 and f is 1, all I had to do is plug these numbers into the definition for odds.
3. **Odds** = $\frac{\text{Success}}{\text{failure}}$
4. **Odds** = $\frac{3}{1}$

Example: Find the probability of an event if the odds are given by 2 to 3.

1. **Odds** = $\frac{\text{success}}{\text{failure}} \rightarrow \frac{2}{3}$.
2. $s = 2$ and $f = 3$
3. $P = \frac{\text{success}}{\text{success} + \text{failure}}$.
4. **Pr obability** = $\frac{2}{2+3} = \frac{2}{5}$

The concept of probability is very simple, a ratio, the number of ways you can succeed over the total number of ways something can happen.

Before we go too far, let's define some terms.

Experiment – is an activity under consideration, such as flipping a coin.

Outcome – is a list of possibilities.

Sample space - the set of all possible outcomes in an experiment.

Event - a list of specific outcomes within a sample space.

Example: Now, let's say we flip a coin, we'll call that the experiment. The set of all possible outcomes – the sample space consists of heads or tails.

If I wanted to know the probability of getting a head, then I use the definition of probability.

$$\text{Probability} = \frac{\textit{success}}{\textit{success} + \textit{failure}}$$

I can succeed one way (getting a head on the toss), over the total number of things that can happen (sample space, success + failure).

$$\text{Probability (heads)} = \frac{1}{2}$$

Pretty easy, don't you think?

Example A card is drawn from an ordinary deck of cards, what is the probability that it is an ace?

In a deck of cards, there are 52 cards. There are 4 aces, so there are 4 ways you can succeed. Therefore, the Probability (ace) = 4/52

This is important, the probability of an event A is equal to the sum of the probabilities of all the outcomes in set A. So a different approach to the last problem, since the probability of drawing any of the four aces is 1/52, then

$$\text{Probability (ace)} = \frac{1}{52} + \frac{1}{52} + \frac{1}{52} + \frac{1}{52} = \frac{4}{52}$$

While the concept of probability is a simple one, the difficulty one runs into is knowing all the possible things (outcomes in the sample space) that can happen.

If I were to ask you what the probability of picking your name out of a hat that contained ten names, you would quickly see that $P = 1/10$, because you knew there were only ten names.

If we are familiar with certain activities, that's very straightforward. However, sometimes knowing all the possible outcomes is not as easy as it sounds.

Example: Put all the names of the 50 states in a hat and find the probability of picking a state that begins with the letter A.

If you didn't know all the states, you might not get the correct answer. However, if you know there are 50 states and there are 4 states that begin with A, then finding the probability is easy, it's 4/50.

The states are Alabama, Alaska, Arkansas, and Arizona. Each state has a probability of 1/50 to be picked out of the hat.

$$\frac{1}{50} + \frac{1}{50} + \frac{1}{50} + \frac{1}{50} = \frac{4}{50}$$

Example: If I flip two coins, a quarter and a dime, what's the probability of getting two heads? What are the possible outcomes?

I'm going to use capital letters for the quarter, small letters for the dime and list them.

Here's the list: *Hh, Ht, Th, Tt.*

There are four outcomes. Now, answer the question, what is the probability of tossing two coins and have them both come up heads? From my list I see that happens once out of four times.

$$\text{Probability (both heads)} = \frac{1}{4}$$

Again, if we know how large the sample space is, then finding the probability is as simple as plugging in the number of ways you can succeed over the total number of outcomes. Did you know there were four different things that could happen if you flipped two coins? How about flipping three coins, how many different things could happen?

The problem then, is finding the total number of ways something can happen. Finding the sample space.

Finding the Sample Space Listing

One way to find everything in a Sample Space is to list them as I did in the last example.

Example If I asked what are all the possible outcomes when I flip a coin and roll a die.

By listing, I have H1, H2, H3, H4, H5, H6
 T1, T2, T3, T4, T5, T6

We can see there are 12 things that could happen. Now, if I asked the probability of getting a head and a 5, from the list you can see I can succeed one way out of a total of 12.

Therefore, the $P(H,5) = 1/12$

Again, the concept of probability is quite simple. If you know all the possible outcomes, then all you do is put the number of ways you can succeed over the total number of things that could happen. Piece of cake!

Example: In a committee of 5 people; A,B,C,D,E, what is the probability that A will be chosen president and C vice president, if they were chosen at random.

Again, the question comes down to; what is the sample space? I could try to list those as I did before or I could use a more systematic method.

The probability of choosing A, president followed by C as vice president is $1/20$. Try to list all those possibilities. After doing that, we might see a need for a better approach than just trying to list all the possible outcomes.

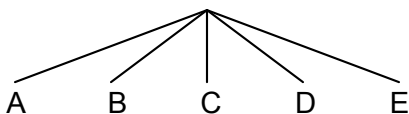
Let's look at some problems.

1. Find the probability of drawing the 2 of Diamonds from a deck of 52 cards.
2. Find the probability of drawing a King from an ordinary deck of cards.
3. Each letter of the alphabet is written on a separate sheet of paper and placed in a box, then one piece of paper is drawn randomly, what is the probability the paper drawn is a vowel?
4. What is the probability of picking a month of the year at random that begins with the letter "J"?
5. If student council has 15 members, 7 boys and 8 girls, what is the probability that a boy will be selected if someone has to be chosen at random to run an errand?
6. A golf course has eighteen holes, if 5 of them are considered water holes, what is the probability that a person playing on that course is not on a water hole?
7. The lunch wagon has 4 tuna sandwiches, 5 egg salad, 2 ham, and three turkey sandwiches. If you reached in at random to select a sandwich, what is the probability of selecting a tuna or egg salad sandwich?
8. If the probability that it will rain tomorrow is .6, what are the odds it will rain?
9. If the odds in favor of the Red Sox winning the series is 2 to 5, what is the probability they will win the series?

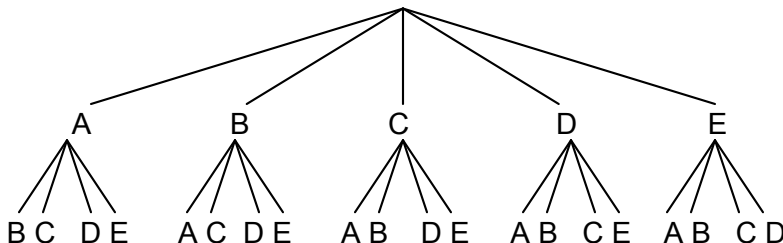
Tree Diagrams

A tree diagram is just a systematic way of listing all the possible outcomes.

To draw the tree, I first list all the ways I could select the president of the committee. The president could be **A,B,C,D** or **E**.



After I list all of those, then I list the ways I could pick the vice president, given the previous selection of the president. So if A was chosen president, then B,C,D, or E could be vice president. Continuing that process for the other branches, if B was chosen president, then A,C,D, or E could be vice president.



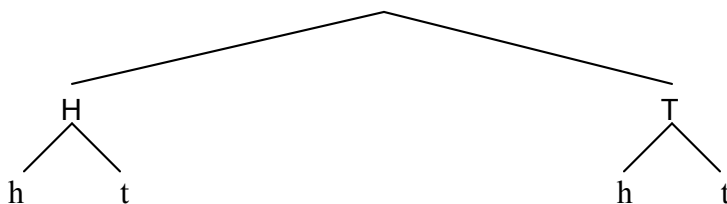
Now, reading down the branches gives me all the possible things that could happen. I could have had AB, AC, AD, or AE. If B was chosen as president, I could have BA, BC, BD, or BE. If C was chosen first, then the outcomes could be CA, CB, CD, or CE. With D being chosen first, we'd have DA, DB, DC, or DE. And if E was chosen present, then the results could have been EA, EB, EC, or ED.

That's a total of 20 things that could happen, the Sample Space. Neat stuff, don't you think? I have only one AC, therefore the probability is $1/20$.

Example Make a tree diagram for tossing the quarter and the dime we did on the last page.

Quarter

Dime



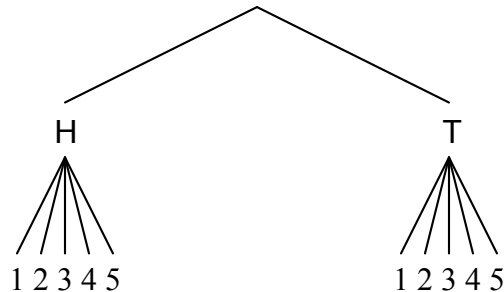
The capital letters represent the quarters, the small letters represent the dimes. Reading down the tree diagram, we have Hh, Ht, Th, and TT. There are 4 things that could happen .

The tree diagram is a very important tool in solving beginning probability problems. To use it, you list all the ways the first even could happen. Under each of those branches, you list all the ways the next event can occur given the first one has already occurred.

Example Draw a tree diagram to show all the possible outcomes of tossing a coin, then throwing a five sided die.

What can happen when you toss a coin? You either get a head or a tail, that's your first branch. If you get a head, you toss the die, what could happen? You get a 1, 2, 3, 4, or 5, What would happen if you would have gotten a tail? You would have thrown the die and it would have turned up a 1, 2, 3, 4, or 5.

Let's draw the tree diagram.



The total number of outcomes, the sample space, is 12. Flipping a coin and rolling a die can may result in any one of the following; H1, H2, H3, H4, H5, T1, T2, T3, T4, T5.

Draw a tree diagram for each of the following problems.

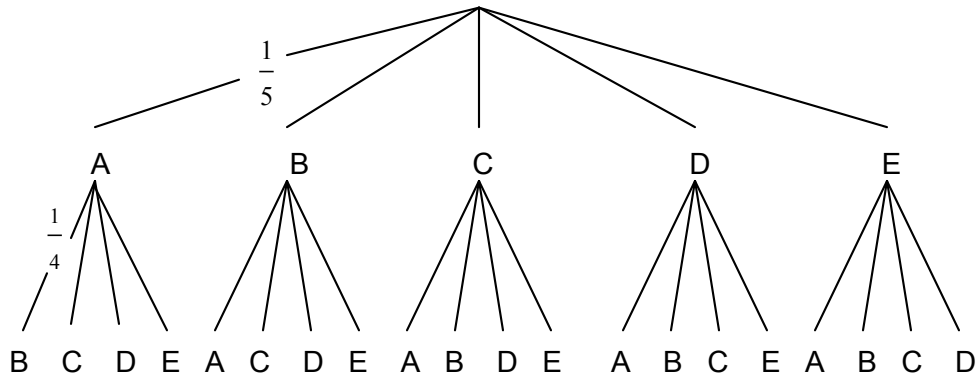
1. Draw a tree diagram and find the probability of having 2 boys in a family of 3 children?
2. On a three question true-false test, draw a tree diagram and find the probability of getting 100% if all the answers were chosen at random.
3. A sports car can be ordered in 5 colors and two types of transmissions, manual and automatic, draw a tree diagram to show how many types of cars can be ordered.
4. Three coins are tossed, what are the possible outcomes?

Assigning Probabilities to Each Branch of a Tree Diagram

If we were to go back and look at the tree diagrams we already drew and made a few more, we might see a pattern develop. The pattern that might jump out at you is the total number of outcomes in a sample space could be determined by multiplying the number of outcomes in each stage of an experiment. For instance, when we flipped the coin, we saw two outcomes. When we rolled the die, 6 things could happen. By drawing the tree diagram, we found there were 12 different outcomes – or 2×6 .

Using that observation gives us some good news. If we were to assign probabilities to each branch of the tree diagram using the number of outcomes, we might see something interesting.

Let's copy the tree with the president and vice president and assign probabilities to each branch.



Because there are 5 ways to choose the president, the probability of each of the top branches is $\frac{1}{5}$. I pick the vice president after the president is chosen, so each of those branches is $\frac{1}{4}$.

Notice, if we multiply down the branches to get A followed by B, we get the probability of that particular outcome. So the probability of having A president and B vice president is

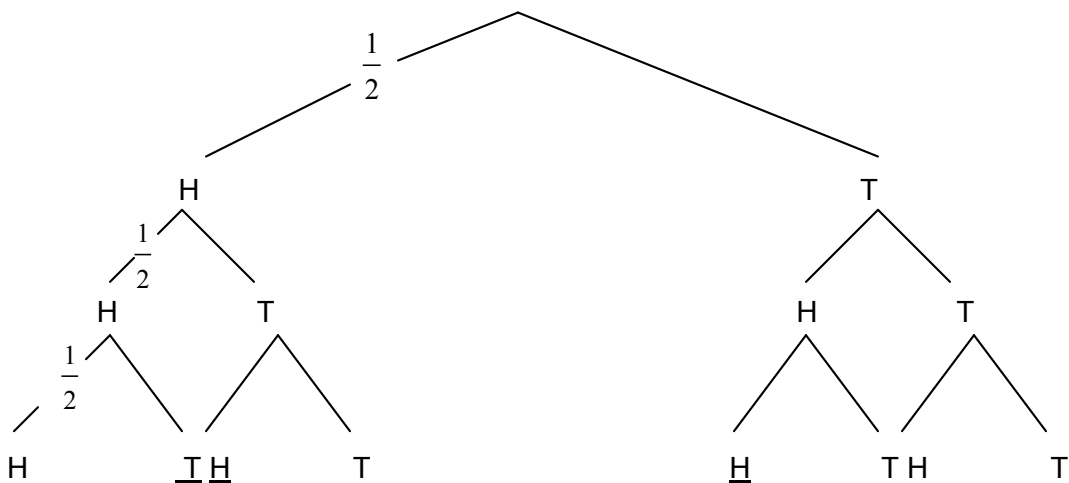
$$\left(\frac{1}{5}\right)\left(\frac{1}{4}\right) = \frac{1}{20} \quad \text{That leads us to this very important note.}$$

For all multistage experiments, the probability of an outcome along any path of a tree diagram is equal to the product of all the probabilities along the path.

Let's check that out with another example.

Example: Tossing three coins, a quarter, dime and nickel, what's the probability of getting 2 heads and a tail?

Drawing the tree diagram and assigning the probabilities, we have



Looking at that and using the rule for all multistage experiments that the probability of an outcome along a path is equal to the product of the probabilities along the path that lead to that outcome, we can quickly see the probability of getting 2 heads and one tail is. I've underlined each outcome that gives us two heads and a tail. The probability of that outcome is found by multiplying down the branches.

$$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{8}$$

The question we were asked was to find the probability of getting 2 heads and a tail. So what we have to do is look at the tree diagram to determine which outcomes satisfy that event occurring. There are three outcomes that result in 2 heads and a tail. The probability of each of those outcomes is $\frac{1}{8}$. So the probability of the event is equal to the sum of the outcomes that make up the event. That is $\frac{3}{8}$

Isn't this a blast!

What's nice about math is I cannot make these problems more difficult, I can only make them longer.

To find probabilities using a tree diagram:

1. Make the tree diagram.
2. Label each branch's probability
3. Multiply down the branches to find the probability of the desired outcome.

To find the probability of the event, you take the sum of the outcomes that make up the event.

Working with tree diagrams can become very cumbersome, particularly by the third or fourth stage of an experiment. Because of this, we will soon discover other ways of determining probabilities using different counting methods.

PROBABILITY

Student should be able to:

- Find the probability of a simple event by counting
- Find the odds of an event.
- Find the probability of a multi-stage experiment using a tree diagram
- Use the Fundamental Counting Principle to determine the number of outcomes.

Problem Set

1. What is the probability of picking a month of the year at random that begins with the letter "J"?
2. Find the probability of selecting a state at random that begins with the letter "A".
3. In the set $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, find the probability of selecting a prime number if a number was chosen at random?
4. If a card is selected at random from an ordinary deck of playing cards, what is the probability that the card is a king?
5. A drawer contains six black socks, four brown socks, and two green socks. Suppose one sock is drawn at random and it is equally likely that any of the socks will be picked, find the probability that the sock is either black or brown.
6. If the student council has 15 members, 7 boys and 8 girls, what is the probability that a boy will be selected president, if the president is chosen at random?
7. What is the probability of selecting a number that is divisible by 4 from the following set; $\{3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36\}$
8. In problem 7, what is the probability of NOT picking a number divisible by 4?
9. A golf course has eighteen holes, if 5 of them are water holes, what is the probability that a person playing golf is on a water hole?
10. The lunch wagon has 4 tuna sandwiches, 5 egg salad, 2 ham, and 3 turkey sandwiches. If you reached in at random to select a sandwich, what is the probability of selecting tuna or egg salad sandwich?
11. A coin is flipped twice, what is the probability that 2 heads come up?

12. A coin is tossed in the air, then a die is rolled. What is the probability of getting a head and a 5? What is the probability of getting a tail and an even number?
13. If Bob's batting average is .300, what is the probability that he will get two hits in a row?
14. A study of people with an MBA degree shows that it is reasonable to assign a probability of .800 that such a person will have an annual salary in excess of \$30,000. Find the probability that such a person will earn \$30,000 or less?
15. If the odds in favor of the Red Sox winning the series is 2 to 5, find the probability that they will win?
16. If the probability that it will rain tomorrow is .6, what are the odds it will NOT rain?
17. On a trip a man took 3 suits, 2 ties, and 2 hats. How many different choices of these items items of clothing can he wear?
18. Any one of three trains will take Bill from his home to school, any one of four elevators will carry him to his floor. How many possible choices of transportation by train and elevator can he select?
19. A restaurant has 4 main courses, 3 desserts, and 5 drinks available for the lunch special. How many different lunches made up of a main course, dessert, and drink can one choose?
20. A girl has 5 dresses, 3 pairs of shoes, and 3 coats suitable for wear at work. How many choices of outfits with these items are possible?

PROBLEM SOLVING

(Probability)

1. Four people attend the symphony together. The coat checker is on a break, so an usher returns their coats. In how many different ways can the coats be returned so that no one receives the correct coat?
2. How many different four-digit numbers can be formed using the digits 1, 1, 9, and 9?
3. Two different prime numbers are selected at random from among the first ten prime numbers. What is the probability that the sum of the two primes is 24? Express your answer as a common fraction.
4. A drawer contains a mixture of red socks and blue socks, at most 1991 in all. It so happens that when 2 socks are selected randomly without replacement, the probability is exactly one-half that both are red or both are blue. What is the largest possible number of red socks in the drawer?
5. A bag contains one marble known to be either red or green. A green marble is put in put in the bag and it is shaken well. A marble is drawn randomly, and it is green. What is the probability that the remaining marble is also green?
6. Two ham sandwiches and two jelly sandwiches are wrapped in foil and put randomly into a bag. When they are taken out, what is the probably that the first two will be alike? What is the probability that both will be ham sandwiches?
7. A bag contains one marble known to be either red or green. A green marble is put into the bag, and it is shaken well. One marble is drawn randomly, and it is green. This marble is returned, the bag shaken, and a second draw made – also green. What is the probability that the marble still in the bag is also green?
8. Consider a pair of ordinary dice and a spinner divided into twelve equal sections numbered from 1 to 12. Compare the probability of getting a prime number using the spinner with the chances of getting a prime number as the sum when you toss the two dice.
9. If the six letters of the word “pencil” are arranged at random, what is the probability that the first and last letters will be vowels?