Factoring, combined

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We have learned to factor polynomials using the Distributive Property, Difference of Two Squares, Addition Method, Trial & Error and Grouping. Now, do you know which method to use by looking at the polynomial?

If we go back to our diagram, we see we should try to factor a polynomial first by using the Distributive Property. That will make the numbers more manageable.

Look at the following polynomials and select the method that should be used to factor it.

a. $8y^2 + 2y - 3$
b. $x^2 - 25$
c. $x^2 - 7x - 12$
d. $2x^2 + 18x$
e. $a^3 - 3a^2 + 9a - 27$

Let's see how you did. Answer a. is Trial & Error because the coefficient of the quadratic term is **NOT** one. Answer b. is the Difference of Two Squares because you have a binomial, both terms are perfect squares, and it is a difference. Answer c, is the Addition Method because the coefficient of the quadratic term is one. Answer d. is the Distributive Property and answer e. is Grouping since it is not a binomial or trinomial.

If you can discriminate between the polynomials, that's half the battle. Because once you know what method to use, you just follow the procedures you have already learned.

Now let's see how you go about factoring polynomials by **GROUPING**.

To be able to factor using Grouping requires you to know how to factor using other methods. Then using those methods, we group terms together.

EXAMPLE Factor $a^3 - 3a^2 + 9a - 27$

I can not use the Distributive Property, nor do I have a binomial of trinomial, that's a pretty good indication that I have to factor by Grouping.

The question is, how do I group? The first two and the last two terms, the first three terms and the fourth term? The first and third term and the second and fourth?

Well, we are going to group them to try and find common factors.

Notice, if I took an a^2 out of the first two terms and a 9 out of the third and fourth terms, I have a common factor of $(a - 3)$

$$a^3 - 3a^2 + 9a - 27 = a^2(a - 3) + 9(a - 3)$$

Factoring out the $(a - 3)$ from both terms, we have

$$(a - 3)(a^2 + 9)$$

Yep, that was fun.

EXAMPLE Factor $x^2 - 6x + 9 - y^2$

This again is a Grouping problem. In this case, I recognize $x^2 - 6x + 9$ as a perfect square. Since I see this pattern, I will group the first three terms together and factor.

$$x^2 - 6x + 9 - y^2 = (x - 3)^2 - y^2$$

If I looked at this result, I might recognize that as the Difference of Two Squares.

Using that method, I take the square root of each term and write them as factors.

$$[(x - 3) \quad y]\ \{(x - 3) \quad y]$$

I then put a "+" sign in one of the factors, a " – " sign in the other. That gives me

$$[(x - 3) + y]\ [(x - 3 - y]$$

It's important that you can recognize special products when factoring by grouping. That information will give you a clue on how to group the terms.

