Geo Ch 2 Reasoning & Proof Notes

Statements

Conditional statement- has two parts, a hypothesis and a conclusion. When it is written in “if, then” form, the “if” part is the hypothesis, and the “then” part is the conclusion.

Mathematically, we write $A \to B$. Which is read, “if A, then B” or “A implies B”.

Ex. If it rains outside, then the sidewalks are wet.

It rains outside – hypothesis
Sidewalks are wet – conclusion

Converting to “if, then” statements

Ex. Rewrite in “if, then” form. Two points are collinear if they lie on the same line.

If two points lie on the same line, then they are collinear.

Ex. Rewrite in “if, then” form. All dogs bark

If the animal is a dog, then it barks.

Ex. Rewrite in “if, then” form. A number that is divisible by 6 is also divisible by 3.

If a number is divisible by 6, then it is divisible by 3.

Converse- The converse of a statement is formed by switching the hypothesis and conclusions.

Ex. Write the converse of ; If it rains outside, then the sidewalks are wet.

Converse: If the sidewalks are wet, then it raining outside.

If the statement is true, the converse of the state does not have to be true.

Statement $A \to B$

Converse $B \to A$

Negation

A statement can be altered by negation. To write the negation of a statement, you write the opposite.

Ex. Write the negation of "$\angle A$ is acute".

$\angle A$ is not acute.

Inverse the inverse of a statement is formed when you negate the hypothesis and conclusion of a statement.

Statement - $A \to B$
Inverse - $\sim A \to \sim B$, or $A' \to B'$

Contrapositive the contrapositive of a statement is formed when you negate the hypothesis and conclusion of the converse.

Statement - $A \to B$
Converse - $B \to A$

Contrapositive - $B' \to A'$

The conditional statement and its contrapositive are equivalent statements. That means they are logically equivalent.

Definitions & Biconditional Statements

All definitions can be read forward and backward. That is, the statement and converse are both true.

Ex. Definition: All right angles measure 90˚.

That means if an angle is a right angle, then it's measure is 90˚ AND
If the angle's measure is 90˚, then it's a right angle.

Only if statements-

Ex. Rewrite in "only if" form. If it is Sunday, I will be in church.

It is Sunday, only if I am in church.

If and only if statements

Biconditional statements are statements that are equivalent to writing a conditional statement and its converse.

Ex. Rewrite as a conditional statement and its converse. Three lines are coplanar *if and only if* they lie in the same plane.

Statement- If three lines are coplanar, then they are in the same plane.

Converse- If three lines lie in the same plane, then they are coplanar.

Example of an *if and only if* statement written as a postulate -

Segment Addition Postulate Point B lies between points A and C if and only if AB + BC = AC

Properties of Algebra, a, b, and c are real numbers

Addition Property of Equality	if a = b, then a + c = b + c
Subtraction Property of Equality	if a = b, then a – c = b – c
Multiplication Property of Equality	if a = b, then ac = bc
Division Property of Equality	if a = b and c ≠ 0, then a/c = b/c
Reflexive Property	a = a
Symmetric Property	if a = b, then b = a
Transitive Property	if a= b and b = c, then a = c
Substitution Property	if a = b, then a can be substituted for b in any equation or expression

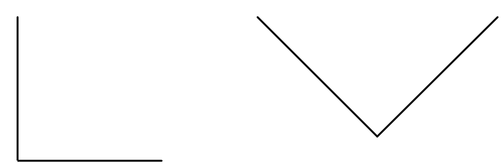
Ex. Solve the equation and give a reason for each step.

3(2x – 1) = 5x + 12	Given
6x – 3 = 5x + 12	Distributive Property
x – 3 = 12	Sub. Prop of Equality
x = 15	Add. Prop of Equality

Proofs

Proofs in math typically have 5 parts; the statement, the Given, the Prove, the picture, and the two columns as shown in the example above.

Theorem – All right angles are congruent.



A B

To prove this, we have the statement.

Given: $\angle A$ and $\angle B$ are right angles

Prove: $\angle A \cong \angle B$

Statements	Reasons
1. $\angle A$ and $\angle B$ are right angles	Given
2. $m\angle A = 90^\circ, m\angle B = 90^\circ$	Def. Rt. Angle
3. $m\angle A = m\angle B$	Substitution
4. $m\angle A \cong m\angle B$	Def. $\simeq$ angles

Theorem- If 2 angles are supplementary to the same or congruent angles, then they are congruent.

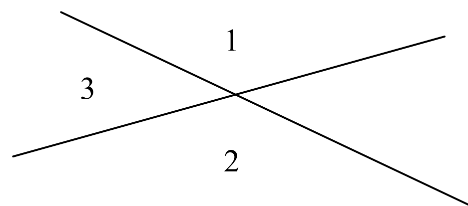
Theorem- If 2 angles are complementary to the same or congruent angles, then the angles are congruent.

Linear Pair Postulate If 2 angles forma a linear pair, then they are supplementary.

Theorem- Vertical angles are congruent.

Given: $\angle 1$ and $\angle 2$ are vertical angles

Prove: Prove: $\angle 1 \cong \angle 2$



Statements	Reasons
1. $\angle 1$ and $\angle 2$ are vert$\angle$'s	Given
2. $\angle 1$ and $\angle 3$ are supp$\angle$'s	Ext sides, 2 adj $\angle$'s in a line (linear pair)
3. $\angle 2$ and $\angle 3$ are supp$\angle$'s	Same as #2
4. $\angle 1 \cong \angle 2$	Two$\angle$'s supp to same $\angle$