

Algebra,

Been there – Done that

Quadratic Formula

Mathematical Systems

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Algebra, Been there –Done that is a newsletter that links algebra to previously learned concepts and skills or outside experiences

Looking at special products led us to see the relationship between the linear term and the constant when a binomial was being squared.

Special Product

$$(x+10)^2 = x^2 + 20x + 100$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

The pattern we saw, taking half the coefficient of the linear term and squaring it, allowed us to solve quadratic equations that we could not factor.

Completing the Square Relationship

For example, we could not solve an equation such as; $x^2 + 6x - 11 = 0$ because we could not factor it. However, if we use the special product pattern for squaring binomials, we can solve this problem by “completing the square”.

Half the middle term and square.

$$x^2 + 6x - 11 = 0$$

$$x^2 + 6x + _ = 11 + _$$

Coefficient of quadratic term MUST be 1.

$$x^2 + 6x + 9 = 11 + 9$$

$$(x+3)^2 = 20$$

$$x+3 = \pm\sqrt{20}$$

Generalization of Completing the Square leads to the

$$x = \pm\sqrt{20} - 3$$

$$x = \pm 2\sqrt{5} - 3$$

Quadratic Formula

If we applied our knowledge of special products with solving quadratic equations, we would notice the relationship between the linear term and the constant only occurs when the coefficient of the quadratic term is one.

So, to solve an equation in the form $ax^2 + bx + c = 0$ by completing the square, we would have to divide both sides of the equation by a .

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = \frac{0}{a}$$

By completing the square, we would end up with a formula that allows us to solve quadratic equations quickly by just using the coefficients of the equation. That formula is called the Quadratic Formula

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

To use the Quadratic Formula, just plug in the values for a, b, and c from the equation;

$$ax^2 + bx + c = 0$$

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The Quadratic Formula is derived from completing the square which was linked to the patterns seen in special products.