

## Simplifying Rational Expressions: + - Bill Hanlon

You add and subtract rational expressions the same way you added and subtracted fractions. That is; 1. you find the least common denominator, 2. you make equivalent fractions, 3. you add or subtract the numerators, 4. you bring down the denominator and 5. you reduce.

That's one of the real treats about math, once you've learned something, the concepts and strategies remain the same.

Let's say I wanted to add  $\frac{x-2}{3}$  and  $\frac{x+5}{2}$

First, I find the least common denominator. In this case its 6. I then make equivalent fractions.

$$\frac{x-2}{3} = \frac{2(x-2)}{6} \text{ and } \frac{x+5}{2} = \frac{3(x+5)}{6}$$

I add my numerators;  $2(x-2) + 3(x+5)$   
 $= 2x - 4 + 3x + 15$  D-Prop  
 $= 5x + 11$  Combining terms

Therefore,  $\frac{x-2}{3} + \frac{x+5}{2} = \frac{5x+11}{6}$

Typically, instead of doing those in separate steps, we would do the problem like this,

$$\begin{aligned} \frac{x-2}{3} + \frac{x+5}{2} &= \frac{2(x-2) + 3(x+5)}{6} && \text{Find the LCD and make equivalent fractions} \\ &= \frac{2x - 4 + 3x + 15}{6} && \text{Distributive Property} \\ &= \frac{5x + 11}{6} && \text{Combine like terms} \end{aligned}$$

The least common denominator (LCD) is the smallest denominator that all other denominators will divide into. Another way of saying that is the LCD will be made up of all the factors of the other denominators.

Let's take a look at a couple.

**EXAMPLE:**

$$\frac{5}{x+2} + \frac{3}{x+1}$$

The LCD must have all the factors of those two denominators, therefore the LCD must be  $(x-2)(x+1)$ . That's easy.

Now to make equivalent fractions.

$$\frac{5}{x-2} = \frac{\quad}{(x-2)(x+1)},$$

Since you multiplied  $(x - 2)$  by  $(x + 1)$ , you have to multiply the numerator by  $(x + 1)$ .

Therefore:

$$\frac{5}{x - 2} = \frac{5(x + 1)}{(x - 2)(x + 1)}$$

Doing exactly the same thing for the other fraction, we have

$$\frac{3}{x + 1} = \frac{3(x - 2)}{(x - 2)(x + 1)}$$

Now adding the numerators,

$$\frac{5(x + 1) + 3(x - 2)}{(x - 2)(x + 1)} = \frac{5x + 5 + 3x - 6}{(x - 2)(x + 1)} = \frac{8x - 1}{(x - 2)(x + 1)}$$

You know this is fun. I know what you're thinking, you want to do more. Right? Let's try this one.

$$\frac{x}{x^2 + x - 6} - \frac{2}{x^2 - 5x + 6}$$

To find the least common denominator, I have to have a denominator that contains all the factors of the other denominators. That means I have to factor those denominators.

$$\frac{x}{(x + 3)(x - 2)} - \frac{2}{(x - 3)(x - 2)}$$

The LCD must have all those factors:  $(x + 3)(x - 2)(x - 3)$

$$\frac{x}{(x + 3)(x - 2)} - \frac{2}{(x - 3)(x - 2)} = \frac{\quad}{(x + 3)(x - 2)(x - 3)}$$

So far so good.

$$\frac{x}{(x + 3)(x - 2)} - \frac{2}{(x - 3)(x - 2)} = \frac{x(x - 3) - 2(x + 3)}{(x + 3)(x - 2)(x - 3)}$$

Make = fractions

$$= \frac{x^2 - 3x - 2x - 6}{(x + 3)(x - 2)(x - 3)}$$

$$= \frac{x^2 - 5x - 6}{(x + 3)(x - 2)(x - 3)}$$

We would have to see if that could be reduced.