Solving Quadratic Equations

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The reason we learned how to factor is so we could solve Quadratic and higher degree equations. Those skills are important. If you are not comfortable factoring, then do more problems.

You not only need that skill for solving the following quadratic equations, you'll also need to factor for adding and subtracting rational expressions and reducing later factoring is important to your success.

A quadratic equation is an equation of degree 2, that is, the exponent on the variable is 2.

EXAMPLE: $3x^2 + 5x - 4 = 0$

There are a number of different methods that can be used for solving quadratic equations, we'll look at two of these methods. We'll solve them by FACTORING and the QUADRATIC FORMULA.

Let's make sure we know what factoring is; FACTORING is the process of changing an expression that is essentially a sum into an expression that is essentially a product.

Depending upon what the expression looks like, we may factor using the DISTRIBUTIVE PROPERTY, DIFFERENCE OF 2 SQUARES, LINEAR COMBINATION, TRIAL AND ERROR, GROUPING AND THE SUM OR DIFFERENCE OF TWO CUBES. It is very, very important that you can recognize these patterns for factoring.

EXAMPLE: Factor $x^2 + 7x + 12$

We want to change this addition problem into a multiplication problem by factoring

$$(x + 4)(x + 3) = x^2 + 7x + 12$$

Now, to solve QUADRATIC EQUATIONS by FACTORING we'll use this strategy.

1. Put everything on one side of the equal sign, zero on the other side.
2. Factor the expression completely.
3. Set each factor equal to zero.
4. Solve the resulting equations.

EXAMPLE: $x^2 - 35 = 2x$

1. Put everything on one side, zero on the other side. $x^2 - 2x - 35 = 0$
2. Factoring $(x + 5)(x - 7) = 0$
3. Set each factor equal to zero $x + 5 = 0 \quad x - 7 = 0$
4. Solve $x = -5 \cup x = 7$

Let's look at the reason why this method works. By putting everything on one side and factoring, we are looking for two numbers when multiplied together equal zero.

If you think for a minute, you'll remember the only time you can get an answer of zero in a multiplication problem is when one or both of the original numbers is zero. That is such an important concept in math that we give it a name; the Zero Product Property. It states if $ab = 0$, then $a = 0$ or $b = 0$ or both a and b are zero.

EXAMPLE: $8y^2 + 2y = 3$

1. Put everything on one side $8y^2 + 2y - 3 = 0$
2. Factor $(4y + 3)(2y - 1) = 0$

3. Use the Zero Product Property $4y + 3 = 0$ or $2y - 1 = 0$
4. Solve: $4y = -3$ $2y = 1$

$y = -3/4$ or $y = 1/2$

Knowing what factoring is and being comfortable with the different patterns involved in factoring will either make or break you with quadratic equations. Practice with factoring is essential.

If we had a cubic equation to solve, factoring is an appropriate method to use.

EXAMPLE: $x^3 + 12x = 7x^2$

Putting everything on the side we have:

$$x^3 - 7x^2 + 12x = 0$$

Factoring $x(x^2 - 7x + 12) = 0$

$$x(x - 3)(x - 4) = 0$$

Now I have three numbers multiplied together that equals zero. That means at least one of those numbers; x, (x – 3) or (x – 4) must be zero.

That's an extension of the Zero Product Property. Setting each factor equal to zero we have:

$x = 0$ $x - 3 = 0$ $x - 4 = 0$

The solutions are

$$x = 0 \ \bigcup \ x = 3 \ \bigcup \text{or } x = 4$$

Plug these back into the original equation and you see they work.

Piece of cake, don't you think? Notice we have changed quadratic and higher degree equations into a series of linear equations in the $ax + b = c$ format.

