Systems; Linear – Quadratic

You might remember when we solved systems of linear equation earlier. We had the opportunity to choose an appropriate method; linear combination, substitution or through the use of matrices.

To solve systems of higher degree equations, we are going to use the **substitution method**.

Recall, using the substitution method, you had to solve for one of the variables in one of the equations and substitute that into the other equation and solve. Sounds easy enough.

Let's just see how easy.

Solve the following system.

$$x^2 + y^2 = 13$$

$$x + y = 5$$

If I solve for x in the second equation I have $x = 5 - y$.

Now substitute that into the top equation everywhere you see an x.

Doing that,

$$x^2 + y^2 = 13$$

$$(5 - y)^2 + y^2 = 13$$

$$25 - 10y + y^2 + y^2 = 13$$

$$25 - 10y + 2y^2 = 13 \quad \rightarrow \quad 2y^2 - 10y + 12 = 0$$

$$2(y^2 - 5y + 6) = 0 \quad \rightarrow \quad 2(y - 3)(y - 2) = 0$$

Therefore, $y = 3$ or $y = 2$

What about the values of x ? Plug those numbers back into the equation $x = 5 - y$.

When y is 3, x is 2. When y is 2, x is 3.

Writing those in ordered pairs we have (2, 3) and (3, 2).

Notice we had two different answers. That is because the first equation was an equation of a circle and the second equation was an equation of a line. The line intersected the circle twice, therefore I had 2 points of intersection.

Let's try another.

$$x^2 + y^2 = 4$$

$$x^2 + 4y^2 = 8$$

The first equation is a circle, the second is an ellipse. How many points of intersection might we have?

Well we could graph and find out or we could solve the system by substitution.

Solving for x^2 in the second equation, we have

$$x^2 = 8 - 4y^2$$

Now everywhere I see an x^2 in the other equation, I substitute $8 - 4y^2$.

The first equation was $x^2 + y^2 = 4$, substituting, we have:

$$8 - 4y^2 + y^2 = 4 \qquad \text{Solving}$$
$$8 - 3y^2 = 4$$
$$-3y^2 = -4$$
$$y^2 = +\frac{4}{3}$$

Simplifying, y = $\pm\sqrt{\frac{4}{3}}$ or

$$y = \pm\frac{2}{\sqrt{3}} \rightarrow y = \pm\frac{2\sqrt{3}}{3}$$

Now plug these values for y back into $x^2 = 8 - y^2$ to find the corresponding values of x and the resulting ordered pairs.