

# Algebra,

*Been there – Done that*

## Special Products

Mental Math

### *Mathematical Systems*

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Algebra, Been there –Done that is a newsletter that links algebra to previously learned concepts and skills or outside experiences

Mental math, knowing some patterns allows students to compute in their head. For instance, after looking at the following multiplications, some people would be able to square numbers in their head. Let's look:

$$25 \times 25 = 625 \quad 35 \times 35 = 1225 \quad 65 \times 65 = 4225 \quad 95 \times 95 = 9025$$

#### Special Products

$$(a-b)(a+b) = a^2 - b^2$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

Do you see a pattern that would allow you to multiply  $75 \times 75$ ?

In algebra, you will study special products, its nothing more than mental math described by patterns.

Let's look at some:

$$(a-b)(a+b) = a^2 - b^2$$

What that pattern allows you to do is multiply numbers like  $32 \times 28$  in your head.

$$32 \times 28 \text{ becomes } (30+2)(30-2) = 30^2 - 2^2 \text{ or } 900 - 4 = 896.$$

#### FOIL

$$(a+b)(x+y) =$$

$$= ax + (ay + bx) + by$$

Compute  $47 \times 53$  in your head. If you recognize the pattern, both numbers are three from 50, I can rewrite that as  $(50-3)(50+3) = 50^2 - 3^2$  or  $2500 - 9 = 2491$ .

Not only does that pattern allow us, with practice to multiply in our head, it will also help us factor polynomials – if we take the time to recognize the pattern.

Let's look at another special product:

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$25 \times 32 = (20+5)(30+2)$$

$$(20+5)(30+2) =$$

$$= 600 + (40+150) + 10$$

$$= 800$$

That pattern allows us to square numbers mentally.  $35^2 = (30+5)(30+5)$ . Using that pattern, we have  $30^2 + 2x(30x5) + 5^2$  which is  $900 + 300 + 25$  or 1225.

Another example  $43^2 \rightarrow (40+3)(40+3)$ . Using the pattern, I would have

$$40^2 + 2x(40x3) + 3^2 = 1600 + 240 + 9 \text{ or } 1849$$

Using subtraction, we have  $(a-b)^2 = a^2 - 2ab + b^2$ , which is almost the same pattern we just used. Using this pattern, we can simplify  $37^2$  mentally.

$$37^2 = (40-3)^2$$

$$(40-3)^2 = 40^2 - 2x(40x3) + 3^2$$

$$= 1600 - 240 + 9$$

$$= 1369$$

Recognizing the patterns in these computations will help students recognize the same patterns in algebra that will help them factor algebraic expressions and solve higher degree equations.