Fractional Equations

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Your ability to solve fractional equations depends greatly on whether you can add or subtract rational expressions.

You might remember, to add or subtract rational expression; you find a common denominator, make equivalent fractions, add or subtract the numerators, bring down the denominator and reduce.

Have you ever noticed most people prefer not to work with fractions? We are going to adopt that same preference. Our strategy when we see a fractional equation is to get an equivalent equation that is not fractional. Neat, you're thinking.

How do we do that? Well, just like when we added rational expressions we found a common denominator. But rather than making equivalent fractions, we are going to multiply both sides of the equation by the least common denominator. That gets rid of the denominators, which means we don't have fractions any more.

Remember, the least common denominator must have all the factors of the other denominators. So again, you get to factor some polynomial expressions. Excited, aren't you? Let's start with an easy one.

EXAMPLE: Find the solution set

$$\frac{x}{x-6} = 3$$

Since there is only one fraction, the LCD must be x-6. Now, multiplying both sides b the LCD, we have

$$(x - 6)\frac{x}{x-6} = (x - 6)\ 3$$

The (x – 6)'s cancel

$$x = 3x - 18 \quad \text{Solving}$$
$$-2x = -18$$
$$x = +9$$

Can I make those problems more difficult? Absolutely not. All I can do is make them longer. The strategy is very straight forward, multiply by the common denominator to get rid of the fractions, and then solve the resulting equation.

EXAMPLE: Find the solution set

$$\frac{x}{x-2} + \frac{2}{x+2} = \frac{x^2+4}{x^2-4}$$

Factor $x^2 - 4$. The LCD is (x + 2)(x – 2), so let's multiply both sides by the LCD to get rid of the fractions.

$$(x + 2)(x - 2)\ \left(\frac{x}{x-2} + \frac{2}{x+2}\right) = (x + 2)(x - 2)\ \frac{x^2+4}{(x+2)(x-2)}$$

Now, using the Distributive Property and dividing out, we have:

$$x(x + 2) + 2(x - 2) = x^2 + 4 \quad \text{Solving}$$
$$x^2 + 2x + 2x - 4 = x^2 + 4$$
$$x^2 + 4x - 4 = x^2 + 4$$
$$4x - 4 = 4$$
$$4x = 8$$
$$x = 2$$

Piece of cake, you multiply by the LCD getting rid of the fractions and solve the resulting equation using previously learned strategies. But I have a minor glitch. The answer, the solution, is the value of the variable that makes the open sentence (original equation) true. If we were to plug 2 back into the original equation, we would get a zero in the denominator. In math, we can't let this happen.

That means 2 does not work. It is not a value of the variable that makes the equation true. So we end up with a problem that does not have an answer. The way we say that mathematically, we say the answer is the empty set, written $\varnothing$.

That is the first time something like that has happened to us. The question is, when does such a thing happen. To answer that we have to know what caused the problem. The 2 made the denominator zero. The problem occurs when you have fractions the solution you find makes a denominator zero.

From this point on, anytime you solve a fractional equation, you are going to have to check your answer(s) to make sure they don't make denominators zero. That's real important!

Let's do another problem.

EXAMPLE: Find the solution set

$$\frac{2x+1}{x-1} - \frac{3x}{x+2} = \frac{5x-2}{x^2+x-2}$$

Factor the denominators; x^2+x-2 = (x + 2)(x – 1). The LCD is (x + 2)(x – 1). Now multiply both sides by the LCD.

$$(x-1)(x+2)\left(\frac{2x+1}{x-1} - \frac{3x}{x+2}\right) = (x-1)(x+2)\left(\frac{5x-2}{x^2+x-2}\right)$$

$$(2x+1)(x+2) - 3x(x-1) = 5x-2$$

Now multiply and combine terms.

$$2x^2+5x+2-3x^2+3x = 5x-2$$
$$-x^2+8x+2 = 5x-2$$

Solve the Quad. Equation.

$$0 = x^2-3x-4$$
$$0 = (x-4)(x+1)$$

Therefore, the two solutions are x = 4 or x = –1.

Don't forget to check your answers!

