

Algebra,

Been there – Done that

Graphing Equations of Lines

Mathematical Systems

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Algebra, Been there –Done that is a newsletter that links algebra to previously learned concepts and skills or outside experiences

We were able to find equations of lines by manipulating the slope formula and rewriting it as the Point-Slope Form of an equation of a Line.

$$m = \frac{y - y_1}{x - x_1}$$

$$y - y_1 = m(x - x_1)$$

Finding
Equations of Lines

The Point-Slope Form of an Equation of a Line allows us to find equations of lines.

If we manipulate the Point-Slope form, we will see other forms of equations of lines that result in graphing linear equations in two variables by inspection.

Point-Slope
Equation of a Line

If you were asked to find an equation of a line passing through (1,8) having slope 5, we'd plug those numbers into $y - y_1 = m(x - x_1)$.

$$y - 8 = 5(x - 1)$$

$$y - y_1 = m(x - x_1)$$

Solving for y, we have $y = 5x + 3$

Graphing
Equations of Lines

If I did enough problems like that and solved for y, I would begin to notice the coefficient of the x is the slope and the constant is where the graph crosses the y-axis. That leads us to the Slope-Intercept Form of an Equation of a Line.

Slope- Intercept
Equation of a Line

$$y = mx + b$$

m is the slope, b is the y- intercept

$$y = mx + b$$

To use that Slope-Intercept to graph, use the y intercept as the first point on the graph and then use the slope to go up and over to find the second point.

m is the slope
b is the y- intercept

Instead of solving for y, if we decided to put the x's and y's on the same side of the equality, that would result in another form of an equation of a line, called the General Form of an equation of a Line.

General Form
Equation of a Line

$$Ax + By = C$$

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If we studied enough graphs, we would see that the graph crosses the y-axis when $x = 0$. We would also see the graph crosses the x-axis when y is 0. That leads us to be able to find two points by just looking at an equation, which allows us to graph equations of lines written in the General Form by inspection.

$$y_{\text{int}} = \frac{C}{B}$$

$$Ax + By = C, \text{ if } x = 0, \text{ then } By = C \rightarrow y_{\text{int}} = \frac{C}{B}$$

$$x_{\text{int}} = \frac{C}{A}$$

$$Ax + By = C, \text{ if } y = 0, \text{ then } Ax = C \rightarrow x_{\text{int}} = \frac{C}{A}$$

Upon further inspection, we would find the slope of a line in General Form can be found by $-A/B$.

Just as importantly, it should be noted that the Point-Slope Form of a Line came directly from the definition of slope. The Slope-Intercept was derived from the Point-Slope, and the General Equation came from the Slope-Intercept. They are all linked.