

Synthetic Substitution

The first way you were taught to evaluate a polynomial for a given value of a variable was direct substitution. Simply put, that means you plugged that value into the expression and found the result.

EXAMPLE: If $P(x) = 2x^2 + 3x - 10$, find P when $x = 5$. By substituting, $P = 2(5)^2 + 3(5) - 10$
 $P(5) = 55$

The process of direct substitution can become a real pain for higher degree polynomial expressions. There is another method called SYNTHETIC SUBSTITUTION that will make evaluating a polynomial a very simple process.

Given some polynomial $Q = 3x^3 + 10x^2 - 5x - 4$ in one variable. You can evaluate Q when $x = 2$ by plugging in that value as we did before.

$$\begin{aligned} Q(x) &= 3x^3 + 10x^2 - 5x - 4 \\ &= 3(2)^3 + 10(2)^2 - 5(2) - 4 \\ &= 3(8) + 10(4) - 10 - 4 \\ &= 24 + 40 - 10 - 4 \\ &= 50 \end{aligned}$$

So the value of Q is 50 when x is 2.

Or by using SYNTHETIC SUBSTITUTION, we would write the coefficients of Q :

$$\begin{array}{cccc} Q = 3x^3 + 10x^2 - 5x - 4 \\ & 3 & 10 & -5 & -4 \end{array}$$

Now, we'll leave a space under those coefficients and draw a line. We will also write down the value of the variable to be plugged in.

$$\begin{array}{r|cccc} 2 & 3 & 10 & -5 & -4 \\ \hline & & & & \end{array}$$

Once we do that, we are set up to evaluate Q when $x = 2$.

To accomplish that, we bring down the first number, 3, and multiply by 2, then add. Keep repeating this process. The last value will be the value of Q when x is 2.

$$\begin{array}{r|cccc} 2 & 3 & 10 & -5 & -4 \\ & & 6 & 32 & 54 \\ \hline & 3 & 16 & 27 & 50 \end{array}$$

Notice, we did get 50 as we did before.

EXAMPLE: Evaluate $2x^4 - x^3 + 5x + 3$ when $x = 3$

In this example, notice there is no quadratic term, no x^2 . When we write the coefficients, we'll need to write zero for the coefficient of that missing term.

$$\begin{array}{r|ccccc} 3 & 2 & -1 & 0 & 5 & 3 \\ & & 6 & 15 & 45 & 150 \\ \hline & 2 & 5 & 15 & 50 & 153 \end{array}$$

The value of that polynomial expression when $x = 3$ is 153.

Let's evaluate the same expression when x is -3

$$\begin{array}{r|rrrrr}
 -3 & 2 & -1 & 0 & 5 & 3 \\
 & & -6 & 21 & -63 & 174 \\
 \hline
 & 2 & -7 & 21 & -58 & 177
 \end{array}$$

The value of the polynomial expression when $x = -3$ is 177.

Piece of cake. This could be written as the ordered pair, $(-3, 177)$ if we wanted to graph this relation.

Let's try another one you say, OK!

EXAMPLE: Let $y = 2x^4 + x^3 - 11x^2 - 4x - 12$, find the value of y when $x = 2$.

$$\begin{array}{r|rrrrr}
 2 & 2 & 1 & -11 & -4 & 12 \\
 & & 4 & 10 & -2 & -12 \\
 \hline
 & 2 & 5 & -1 & -6 & 0
 \end{array}$$

The value of y when $x = 2$ is 0.

If we were to write this as an ordered pair, we'd have $(2, 0)$.

If we were to graph that, we'd also notice that $(2, 0)$ is an x -intercept, where a graph crosses the x -axis.

In algebra, when we solve quadratic or higher degree equations, we set the equations equal to zero, then find values of the variable that will make the equation true.

Notice, in the last example $x = 2$ made $y = 0$. Oh wow! What that means is $x = 2$ is a solution to the equation.

$$2x^4 + x^3 - 11x^2 - 4x + 12 = 0$$

In fact, anytime the last number in a synthetic substitution problem is zero, The value we plugged in represents a solution, a zero, or what most of us might call an answer, if we had an equation.

Can you feel the excitement running through your veins?

If you put this together with the Rational Root Theorem, we now have a very simple method of finding rational roots and if you think about it, another method of factoring.

If $x = 2$ is a zero, a solution, then $x - 2$ must be a factor of the polynomial expression.

Now don't you just love how math seems to come together.