Algebra,

Been there – Done that

Higher Degree Equations

Zero Product Property

Mathematical Systems

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Algebra, Been there –Done that is a newsletter that links algebra to previously learned concepts and skills or outside experiences

Back in the third grade students were taught when they multiplied a number by zero, the product would be zero. In algebra, that's an extremely important property, so important it has a name, the Zero Product Property.

Math Facts
4 x 0 = 0

0 x 5 = 0

While teachers may not have used that name, the point they tried to get across was that 5 x 0 = 0, 123 x 0 = 0, and 0 x 4.32 = 0. It's a pretty straight forward concept.

Extending the thinking, we found that in order for the product of 4 and some number n to be zero, n would have to be zero.

$$4n = 0$$
$$n = 0$$

4 x n = 0, then n has to be 0.

If I multiplied two numbers, x and y, and found their product to be zero, then one of those two numbers would have to be zero. Mathematically, we'd write

$$xy = 0$$

then x = 0 or y = 0. One of the two numbers would be equal to zero. But, there is another possibility, both x and y could be equal to zero.

So a list of all possible answers would include, x = 0, y = 0, or x = 0 and y = 0.

Zero Product Property

If AB = 0, then A = 0, or B = 0, or A and B equal zero

An application of the Zero Product Property will allow us to solve higher degree equations by changing them into multiplication problems whose product is zero.

For example, if I were asked to solve, $x^2 - x = 12$ I might have a little trouble. If I changed the problem into a multiplication problem (factoring), I would have

$$x(x - 1) = 12$$

I have two numbers multiplied together that equal 12. Through a trial and error process, I might find values of x that would satisfy the original equation. But, if I used the Zero Product Property, I would be able to solve the problem using the logic we used in third grade – the product is zero if I multiplied by zero.

Solve:
(x-3)(x+2)(x-5) = 0

Set each factor equal to zero, sole the resulting equations.

If x-3 = 0 , then x = 3
If x+2 = 0, then x = -2
If x-5 = 0, then x = 5

The solution set is {3,-2,5}

Rewriting $x^2 - x = 12$ to $x^2 - x - 12 = 0$, then factoring, I would have the product of two numbers equaling zero rather than 12. By using the third grade logic, I know that one of the two numbers or maybe both of them are zero since the product is zero.

In other words, we would have (x - 4)(x + 3) = 0.

When is x - 4 = 0? When is the other number x + 3 = 0?

When you answer those questions, you have found the values of x that make the open sentence true. Simply put, you solved the equation.

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The answer, the solution, the zeros are x = 4 or x = -3.