Rational Root Theorem

If P (x) = 0 is a polynomial equation with integral coefficients of degree **n** in which a_0 is the coefficients of x^n, and a_n is the constant term, then for any rational root p/q, where p and q are relatively prime integers, p is a factor of a_n and q is a factor of a_0

$$a_0 x^n + a_1 x^{n-1} + \ldots + a_{n-1} x + a_n = 0$$

That's math talk. What that means is you have to start with an equation without fractions, and "if" there are rational answers (roots), the numerator of the answer must be a factor of the constant (a_n) and the denominator will be a factor of the leading coefficient (a_0).

EXAMPLE: Find the rational roots of

$$2x^2 + 5x - 12 = 0$$

$a_0 = 2$ $a_n = -12$

Notice, no fractions, we have integral coefficients. The factors of the leading coefficient 2 are; $\pm 2, 1$

The factors of the constant –12 are; $\pm 12, 6, 4, 3, 2, 1$.

Now, we take each of the factors of the constant and put them over each of the factors of the leading coefficient (p/q). All of those fractions will be possible answers (roots) along with their negatives.

$$\pm 12/2, 12/1, 6/2, 6/1, 4/2, 4/1, 3/2, 3/1, 2/2, 2/1, 1/2, 1/1.$$

Oh yes – you are having fun. Math is your life.

The Rational Root Theorem says "if" there is a rational answer, it must be one of those numbers. Some of those possible answers repeat.

To find which, or if any of those fractions are answer, you have to plug each one into the original equation to see if any of them make the open sentence true.

It turns out $3/2$ and – 4 are solutions.

That's alot of plugging in. However, if we were able to plug in using SYNTHETIC SUBSTITUTION our work would be much, much easier.

One side note, we could have solved this particular equation more efficiently by factoring or the Quadratic Formula.

The question is, why then am I solving it by the Rational Root Theorem? For practice. For practice. Typically you use the Rational Root Theorem when you have higher degree equations or when you can't factor the polynomial.

This is an importance theorem because before knowing it, if you could not factor the polynomial, you could not solve the equation.

Yes, math is a blast!!

EXAMPLE: Find the possible rational solutions of: $x^3 + 6x^2 + 3x - 10 = 0$

Can you use the Quadratic Formula?

Can you factor the polynomials?

The answer to both these questions is NO. The only other method we know at this time to solve this equation is by using the Rational Root Theorem.

The factors of the leading coefficient are; ± 1

The factors of the constant are; $\pm 10, 5, 2, 1$

Placing the factors of the constant over the factors of the leading coefficient, the possible solutions are:

$$\pm 10, 5, 2, 1$$

Again, to find out if any of these are solutions, I would have to plug them back into the original equation.

The Rational Root Theorem does not guarantee that there is a rational solution. So, there are times when none of the possible solutions will work. The equation will have a solution, it just won't be rational.

See if you can determine possible rational roots of the following equation just by looking.

$$x^4 - 5x^3 + 9x^2 - 7x + 2 = 0$$

The equation looks longer, but it is not more difficult. The only possible rational solutions are $\pm 2, 1$.

Try this one.

$$2x^3 + 3x^2 - 8x + 3 = 0$$

What are the factors of 2?

What are the factors of 3?

Therefore, the possible solutions are $\pm\ 3/2,\ 3/1,\ 1/2,\ 1/1$.

The factor and Remainder Theorems are two very closely related theorems; you might want to look them up for the fun of it.