Algebra II Notes – Unit One

Syllabus Objective 1.1 – The student will differentiate among subsets of the real number system.

Real Numbers: Numbers that can be graphed on the number line

Ex: $3,-10,\sqrt{2},\frac{8}{5},4.2,\pi$ Put the numbers in order and graph on the real number line.

The real number system is made up of the following subsets:

- Whole Numbers: the set of the counting numbers including 0

  $\{0,1,2,3,...\}$

- Integers: the whole numbers and their opposites

  $\{...-3,-2,-1,0,1,2,3,...\}$

- Rational Numbers: numbers that can be written as the ratio of two integers

  Ex: $5,-2,\frac{1}{6},0.\overline{3}$

- Irrational numbers: numbers that are not rational. Decimals that do not terminate and do not repeat.

  Ex: $\sqrt{3},\pi \approx 3.14159...,-\sqrt[3]{7}$

Properties of Real Numbers:

Let *a, b,* and *c* be real numbers.

- Commutative Properties
  - Addition: $a+b=b+a$ **Ex:** $3+-5=-5+3$
  - Multiplication: $a \bullet b = b \bullet a$ **Ex:** $4\cdot 2 = 2\cdot 4$
- Associative Properties
  - Addition: $(a+b)+c=a+(b+c)$ **Ex:** $(-3+2)+8=-3+(2+8)$
  - Multiplication: $(ab)c=a(bc)$ **Ex:** $\left(4\cdot\frac{1}{3}\right)\cdot 9 = 4\cdot\left(\frac{1}{3}\cdot 9\right)$

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- Identity Properties
  - Addition (Note: 0 is the "**identity**" of addition): $a+0=a$ **Ex:** $0+-3=-3$
  - Multiplication (Note: 1 is the "**identity**" of multiplication): $1\cdot a=a$ **Ex:** $1\cdot\pi=\pi$
- Inverse Properties
  - Addition (Note: $-a$ is called the "**opposite**" of a): $a+-a=0$ **Ex:** $-0.5+0.5=0$
  - Multiplication (Note: $\frac{1}{a}$ is called the "**reciprocal**" of a): $a\cdot\frac{1}{a}=1$ **Ex:** $\frac{3}{4}\cdot\frac{4}{3}=1$
- Distributive Property

$a(b+c)=ab+ac$ $a(b-c)=ab-ac$ **Ex:** $-2(3+8)=-2\cdot 3+-2\cdot 8$

Review: Operations with real numbers.

1. Find the product of 3 and $-\frac{2}{9}$.
2. Find the sum of -8 and -4.2.
3. Find the difference of 6 and -23.
4. Find the quotient of $-\frac{1}{3}$ and 4.

<u>You Try:</u> Fill in the chart using the following subsets of numbers. Give examples in each box.

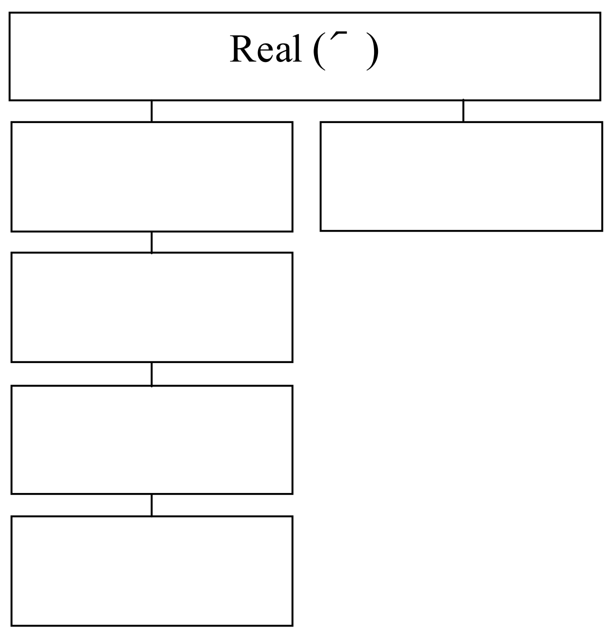
Counting ($\mathbb{N}$)

Integers ($\mathbb{Z}$)

Irrational

Rational ($\mathbb{Q}$)

Whole



<u>QOD</u> (Question of the Day): What is the difference between a rational and irrational number?

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Syllabus Objectives 1.2 – The student will simplify numerical and algebraic expressions applying the appropriate field properties. 1.3 – The student will demonstrate proper techniques for entering data into his/her calculator.

Powers:

$a^b = c$ a is the **base**, b is the **exponent**, and a^b is the **power**

A power is repeated multiplication. **Ex:** $a^5 = a \cdot a \cdot a \cdot a \cdot a$

Caution: Be careful with negative signs.

$$-2^4 = -2 \cdot 2 \cdot 2 \cdot 2 = -16 \qquad \text{and} \qquad (-2)^4 = (-2)(-2)(-2)(-2) = 16$$

Order of Operations:

1. Parentheses – perform any operation in parentheses or grouping symbols
2. Exponents – evaluate any powers
3. Multiplication and Division – perform these two operations from left to right as seen in the expression
4. Addition and Subtraction – perform these two operations from left to right as seen in the expression

Mnemonic device for the Order of Operations: PEMDAS or "Please Excuse My Dear Aunt Sally"

Ex: Evaluate the expression $2x^2 - 6(x+3y) \div 2 + 5$ when $x = -3$ and $y = 5$.

Ex: Write an algebraic expression for the following real-life situation. Suppose you have \$25 to spend for music on a website that charges \$1.50 per song. Write an expression that shows how much money you have left after buying n songs. Evaluate the expression for $n = 6$.

Ex: Write an algebraic expression for the area of a triangle that has a base of $w - 6$ ft. and a height of w ft. Find the area when $w = 14$. Label your answer with correct units.

Simplifying Algebraic Expressions: To simplify an algebraic expression, eliminate parentheses using the distributive property and combine like terms. Terms are separated by + or – signs.

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Term: $3x^4$ 3 is the **coefficient**, and x^4 is the **variable part**

Like terms are terms with the same variable part. A **constant term** is a term without a variable part.

Ex: $4x^2 - 3\left(2x^3 + 6x^2 - y^2 + xy + 1\right)$

Step One: Use the Distributive Property. $4x^2 - 6x^3 - 18x^2 + 3y^2 - 3xy - 3$

Step Two: Combine like terms. $-6x^3 - 14x^2 + 3y^2 - 3xy - 3$

How many terms are in your simplified expression? 5 How many are constant terms? 1

Evaluate the expression when $x = -1$ and $y = 7$.

Step One: Substitute the given values in for the variables.

$-6\left(-1\right)^3 - 14\left(-1\right)^2 + 3\left(7\right)^2 - 3\left(-1\right)\left(7\right) - 3$

Step Two: Simplify using the order of operations.

Exponents: $-6\left(-1\right) - 14\left(1\right) + 3\left(49\right) - 3\left(-1\right)\left(7\right) - 3$

Multiplication: $6 - 14 + 147 + 21 - 3$

Addition/Subtraction (left to right) $-8 + 147 + 21 - 3$

$139 + 21 - 3$ $160 - 3 = 157$

Calculator Activity: Does your calculator "know" the order of operations?

Evaluate the following expression on the calculator. Write down the keystrokes necessary for the calculator to obtain the correct answer according to the order of operations.

$$5\left(\frac{32-2}{3^2+1}\right)^3$$

<u>You Try:</u> Evaluate the expressions when $x = -5$ and $y = -1$.

a. $\dfrac{x+y}{y+5}$ b. $-2x^2 - y$ c. $\dfrac{1}{2}\left(3x - 5y\right) + 4$

<u>QOD:</u> Describe what it means for terms to be like terms and give an example.

Syllabus Objective 1.4 – The student will solve linear equations and inequalities, and absolute value equations in one variable.

Equation: an algebraic "sentence" that consists of two equal algebraic expressions

Linear Equation: an equation that can be written in the form $ax+b=c$

Note: To solve an equation, we use the properties of equality to isolate the variable on one side of the equation. When isolating the variable, we will "undo" addition/subtraction first, then multiplication/division using inverse operations. This is the order of operations in reverse!

Ex: Solve the equation $-8x+6=5$.

Step One: Subtract 6 from both sides. $-8x=-1$

Step Two: Divide both sides by -8. $x=\frac{1}{8}$

For more complicated equations, you may have to simplify both sides of the equation first using the distributive property and combining like terms.

Ex: Solve the equation $6(x-4)-2x=24$

Step One: Use the distributive property. $6x-24-2x=24$

Step Two: Combine like terms. $4x-24=24$

Step Three: Add 24 to both sides. $4x=48$

Step Four: Divide both sides by 4. $x=12$

If there are variables on both sides, you will need to simplify both sides of the equation and then bring all variables to the same side using inverse operations.

Ex: Solve the equation $3x-2=4(x+6)$

Step One: Use the distributive property. $3x-2=4x+24$

Step Two: Subtract $3x$ from both sides. $-2=x+24$

Step Three: Subtract 24 from both sides. $-26=x$

Note: We can also write the answer in set notation. The solution set to the equation above is $\{-26\}$.

When solving an equation with fractions, it saves time to "wipe-out" the fractions by multiplying both sides by the LCD (lowest common denominator).

Ex: Solve the equation: $\frac{3}{5}x-\frac{9}{10}=\frac{1}{3}+2x$

The LCD of the fractions is 30. Multiply every term in the equation by 30:

$$30\cdot\frac{3}{5}x-30\cdot\frac{9}{10}=30\cdot\frac{1}{3}+30\cdot 2x \quad\rightarrow\quad 18x-27=10+60x$$

Now use the steps illustrated in the previous examples to solve the equation.

Using equations in real-life:

Ex: Ann earns $8 per hour for every 40 hours worked per week, and time-and-a-half for every hour over 40 hours. If Ann earned $380 last week, how many hours did she work?

Solution: Let h be the number of hours of overtime (over 40) Ann worked last week.

$$8\cdot 40+1.5(8h)=380$$

$$320+12h=380$$

$$12h=60$$

$h=5$ Therefore, Ann worked a total of 45 hours last week.

<u>You Try:</u> In 2005, 176 students bought non-spiral notebooks, and this number is increasing by 24 per year. That same year, 331 students bought spiral notebooks, and that number is decreasing by 15 per year. When will the number of spiral notebooks be half of the non-spirals?

<u>QOD:</u> What role does the order of operations play when solving an equation?

Syllabus Objective 1.5 – The student will solve for a given variable in an equation with more than one variable.

The equation $3x+2y=5$ is an example of an equation that is **implicitly defined.** Sometimes it is useful to rewrite the equation so that it is **explicitly defined**, which means it is solved for one of the variables.

Ex: Solve the equation $3x+2y=5$ for y.

Step One: Subtract $3x$ from both sides. $2y=5-3x$

Step Two: Divide both sides by 2. $y=\frac{5}{2}-\frac{3}{2}x$

Note: There are many ways to write the correct answer. This equation is equivalent to the following:

$y=-\frac{3}{2}x+\frac{5}{2}$ and $y=\frac{5-3x}{2}$

Sometimes it is useful to solve formulas for one of the variables.

Ex: Solve the formula for the area of a trapezoid for b_1.

$$A=\frac{1}{2}h\left(b_1+b_2\right)$$

Step One: Multiply both sides by 2 (LCD). $2A=h\left(b_1+b_2\right)$

Step Two: Use the distributive property. $2A=hb_1+hb_2$

Step Three: Subtract hb_2 from both sides. $2A-hb_2=hb_1$

Step Four: Divide both sides by h. $\frac{2A-hb_2}{h}=b_1$ or $\frac{2A}{h}-b_2=b_1$

<u>You Try:</u> Solve the temperature conversion formula for C. $F=\frac{9}{5}C+32$

<u>QOD:</u> Describe a real-life situation where you would need to be able to solve the area formula of a circle for r.

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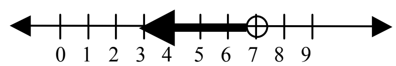
Syllabus Objective 1.4 – The student will solve linear equations and <u>inequalities</u>, and absolute value equations in one variable.

Inequality symbols:

$>$	Greater Than	$\geq$	Greater Than or Equal To
$<$	Less Than	$\leq$	Less Than or Equal To

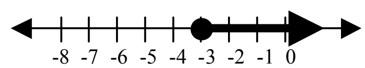
Graphing an inequality: Use an open circle for greater than (>) or less than (<). Use a closed circle for greater than or equal to ($\geq$) or less than or equal to ($\leq$).

Ex: Graph the inequality $x < 8$



Ex: Graph the inequality $-3 \leq x$

Note: It may help to rewrite as $x \geq -3$



Ex: Write an inequality for the situation. Zac needed to score at least a 76% on his Algebra exam to get a B in the class. $s \geq 76$ (where s is Zac's exam score)

Solving a linear inequality: solving an inequality is similar to solving an equation, with one exception. *When multiplying or dividing by a negative number in an inequality, you must FLIP the inequality sign*

Ex: Solve the inequality $-x < 4$.

To isolate the variable, we need to switch sides to obtain $-4 < x$.

Which, written with the variable on the left would read $x > -4$.

An alternative way to solve this inequality would be to divide both sides by -1.

$$\frac{-x}{-1} > \frac{4}{-1}$$

Note: We needed to FLIP the inequality sign!

Ex: Solve the inequality $-2x + 1 \leq 3(x - 8)$

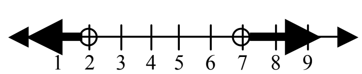
Step One: Distributive property $-2x + 1 \leq 3x - 24$

Step Two: Get variables to one side $-5x + 1 \leq -24$

Step Three: Subtract 1 from both sides $-5x \leq -25$

Step Four: Divide both sides by NEGATIVE 5 (FLIP the inequality) $x \geq 5$

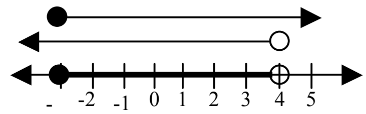
Compound Inequalities: two inequalities joined by "and" or "or"



Ex: $x > 5$ or $x < 2$

Ex: $x \geq -3$ and $x < 4$

Note: This can also be written as $-3 \geq x < 4$



Solving a compound inequality: Isolate the variable. In an "and" statement, isolate the variable between the two inequality signs. What you do to the middle, you must do to both sides!

Ex: Solve and graph: $-3 < -1 - 2x \leq 5$

Step One: Add 1 to the middle and both sides $\quad -2 < -2x \leq 6$

Step Two: Divide the middle and both sides by -2 $\quad 1 > x \geq -6$ Don't forget to FLIP!

Step Three: Rewrite the inequality $\quad -6 \leq x < 1$

Step Four: Graph on a number line (closed circle on -6, open circle on 1, shade in between)

Ex: Solve and graph: $6x - 5 < 7$ or $8x + 1 > 25$

Step One: Solve both inequalities separately $\quad 6x < 12 \quad 8x > 24$

$x < 2 \quad x > 3$

Step Two: Write the solutions as a compound inequality $\quad x < 2$ or $x > 3$

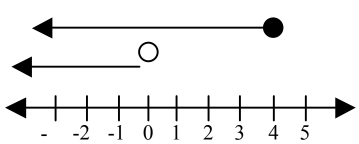
Step Three: Graph on a number line (open circle on 2, shade left; open circle on 3, shade right)

Ex: Application Problem – There are 8 sections of seats in an auditorium. Each section contains at least 150 seats, but not more than 200 seats. Write a compound inequality that represents the number of seats (s) that could be in the auditorium.

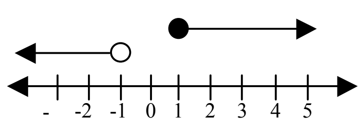
Solution: $8(150) \leq s < 8(200) \quad \rightarrow \quad 1200 \leq s < 1600$

<u>Challenge</u>: What about special cases?

$x \leq 4$ and $x < 0$

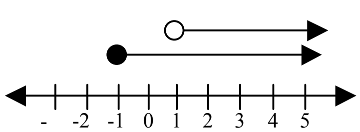


$x \geq 1$ and $x < -1$

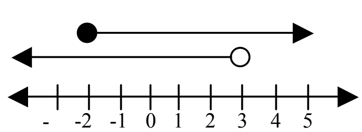


Special cases (continued):

$x > 1$ or $x \geq -1$

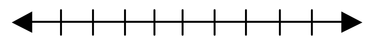


$x \geq -2$ or $x < 3$

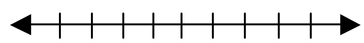


<u>You Try:</u> Solve and graph the inequalities. Is the indicated value a solution?

a) $3 \le 2x - 7 < 11;\ x = 9$



b) $5x - 3 \le 6$ or $7 < 4x - 9;\ x = 1.9$



<u>QOD:</u> When solving an inequality, when must you flip the sign, and why?

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Syllabus Objective 1.4 – The student will solve linear equations and inequalities, and <u>absolute value equations</u> in one variable.

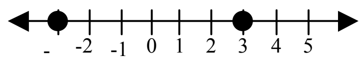
Absolute Value: the distance a number is away from the origin on the number line

Algebraic definition of absolute value: $|a| = \begin{cases} a, & a \geq 0 \\ -a, & a < 0 \end{cases}$

Ex: Solve the absolute value equation: $|x| = 3$

Note: This means that the distance from the origin is 3.

Solution: There are two numbers that are 3 units away from the origin: $x = 3$ and $x = -3$



When solving an absolute value equation in the form $|ax+b| = c$, you must solve the equations $ax+b=c$ and $ax+b=-c$.

Ex: Solve the absolute value equation: $|5x+1| + 3 = 14$

Step One: Put the equation in $|ax+b| = c$ form. $|5x+1| = 11$

Step Two: Rewrite as two equations. $5x+1=11$ $5x+1=-11$

Step Three: Solve both equations. $5x=10$ $5x=-12$

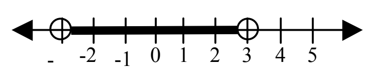
Solutions: $x = 2$ or $x = \frac{-12}{5}$

Note: You can check your solutions by substituting them back into the original equation.

Absolute Value Inequalities:

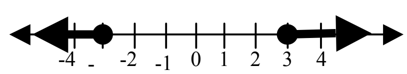
Ex: $|x| < 3$ means that the distance x is away from the origin is less than 3.

So, $x > -3$ and $x < 3$. We write this as $-3 < x < 3$



Ex: $|x| \geq 3$ means that the distance x is away from the origin is greater than 3.

So, $x \leq -3$ or $x \geq 3$.



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Solving absolute value inequalities:

Ex: Solve and graph the solutions to the inequality: $|x+6|<8$

Note: This means that the distance $x+6$ is away from the origin is less than 8.

Step One: Rewrite as two inequalities $\quad x+6<8 \quad\quad x+6>-8$

Step Two: Solve each inequality $\quad x<2 \quad\quad x>-14$

Step Three: Rewrite as a compound inequality $\quad$ Solution: $-14<x<2$

Step Four: Graph on a number line (open circle on -14 and 2, shade between)

Ex: Solve the inequality: $|3x-3|+4\ge 10$

Step One: Rewrite in $|ax+b|\ge c$ form $\quad |3x-3|\ge 6$

(Note: This means that the distance $|3x-3|$ is away from the origin is greater than or equal to 6.)

Step Two: Rewrite as two inequalities $\quad 3x-3\ge 6 \quad\quad 3x-3\le -6$

Step Three: Solve each inequality

$$3x\ge 9 \quad\quad 3x\le -3$$
$$x\ge 3 \quad\quad x\le -1$$

Step Four: Rewrite as a compound inequality $\quad$ Solution: $x\le -1$ or $x\ge 3$

Application of Absolute Value Inequalities:

Ex: At a bottling company, machine A fills a bottle with spring water and machine B accepts bottles only if the number of fluid ounces is between $17\frac{8}{9}$ and $18\frac{1}{9}$. If machine B accepts a bottle containing n fluid ounces, write an absolute value inequality that describes all possible values of n.

Solution: $|n-18|<\frac{1}{9}$

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Challenge: Special Cases

a) $|1-2x|=0$ This has one solution: $x=\frac{1}{2}$

b) $|3x-7|=-5$ This has no solution. (The absolute value cannot be negative, because it is a distance.)

c) $|x+3|<-4$ This has no solution. (The absolute value less than -4, because it would have to be a negative number.)

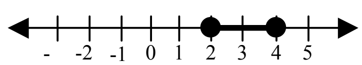
d) $|2x+1|>-9$ The solutions are restricted to when the absolute value is nonnegative (greater than or equal to zero). Therefore, we will solve the inequality $|2x+1|\ge 0$, which gives us $x\ge -\frac{1}{2}$ as our solution.

You Try:

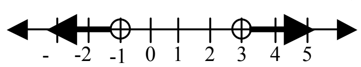
1. Solve and graph: $3|x-6|\le 9$

2. Solve and graph: $|3x+2|-6>2$

3. Write an absolute value inequality for the graph:



4. Write an absolute value inequality for the graph:



QOD: How are the graphs of the solutions of absolute value inequalities with a $<$ or $\le$ different from absolute value inequalities with a $>$ or $\ge$?

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Syllabus Objective 1.7 – The student will develop a mathematical model to solve real-world problems.

When solving an application problem, it is important to have a plan. We will use the following steps to solve the problems that follow.

1. Write a verbal model.
2. Assign labels.
3. Write an algebraic model.
4. Solve the algebraic model.
5. Answer the question.

Ex: A gym offers 2 membership packages: $50 initial fee and $5 each visit or $200 initial fee and $2 each visit. How many visits will it take for the two packages to have equal cost?

1. Initial Fee + Cost Per Visit $\cdot$ Number of Visits = Initial Fee + Cost Per Visit $\cdot$ Number of Visits

   (Package One) (Package Two)

2. Package One: Initial Fee = 50 | Package Two: Initial Fee = 200

   Cost Per Visit = 5 | Cost Per Visit = 2

   Number of Visits = v | Number of Visits = v

3. $50+5v=200+2v$

4. $3v=150$
   $v=50$

5. It will take 50 visits for the two packages to have equal cost.

Ex: Find three consecutive integers with a sum of 84.

1. First Integer + Second Integer + Third Integer = 84
2. First Integer = n, Second Integer = $n+1$, Third Integer = $n+2$
3. $n+(n+1)+(n+2)=84$

   $3n+3=84$
4. $3n=81$
   $n=27$
5. The three consecutive integers are 27,28, and 29.

<u>You Try:</u>

a) If 4 less than 3 times a number is 2 more than the number, what is the number?

b) Joan has a monthly base salary of \$200 and earns a \$10 commission for each item she sells. Jimmy has a monthly base salary of \$300 and earns a \$5 commission for each item he sells. How many items would each need to sell to earn the same amount?

<u>QOD:</u> List and describe the steps of a problem solving plan.