

Writing Equations of Lines

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When we graphed linear equations earlier, we used two equations, the General Form of the Equation of a Line, $Ax + By = C$ and the Slope Intercept Form, $y = mx + b$. Both of those equations allow us to graph a linear equation by inspection.

Now the question arises, what if I have the graph or know something special about the graph, can you find an equation to describe that graph. I know what you're thinking, I wouldn't have asked unless I could.

Taking information we already know, the slope m equals the change in y over the change in x .

Mathematically, we write

$$m = \frac{y - y_1}{x - x_1}$$

Multiplying both sides by the denominator, we have

$$m(x - x_1) = \frac{y - y_1}{x - x_1}(x - x_1)$$

$$m(x - x_1) = y - y_1 \quad \text{or} \quad y - y_1 = m(x - x_1)$$

This is called the **Point Slope Form of a Line**. Its generally used to find an equation of a line. The "m" stands for the slope, the (x_1, y_1) represents a point.

To use the Point Slope Form of an Equation, you must know at least one point and be able to find the slope.

Let's look at a few. Find an equation of a line that passes through the points (2, 5) and (4, 13).

First, we'll break out the formula for finding equations of lines.

$$y - y_1 = m(x - x_1)$$

We do have a point, so I know (x_1, y_1) . The slope, m , was not given to me. Can I find the slope given that information? The answer is yes. The slope is the change in y over the change in x .

Using the two ordered pairs (2, 5) and (4, 13), we have:

$$m = \frac{13 - 5}{4 - 2} = 4$$

Now that I know a point and the slope, I just plug those into the Point Slope Equation.

It doesn't matter which point you use. Using either point, you will arrive at the same answer. I'll use (2, 5).

$$\begin{aligned}y - y_1 &= m(x - x_1) \\y - 5 &= 4(x - 2) \\y - 5 &= 4x - 8 \\y &= 4x - 3\end{aligned}$$

Often times the slope is not just given to you, you need to find it as we just did. Things you need to remember are parallel lines have the same slope, perpendicular lines have negative reciprocal slopes.

As soon as someone says find an equation of a line, no matter what comes next, you should write the Point Slope Form of a Line, and plug in the appropriate numbers.

Try a couple

1. Find an equation of a line passing through $(2, 3)$ and $(3, 7)$.
2. Find an equation of a line passing through $(4, 5)$ having a slope 3.
3. Find an equation of a line passing through $(3, 1)$ that is parallel to $4x - 2y = 6$.