

Simplifying Rational Expressions; $x \div$ Bill Hanlon

Since you thought adding and subtracting polynomials expressions was easy, then you are going to love multiplying and dividing polynomials.

Basically, we have a game of factor and cancel out common factors. Because we have two or three fractions with numerators and denominators, we end up factoring 4, 5, or 6 polynomials before we get to divide out common factors.

EXAMPLE:

Reduce $\frac{x^3 - 9x}{x^3 + x^2 - 6x}$

Factoring, we have

$$\begin{aligned}\frac{x^3 - 9x}{x^3 + x^2 - 6x} &= \frac{x(x^2 - 9)}{x(x^2 + x - 6)} \\ &= \frac{\cancel{x}(x+3)(x-3)}{\cancel{x}(x+3)(x-2)}\end{aligned}$$

The x 's cancel as do the $(x+3)$'s, we are left with $\frac{(x-3)}{(x-2)}$.

What could be easier? You can clearly see the importance in your ability to factor polynomials quickly. If you are still having some difficulty, go back over the tapes on factoring and the printed material that comes with it. Your future success depends on your ability to factor.

Now, if you understand how to reduce, then multiplying and dividing polynomials is just an extension of reducing.

Let's take a look.

EXAMPLE: Simplify

$$\frac{x^2 - 16}{x^2 + 7x + 12} \times \frac{x^2 - 4x - 21}{x^2 - 4x}$$

Factoring

$$\frac{(x+4)(x-4)}{(x+4)(x+3)} \frac{(x-7)(x+3)}{x(x-4)} \quad \text{Canceling, we have}$$

$$\frac{x-7}{x} \quad \text{That's it, we're done.}$$

To multiply polynomials, we factor and divide out like terms. It's that simple. Can I make it more difficult? I wish. All you need to know is how to factor and cancel – just like reducing fractions.

How about dividing polynomials? Certainly that must be difficult. No way. Remember how you divided fractions? You inverted the divisor and multiplied. Well, to divide polynomials, we do the same. Piece of cake.

EXAMPLE: Divide

$$\frac{x^2 - 81}{x^2 - 16} \div \frac{x^2 - x - 20}{x^2 + 5x - 36}$$

Watch me now. I am going to do two steps at once. I'm going to factor each of these polynomials and invert the divisor. Excited?

$$\frac{(x+9)(x-9)}{(x+4)(x-\cancel{4})} \cdot \frac{(x+9)(x-\cancel{4})}{(x-5)(x+4)}$$

we have

$$\frac{(x+9)^2(x-9)}{(x+4)^2(x-5)}$$

What could be more fun?