Solving Exponential Equations

Most of us are familiar and comfortable with problems such as $5^2 = 25$ and $2^3 = 8$.

In algebra, we are sometimes asked to solve equations like $x^2 = 25$. Without too much fanfare, most of us would answer by saying $x = 5$ or $x = -5$. You could have solved that by factoring, then using the Zero Product Property or you could have solved by the definition:

$$\text{if } x^2 = b, \text{ then } x = \pm\sqrt{b}.$$

You'll notice, up to this point, the variables in the equations we have been solving for are not in the exponent. What would happen if the variable was in the exponent, a problem like $3^x = 81$.

One way to solve it is by trying to plug a number in for x by trial and error that would make the equation true. Using intelligent guessing, trial and error is OK, but it's time consuming.

If we worked with enough of these, we might see a way of solving this same equation by rewriting 81 as a power of 3.

For instance; 81 = 3□

By substituting, we have 3^x = 3□. Now, since the numbers are equal and the bases are equal, then the exponents must be equal. In other words, $x = 4$.

Mathematically, we write:

For $b > 0$, $b \neq 1$ $b^x = b^y$ if and only if x =

Don't you just love how we write things mathematically? What's this b has to be greater than zero and not equal to one business?

Let's try b being negative, not greater than zero, and see what happens. $(-2)^2 = 2^2$, the exponents are equal, are the bases then equal?

How about when the base equals one: 1□ = 1^{12} in this case, the bases are equal, do the exponents have to be?

Now you know why $b > 0$ and $b \neq 1$.

EXAMPLE: Solve $2^5 = 2^{2x-1}$

Since the bases are equal, then the exponents must be equal.

Therefore,

$5 = 2x - 1$ Solving

$6 = 2x$

$3 = x$

Plugging in 3 makes the original equation true

EXAMPLE: **Solve** $2^{6x^2} = 4^{5x+2}$

Notice the bases are not the same, we therefore can not set the exponents equal.

That's too bad, things were working out so well, But alas!

I've always wanted to use that expression.

Is it possible to make the bases the same? Can I write the number 4 as a power of 2? You wouldn't have asked if there was not a way you say.

Let's look at the problem;

$$2^{6x^2} = 4^{5x+2}$$

We know $2^2 = 4$ $\qquad 2^{6x^2} = (2^2)^{5x+2}$ $\qquad$ Simplified exponents

$$2^{6x^2} = 2^{10x+4}$$

Wow! Now the bases are the same, we can now set the exponents equal like we did before.

$6x^2 = 10x + 4$

Solving

$6x^2 - 10x - 4 = 0$

$2(3x^2 - 5x - 2) = 0$

$2(3x + 1)(x - 2) = 0$

$X = \frac{1}{3}$ or $x = 2$

Yes, say it, you love math!!!

You might be thinking that the problem was more difficult than the first one; $3^x = 81$. But in reality, it was not. It was longer – not harder.

The strategy we'll use to solve equations that have a variable in the exponent is:

1. Express each side of the equation as a power in the SAME base.
2. Simplify the exponents
3. Set the exponents equal
4. Solve the resulting equation

EXAMPLE: Solve $9^{n-1} = (\frac{1}{3})^{4n-1}$

I have to write each side of the equation using the SAME base. I can write 9 as 9^1 or as 3^2.

What can I do with $\frac{1}{3}$. If you remember, that is equal to 3^{-1}.

Therefore, I can write both sides having base 3

$9^{n-1} = (\frac{1}{3})^{4n-1}$ $\qquad$ Make the bases the SAME

$(3^2)^{n-1} = (3^{-1})\ 4^{n-1}$ $\qquad$ Simplify

$3^{2n-2} = 3^{-4n+1}$ $\qquad$ Set the exponents equal

$2n - 2 = -4n + 1$

$6n = 3$

$n = \frac{1}{2}$

Try this next one on your own.

$9^{3x} = 27^{x-2}$

The answer is -2.