Properties of Real Numbers

Commutative Property – Addition	$a+b=b+a$	Order
Commutative Property – Multiplication	$a \bullet b = b \bullet a$	

Ex. $6 + 4 = 4 + 6$

Ex. $7 \bullet 3 = 3 \bullet 7$

Associative Property - Addition	$(a+b)+c=a+(b+c)$	Grouping
Associative Property – Multiplication	$(a \bullet b) \bullet c = a \bullet (b \bullet c)$	

The properties allow you to manipulate expressions and compute mentally.

Ex. Simplify $4 \bullet 13 \bullet 25$

Using only the Order of Operations to simplify that expression, you would multiply 4 by 13, then multiply that result by 25. Most students would need pencil & paper.

Knowing the properties, I can rewrite $4 \bullet 13 \bullet 25$ as $4 \bullet 25 \bullet 13$ using the commutative property, then group the numbers by using the associative property. $(4 \bullet 25) \bullet 13$.

$100 \bullet 13 = 1300$

Distributive Property - $a \bullet (b \pm c) = a \bullet b \pm a \bullet c$

Ex. Simplify $25 \bullet 12$

That could be simplified by just multiplying 25 by 12 using the standard algorithm. Or, we could use the distributive property to break the number apart to perform the multiplications mentally.

$$\begin{aligned} 25 \bullet 12 &= 25 \bullet (10+2) \\ &= 25 \bullet 10 + 25 \bullet 2 \\ &= 250 + 50 \\ &= 300 \end{aligned}$$

Ex. Use the distributive property to write an equivalent expression; 3(x + 7)

$$\begin{aligned} 3(x + 7) &= 3x + 3(7) \\ &= 3x + 21 \end{aligned}$$

Identity Property for Addition – $a + 0 = a$
Identity Property for Multiplication - $a \cdot 1 = a$

Variable Expressions

Terms are parts of the expression that are added together.

Ex. In the variable expression $2x + 3y - 5$, the 2x, 3y, and –5 are terms of that expression.

Coefficient the number part of the term.

Ex. In the variable expression $2x + 3y - 5$, the 2 is the coefficient of the first term, the x, the 3 is the coefficient of the y – the second term.

Constant is a number without a variable.

Ex. In the expression $2x + 3y - 5$, the only number without a variable attached is –5, negative 5 is the constant.

Simplifying Variable Expressions

Like terms – are terms that have identical variable parts.

Ex. Identify the like terms of $3x + 4 + 5x$

3x and 5x have identical variable parts, they are like terms.

As you would simplify arithmetic expressions by adding 100's to 100's, 10's to 10's and ones to ones, you use that same concept in algebra for simplifying algebraic expressions.

Ex. Simplify $321 + 548$

$= (3 + 5)100 + (2 + 4)10 + (1 + 8)1$
$= 8 (100) + 6 (10) + 9 (1)$
$= 869$

Ex. Simplify $3x + 2y + 1 + 5x + 4y + 8$

As we added 100's to 100's, now we will combine like terms in algebra using the commutative and associative properties.

$3x + 5x + 2y + 4y + 1 + 8$

$8x + 6y + 9$

Notice we got the same coefficients when we added like terms in algebra as we did hen we added the numbers in base 10.

Ex. Simplify $4x + 3 + 5x$

Combining the x's, $9x + 3$

Ex. Simplify $5(x + 4) – 2x + 5$

Using the distributive property to get rid of the parentheses

$5x + 20 – 2x + 5$
Combining like terms
$3x + 25$

Word Translations

STATEMENT	ALGEBRA
twice as much as a number	$2x$
two less than a number	$x – 2$
five more than an unknown	$x + 5$
three more than twice a number	$2x + 3$
a number decreased by 6	$x – 6$
ten decreased by a number	$10 – x$
Tom's age 4 years from now	$x + 4$
Tom's age ten years ago	$x – 10$
number of cents in x quarters	$25x$
number of cents in 2x dimes	$10(2x)$
number of cents in x + 3 nickels	$5(x + 3)$
separate 15 into 2 parts	$x, 15 – x$
distance traveled in x hrs at 50 mph	$50x$
two consecutive integers	$x, x + 1$
two consecutive odd integers	$x, x + 2$
sum of a number and 30	$x + 30$
product of a number and 5	$5x$
quotient of a number and 7	$x \div 7$
four times as much	$4x$
two less than 3 times a number	$3x – 2$

Equation – is a statement of equality between two expressions.

Ex. $3x + 4 = 2x + 8$ represents an equation.

Solution – is a value of a variable that makes an equation (open sentence) true.

Ex. $x = 4$ is a solution to the equation $3x + 4 = 2x + 8$

Properties of Real Numbers

Inverse operations are two operations that undo each other, such as addition and subtraction OR multiplication and division.

Additive Inverse
Multiplicative Inverse

Properties of Equality

Addition Property of Equality	if $a = b$, then $a + c = b + c$
Subtraction Property of Equality	if $a = b$, then $a - c = b - c$
Multiplication Property of Equality	if $a = b$, then $ac = bc$
Division Property of Equality	if $a = b$, $c \neq 0$, then $a/c = b/c$

Solving Equations

We used the Order of Operations to evaluate expressions such as $5 + 3x$ when $x = 4$.

The expression $5 + 3x$ was rewritten substituting 4 for x.

$$
\begin{aligned}
& 5 + 3x \\
&= 5 + 3(4) \\
&= 5 + 12 \\
&= 17
\end{aligned}
$$

So we now know the $5 + 3x = 17$ when $x = 4$.

The question becomes, can we find the value of x, the solution, if we know that

$$5 + 3x = 17?$$

Order of Operations

↑

1. Parentheses
2. Exponentials
3. Multiplication/Division from left to right
4. Addition/Subtraction from left to right

To undo a variable expression and isolate the variable, we must use the Order of Operations in reverse using the opposite (inverse) operations. (Gift-wrapping Analogy)

To isolate the variable means to have all the variables on one side of an equation and the numbers on the other side.

Ex. Solve $x + 5 = 9$

To solve this equation, undo the variable expression on the left side by using the order of operations in reverse using the opposite (inverse) operation.

We have an addition, to get rid of that, we subtract 5 from both sides.

$x + 5 = 9$

$x + 5 - 5 = 9 - 5$

$x = 4$

Ex. Find the solution of $y - 3 = 12$

There is a subtraction of 3, to get rid of that, add 3 to both sides

$$y - 3 = 12$$

$$y - 3 + 3 = 12 + 3$$

$$y = 15$$

Ex. Solve the equation; $-4x = 32$

There are not additions or subtractions. To isolate the variable, divide both sides of the equation by –4.

$$\frac{-4x}{-4} = \frac{32}{-4}$$

$$x = -8$$

Ex. Solve for y. $y/3 = 10$

There are no additions or subtractions, so we multiply both sides by 3 to get rid of the division.

$$\frac{y}{3} = 10$$

$$3\left(\frac{y}{3}\right) = 3 \bullet 10$$

$$y = 30$$

The rules of math do not change when using different number sets. So we solve equations the same whether there are fractions, decimals, or integers in the equation.

Ex. Solve $x + 4.25 = -2.5$

$$x + 4.25 = -2.5$$

$$x + 4.25 - 4.25 = -2.5 - 4.25$$

$$x = -6.75$$