Solving Linear Equations with Rational Coefficients **Bill Hanlon**

Last time we solved linear equations with coefficients that belonged to the set of natural numbers.

Today we will solve the same type of equation, however we will use rational coefficients. That is to say we will involve fractions.

You might recall the overall strategy we used last time was to use the Order of Operations in reverse using the opposite operation to undo an expression in the ax + b = c format. The good news is that does not change.

Having fractional coefficients does not make the problems any more difficult. The question is, can we make equations with fractional coefficients look like the problems we have already solved? I would not have asked unless there was.

If I multiplied both sides of an equation by a common denominator, that will get rid of the fractions.

EXAMPLE: Solve $\frac{x}{2} + \frac{x}{3} = 10$

Using magic, I'll make the denominators 2 and 3 disappear by multiplying both sides by the common denominator – 6

$$6\left\{\frac{x}{2} + \frac{x}{3}\right\} = 6\,(10)$$

Simplifying, I have

$$3x + 2x = 60$$

That gets rid of the fractions. Now I will combine like terms,

$$5x = 60$$
$$x = 12$$

Notice, once we multiplied both sides of the equation by the common denominator, we no longer had fractions and we solved the resulting equation the same way we did last time. Piece of cake! Makes you want to do a longer problem, doesn't it?

EXAMPLE: **Solve** $\frac{x+3}{4} - \frac{4x-5}{5} = -1$

As you can see, these problems with rational coefficients are no big deal. How so I get rid of the denominator? Good, multiply both sides by the common denominator, which is 20.

$$20\left[\frac{(x+3)}{4} - \frac{4x-5}{5}\right] = 20(-1)$$

$5(x + 3) – 4(4x – 5) = –20$ That gets rid of the denominators

Now, how do you get rid of the parentheses? Use the Distributive Property.

$5x + 15 – 16x + 20 = –20$

That equation is still not in the ax + b = c format. So I will need to combine like terms.

$$-11x + 35 = -20$$

When I do that, the equation is now in ax + b = c format. I know how to solve those by undoing the expression by using the Order of Operations in reverse using the opposite operation.

$$-11x = -55$$
$$x = +5$$

The reason this problem seemed longer than the other problem we have solved is because we had to not only multiply both sides by the common denominator to get rid of the fractions, we then had to use the distributive property to get rid of the parentheses, then combine terms. After all that was accomplished, the resulting equation looked like the ones we solved before.

How can you be sure if you got the right answer? A solution should satisfy the original equation. Plug +5 into that equation and check.

Try this one on your own, the answer is 7.

$$\frac{2x-2}{4} + \frac{2x+1}{3} = x + 1$$