

Quadratic Formula

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The reason we learned to simplify radicals is because not all polynomials can be factored over the set of Rational Numbers. What that means is that you might be asked to solve a quadratic equation where the polynomial can not be factored.

Well, that poses a small problem, but not one we can't handle. If we can't factor it, we can solve the quadratic by "Completing the Square". We can generalize that method to come up with a formula.

So, another method for solving QUADRATIC EQUATIONS is the QUADRATIC FORMULA. That formula is derived from "Completing the Square."

The General Form of a Quadratic Equation looks like this: $ax^2 + bx + c = 0$, a is the coefficient of the squared term (quadratic term), b is the coefficient of the x term (linear term) and c is the constant (the number without a variable).

The QUADRATIC FORMULA is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Rather than write it as one fraction, I am going to write it as two fractions

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

That will eliminate some of the difficulties in reducing algebraic fractions that some of the students experience. You need to memorize that formula, say it once, say it twice, say it 20 times if you have to – but know it!

Now, all we have to do is substitute the numbers in those positions in the Quadratic Formula.

EXAMPLE: $2x^2 + 3x - 5 = 0$

In this example $a = 2$, $b = 3$, and $c = -5$

Substituting those values in $x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

we get $x = \frac{-(3)}{2(2)} \pm \frac{\sqrt{3^2 - 4(2)(-5)}}{2(2)}$

simplifying $x = \frac{-3}{4} \pm \frac{\sqrt{9 - (-40)}}{4}$

$$x = \frac{-3}{4} \pm \frac{\sqrt{49}}{4}$$

$$x = \frac{-3}{4} \pm \frac{7}{4}$$

Therefore one answer is $\frac{-3}{4} + \frac{7}{4}$ or 1 and the other answer is $\frac{-3}{4} - \frac{7}{4}$ or $-\frac{10}{4} = \frac{-5}{2}$.

To use the QUADRATIC EQUATION, everything must be on one side and zero on the other side, then use those coefficients to plug into the Quadratic Formula. Watch your signs.

The Quadratic Formula allows you to solve any Quadratic Equation. The bad news is if you use it to solve an equation, it takes 2 or 3 minutes to do. The good news is to solve a quadratic; it takes 2 to 3 minutes to do.

What that means, if you can see the factors immediately – Solve the problem by factoring, its quicker.

However, if you don't see the factors and you don't think the polynomial can be factored, use the Quadratic Formula:

That way you won't spend forever trying to factor the problem.

Try some of these; you should be able to do them both ways:

Factoring then using the Zero Product Property and by the Quadratic Formula.

1. $x^2 - 6x - 55 = 0$

2. $x^2 + 3x = 40$

3. $2x^2 - 12 = 5x$

SOLUTIONS:

1. $x = 11$ or -5

2. $x = -8$ or -5

3. $x = -\frac{3}{2}$ or 4