Increasing Student Achievement in Mathematics "5 + 1"

A GUIDE FOR PRINCIPALS AND TEACHERS

Not only is what students are being taught important but, how they are taught and tested on those concepts and skills is just as important. This book addresses very effective teaching strategies for learning mathematics. The author's knowledge of mathematics combined with his knowledge and insight of working with students of poverty results in recommendations that result in increased student achievement.

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Bill's knowledge of mathematics combined with his knowledge and insight of working with students living in poverty brings uniqueness to his style of professional development. Based on the foundation that students should feel comfortable in their knowledge, understanding, and application of mathematics, Bill provides professional development for teachers that will assist them in helping all students succeed in math.

Bill has published three books, Math, *your students can do it!* , *Algebra, you can do it!* and Accelerating Mathematics Achievement. He has presented at numerous local and national conferences including the Standards & Assessment, National Council of Teachers' of Mathematics, National Association Secondary School Principals as well as providing services to local school districts.

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Preface

Supervising, monitoring, and evaluating math instruction must be focused on student learning. Having said that, I will make the assumption that school administrators who are evaluating and supervising mathematics have taken many of the same classes students in their school are now enrolled, they have graduated high school, graduated college, and have a post graduate degree, and have grown up and become more mature in how they listen and learn. Having that assumption, I believe if a school administrator is observing classroom instruction and doesn't understand the day's lesson, then they should be questioning why the 13, 15, or 17 year old student sitting beside them should be understanding the lesson since it is their very first time the concept or skill is being presented to them. I am not suggesting that principals should be able to teach the lesson or even know if something is being taught incorrectly, what I am suggesting is that school administrators should be able to follow the lesson and it should make sense to them.

There are two standards that serve as a foundation for the recommendations in this book. The first is the "Common Sense" standard. Simply stated, it means just what it says, the suggestions should appeal to a person's common sense. The second standard is the "My Kid" standard. The "My Kid" standard simply states that classroom teachers and administrators should treat the students in their classrooms the same way they would like to have other educators treat their own children. As you read this text, challenge the suggestions to determine if they meet those two standards.

There are no shortcuts to increasing student achievement. The underlying belief is *what works is work*. Increasing student achievement does not just happen. Classroom teachers and administrators must work to achieve their goals. That work should be reflected in teacher knowledge, instructional and assessment practices, and how well they work with their students. If instruction is to improve, supervisors must give specific feedback to classroom teachers.

School administrators would better serve their students by building on the school's strengths – not by trying to change everything that was done by the previous administration. Relentlessly supporting best practices will eventually crowd out poor instructional strategies. The good news is many teachers are already employing the practices needed to increase student achievement. What is typically needed is a reinforcement and refinement of those practices. Teachers and administrators need to come to work happy, with smiles on their faces, knowing they can be successful in the things they do.

There have been a number of major initiatives that have come and gone during the last half century. Many have focused on the identification and testing of outcomes without closely examining student work or classroom practice. While the identification of outcomes is an important step in planning effective instruction, there must also be an emphasis on "how" to improve instruction for increased student achievement. To that end, this document offers proven recommendations to improve student achievement in

mathematics, recommendations that teachers can incorporate and implement in their classrooms. Importantly, these factors can also be readily identified and monitored by an instructional supervisor. Additionally, more focus has to be placed on acquiring the skills students need to develop to be successful in higher-level coursework and college. There should be no doubt that increasing student achievement depends upon students taking more appropriate, more rigorous coursework. The research suggests students completing a more rigorous high school curriculum tend to be more successful in college.

To address these issues, educators need a plan and a support system, a system that is research based, a system that allows them the opportunity to meet regularly with their colleagues to discuss what they teach, how they teach it, student performance, and instructional strategies that will result in increased student achievement. To develop this plan, professional expectancies should be adopted. Expectancies, protocols, are expected behaviors. We see these in doctors' offices, courtrooms, air traffic controllers, and other professions as a way of setting common standards in practice. We will recommend teacher expectancies that should be adopted in education.

Chapter 1

Overview
Organizing Student Learning, "5 + 1"

"5 + 1" Essential Tips to Increasing Student Achievement

To increase student achievement, students need to be taught how to study more effectively and efficiently. Many struggling students honestly don't know how to do this. Classroom teachers interested in accelerating student achievement should take a closer look at their own practices to ensure they are organizing their instruction in ways that help students learn.

The Increasing Student Achievement in Mathematics, 5 + 1 recommendations are minimums teachers should be employing to ensure instruction is designed to help students learn successfully. The "5 + 1" is embedded in the Components of an Effective Lesson and Teacher Expectancies that will also be discussed in this book. Making the connection between the instruction, student notes, homework assignments, test preparation and assessments helps students focus and study more effectively and efficiently resulting in increased student achievement.

My experiences suggest there is a major disconnect between what teachers teach in the classroom, the notes students take away from that instruction, the homework assignments given to support that learning, test preparation and tests used to monitor student learning which ultimately leads to a grade. These five very observable components of an effective lesson and teacher expectancies require high quality instruction based lesson preparation that includes concept development and linkage, 2. Student notes that reflect and reinforce that instruction, 3. Homework that supports and reflects the notes and the instruction, 4. Test preparation, using the pre or practice test that was constructed before instruction began on the unit, and 5. A balanced assessment that reflects state standards, school district curriculum, and reflects and reinforces the instruction, notes, homework, and test preparation. The "+1" reflects the importance of student-teacher relationships to improving student achievement.

Instruction matters! So that means planning matters too!. Teachers should have a very clear idea of what they expect students to learn before they present their lessons. That means that before teachers begin to teach a unit, they should create a specification sheet, assessment blueprint and a test that reflects their expectations and what they value in that unit. Good mathematics instruction requires that ideas are developed and translated to procedures, rules, postulates, theorems and corollaries that students would be able to reconstruct over time.

What should not be typically happening in a classroom is a rule placed on a board that seemingly came out of nowhere, then 4, 5 or 5 examples are provided as examples or student class work. Evidence of pre planned instruction could be a practice test, specification sheet or assessment blueprint that reflects what teachers expect students to know, recognize and be able to do after instruction.

Memory researchers have identified "writing it down" as the number one memory aid they use to remember. In school, we identify "writing it down" as **note taking**. Middle school and high school students are typically taking six, seven or eight classes per day. They are receiving an enormous amount of information daily, information that in many cases is not very well organized. Many classroom teachers believe that it is a student's responsibility to take notes – that they have no responsibility in that endeavor. That's unfortunate.

Teachers should have a good visualization of what they expect students to take away from each class. If teachers can not visualize what student notes should look like before they start to teach the lesson, then it is very doubtful student notes will be organized in a way that will promote student learning. Notes are important in assisting students to study more effectively and efficiently resulting in increased student achievement! To ensure students have a higher probability of studying more effectively and efficiently resulting in increased student achievement, school administrators should examine student notes and look for the components that will be discussed in later chapters.

Many homework assignments in math are nothing more than page numbers and exercises. Homework should encourage studying and support learning. Homework should reflect and support instruction. A school administrator should look at daily homework assignments to ensure they support instruction and increase student performance in class. Homework recommendations that encourages study and helps students become more successful will be discussed in a later chapter. Evidence of well thought out homework assignments will be made clear for supervisors of mathematics.

For struggling students, students who don't typically experience success in a math class, using the pre test as a practice test that was created before instruction began that reflects and supports instruction, notes, and homework will assist students organize their learning that will result in increased student achievement. The practice exam is another tangible that can be viewed by the school administrator to ensure students are learning how to learn and learning the content that is being delivered to them.

If the real test is balanced and is parallel constructed to the practice test, students will be able to walk into an exam situation more confidently because they know what they know and what is expected of them. Again, school administrators can and should examine these exams for curriculum, balance, fairness, and to ensure grades are portable.

There is a great deal more to effective teaching that results in increased student performance than standing and delivering. While preparation and high quality instruction

that is connected to student notes, homework test preparation, and tests constitute the essential 5, the + 1 is also critically important for student buy-in. The +1 stresses the need for positive student teacher relationships. How that alone affects student performance can not be overstated.

The Increasing Student Achievement in Mathematics, "5 + 1" provides many suggestions and recommendations that can be implemented quickly, at no cost and have the advantage of being readily observable when monitoring math instruction. The "5 +1"will be covered in more depth in later chapters along with other components of effective lessons and teacher expectancies.

Chapter 2
1st Essential - Preparation/Instruction

Teachers should know what they expect their students to know, recognize and be able to do based on state standards, school district curriculum documents, high stakes tests and mathematical content before instruction begins. They should also know how they are going to monitor their students' learning during a unit and how they will assess it after the completion of the unit so student grades are both fair and portable.

To accomplish that, teachers should be expected to construct a specification sheet, assessment blueprint, and practice test before instruction begins to ensure they are focused on student outcomes. Creating a practice test before instruction begins requires the teacher to prepare the unit, timeframes to teach the unit, identify areas where students traditionally experience difficulty and identify resources and strategies to overcome those obstacles. Construction of these three documents will be discussed in great detail using the Backward Assessment Model that is fully discussed in Chapter 3.

We have all heard expressions describing the importance of preparation like Today's Preparation Determines Tomorrow's Achievements. The quote I like best comes from former Dallas Cowboy coach Tom Landry; The only thing more important than the willingness to succeed and win is the willingness to *prepare* to succeed and win.

Construction of a practice test for a unit that contains items from the state standards, school district curriculum, the math content in the unit, CRTs, graduation tests, and college entrance exams such as the ACT and SAT suggest the teacher is prepared and will be better able to prepare their students for any test they must face.

Balanced Delivery of Instruction

Some states have experienced math wars between the so-called *traditionalists* and the *constructivists* on the best way to teach math that will result in increased student achievement. They saw it as an either/or proposition. What I profess, recommend and found very successful is a balance in the delivery of instruction matched by a balance in assessment. A balance defined by what we say we value in math education – what we want students to know, recognize and be able to do.

Teacher-made tests are important, they drive instruction, and they should be balanced. Balance in mathematics is defined as:

1. Vocabulary & Notation
2. Conceptual Development & Linkage
3. Memorization of Important Facts & Procedures

4. Problem Solving

5. Appropriate use of technology

Vocabulary and Notation

I encourage teachers to introduce new concepts and skills by linking those concepts to previously learned knowledge using language students are more familiar and comfortable. Having said that, a certain amount of thoroughness, precision, and formality is required in mathematics and specifically in terms of notation and vocabulary; these are the building blocks of concepts and therefore their correct use is vital. So while initially introducing new concepts in familiar language should be encouraged by linking, by the end of the lesson, more formal language should be used to describe the mathematics.

Mathematics notation is a system of shorthand for the language of mathematics. This notation utilizes symbols to denote quantities, relationships, and operations and has evolved over time to enable us to show the manipulation of data and ideas. Notation enables us to designate mathematical concepts and processes with precision and clarity.

Studying test results and student work would suggest to even the casual observer that students miss far too many questions because they simply did not know what was being asked. For instance, to find a degree of a monomial such as $2x^3y^4z^5$, all students need to do is add the exponents. The answer is 12 – not very difficult mathematically speaking. But many students will miss such an easy question. Why? They didn't know or understand the vocabulary. Vocabulary and notation are important and need to be taught explicitly and used in teaching. Stressing vocabulary and notation, language acquisition, is recognized as being important when addressing the needs of English language learners, it is also important for addressing the needs of all students studying mathematics. There is no more single important factor that leads to comprehension than acquiring the necessary vocabulary.

As part of developing a new concept, teachers should take great care in developing good definitions. For example, when first learning to add fractions, many students mistakenly add the numerators together and the denominators together ($\frac{1}{3} + \frac{1}{3} = \frac{2}{6}$). The teacher all too often responds by telling the students to add only the numerators and gives no further explanation. (In other words, students are being asked to memorize a procedure without understanding.) In order to develop understanding properly, a fraction needs to be defined as part of a whole – composed of a numerator and denominator. The denominator tells you how many equal parts make one whole unit; the numerator tells you how many equal pieces are under consideration. When students then try to add denominators, the teacher can then have them analyze their work based upon the defined terms and explain that if they added the denominators they would not have one whole unit.

Knowing and understanding vocabulary and notation require teacher modeling, use and reading. There is, and should be, an expectation that students can understand, read, speak, and write mathematics. Students in elementary school should be able to read 16.023 as sixteen and twenty-three thousandths – not sixteen point zero, two three. Similarly, secondary school students should be able to read ${}_nP_r$ as a permutation of n things being taken r at a time – not as "npr".

I marvel when I realize how much information is not transferred formally to students. For instance, many teachers in elementary schools teaching subtraction don't use words like minuend, subtrahend, and difference to describe numbers in that operation. Not only don't they use that vocabulary, they don't explicitly teach their students words or phrases that would generally indicate a subtraction problem. Words such as *difference, left, more*, words ending in "*er*", words in the comparative form suggest subtraction. Without repeated exposure to vocabulary and notation, students will not acquire the language of mathematics or any other subject and will experience difficulty in class and on high stakes tests.

A simple example to emphasize this point might be to ask students to translate the following expression to mathematics; "four less than a number". Many students are very literal and since they see the four first, they will write 4 – x. Others might write it as 4 < x which is actually translated as 4 "is" less than x. The correct way to translate four less than a number is "x – 4".

Clearly, this falls under the category of language acquisition. Students not acquiring the vocabulary and notation will have great difficulty on high stakes tests. And teachers need to remember, this is not just a problem for non-English speakers, it's a problem for all students. While we often hear of the difficulties English Language Learners (ELL students) encounter while learning, we seldom hear about the problems all students have learning math as a language – all students need to be identified as Math Language Learners (MLL).

The research is clear, there is no more single important factor that affects student achievement than the acquisition of vocabulary and notation.

Concept Development and Linkage

Remember the Law of Cosines? No, then you illustrate my next point. That is, it is not a matter of if students are going to forget much of the information they learn, it's a matter of when they will forget it. If you were to ask me the sine of thirty degrees right now, the fact is, I can't immediately recall it. But, since I understand how it was defined and developed, give me thirty seconds and I can tell you the answer. Understanding and being able to reconstruct information is important in learning and maintaining knowledge and skills over time.

In the late 80's and early 90's, there were math educators who maintained that any type of drill kills. Others suggested that repetition is the mother of learning. If we want students to be problem solvers, critical thinkers, then they must be able to immediately recall important facts and procedures. Memorization is important and teaching students how to memorize will help them in their learning, thinking, and problem solving.

Now, let's look at these components of balanced instruction and assessment individually and see how nicely they all really fit together so students have a greater appreciation of mathematics.

In classrooms lacking sufficient concept development, teachers primarily emphasize memorization of rules and algorithms with little or no attempt made to help students understand the "why" of mathematics processes. Without a thorough understanding of the underlying mathematical concepts, students are not truly learning mathematics. Mathematics becomes an arbitrary set of isolated rules that don't make sense and have a tendency to look like magic. Teachers must do more than just give the rules; they must build student competence through understanding.

Think of the rules we give students. Two negatives are a positive, unless you are adding, then they are negative. Any number to the zero power, except zero, is one. When you add integers, sometimes you add, sometimes you subtract. When you subtract integers, you change the sign and add, but then again, you might subtract. You can't divide by zero. When you divide fractions, you flip and multiply. Who's making up these rules? Were they out of their minds? Standing alone, the rules don't make much sense.

The fact is those rules are just short cuts that allow students to solve problems or compute quickly. Shortcuts in math have fancy names; we call them theorems, corollaries, axioms, postulates, formulas, or rules but- they are nothing more than short cuts. Some of the short cuts we are most comfortable, familiar, and use don't make sense either. For instance, if I asked you to multiply 72 by 10, you'd get 720 very quickly. If I asked how you arrived at the answer, you'd most likely respond by saying you added a zero. That's not true. If you added zero to 72, you'd end up with 72. What really happened that allowed you to instantly perform that computation was a pattern was identified.

As mathematics becomes more abstract, "math anxiety" may develop if short cuts (rules, theorems, and algorithms) have not been developed with an understanding of "why" they work. If this is not addressed, students can become frustrated and eventually quit enrolling in mathematics classes, even though the grade they earned in their last class was average or above. Each lesson should build on and strengthen the students' mathematical foundation. Teachers should not assume students have already seen, let alone remember, an explanation of a particular mathematics concept. Even if they have, a quick refresher may be beneficial.

For example, finding the sum of the interior angles of a triangle might be introduced by having students draw a triangle and cut out the three angles of their triangle and piecing them together to form a straight angle (180°) – suggesting the sum of the three angles is $180^{\cdot}$ The Pythagorean Theorem might be introduced by examining the areas of the

squares formed by the sides of the triangle. Seeing these patterns, students might hypothesize the area of the square formed by the hypotenuse is equal to the area of the squares formed by the other two legs. Teachers should not just draw a right triangle, identify the hypotenuse, give the formula and work out problems. The understanding gained, in combination with sufficient practice and memorization of important information, gives students confidence in their ability to do mathematics. To deliver this balanced approach, time and thoughtful preparation must be given to each and every lesson. This may require teachers to consult a variety of resources, especially professional colleagues, to find applicable concept development activities.

Explaining the "why" will often address different learning modalities. For example, having the students cut out the angles of a triangle to see how the relationship came about would address kinesthetic learners and will help them remember the theorem. Concept development is important because my belief is it is not a matter of "if" students will forget, but rather, "when" they will forget. Developing the concepts will allow students an opportunity to reconstruct ideas. For English language learners, conceptual development of ideas comes under the heading of "building background". It's a necessary step in language acquisition and understanding for all students.

Procedural knowledge, while important, is not enough. Understanding is important. The following questions came from high school exit exams. Each question measures students' abilities to find the measures of central tendency – often referred to as averages. If student understanding of the mean was only memorized information, students would have difficulty answering some of the following questions.

Standard: Finding Measures of Central Tendency

If you look at the following problems, they all involve finding the mean. The first question tests students' procedural knowledge. The p-score on that particular question was approximately 0.8. That means about 80% of the students taking the test got the correct answer. They knew procedurally how to find the mean.

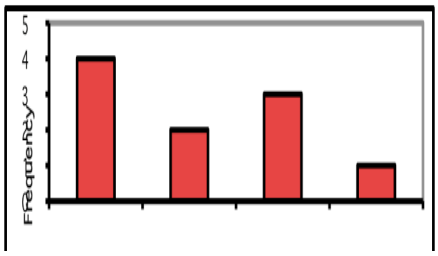
1. Find the mean of the following data: 78, 74, 81, 83, and 82.

2. In Ted's class of forty students, the average on the math exam was 80. Andrew's class of thirty students had an average of 90. What was the mean of the two classes combined?

3. Ted's bowling scores last week were 85, 89, and 101. What score would he have to make on his next game to have a mean of 105?

4. One of your students was absent on the day of the test. The class average for 24 students was 75%. After the other student took the test, the mean increased to 76%, what did the last student make on the test?

5. Use the following graph to find the mean.



On the second question, one of the distracters was 85%. The students that relied only on procedural knowledge, added 80 to 90, divided by two and got the incorrect answer of 85%. The p-score for that question on the exit exam was 0.48. This was a dramatic decrease from finding the mean in the first question. The question was not very difficult, but apparently students need greater understanding to answer the question correctly. Students not understanding they had to re-distribute the total number of points among the seventy students got that question wrong.

Looking at question three, how many times have you been asked what grade a student has to make on their next test to earn a particular letter grade? As you can see from questions two and three, if students do not understand the concept, any variation in problem will cause them difficulty.

Question four caused a tremendous amount of difficulty for students. Many incorrectly think the missing student would have to earn a 77% on the test rather than 100%. Again, not understanding the importance of re-distributing the total number of points led these students to the wrong conclusion. Procedural knowledge is important – but so is understanding. The p-score on this question dropped to .22. What we can quickly see is, without understanding, the slightest variation in a problem will cause students a great deal of difficulty.

Students often interrupt their teachers as they are trying to develop a concept. Why do they interrupt, what's the big deal? The answer is too easy. Typically, they want to know tonight's homework assignment. Students know what their teachers value, even more so than their teachers. Students know if you are not going to ask them a conceptual understanding question on their homework, quiz, or on the test, it must not be important.

Since students value what teachers' grade, concept development and linkage should also be tested. Students should be asked to write a brief explanation of a particular concept as

part of the homework assignment, and then be asked an open-ended question on a test where they must explain the origin of a rule or algorithm. If students are tested on the "why" of mathematics, they will be less likely to "tune out" teachers during concept development. Balanced delivery of instruction requires balanced assessment.

Linkage

As teachers teach mathematics, they should remain cognizant of the fact that the concepts and skills they teach will be used later as building blocks to introduce more abstract concepts. Middle-school teachers use concepts and algorithms taught in elementary school, and high-school teachers continue to build on student knowledge gained in middle school. This process is referred to as "linkage" (connections), the introduction of new material through the use of skills and concepts that have previously been taught. The idea of linkage can also be applied to smaller units of time, including material learned yesterday, last week or last month.

Therefore, as lessons are presented, teachers should link the new material to previously learned concepts or outside experiences. By introducing concepts through the utilization of linkages, teachers enable students to place new ideas into a context of past learning. Students are introduced to new or more abstract concepts using familiar language, thereby not being threatened. Teachers, on the other hand, have an opportunity to review and reinforce previously learned topics, topics and skills they often identify as deficiencies and reasons why they are not successful teaching their assigned curriculum. Teachers can then compare and contrast that information, and students see the idea in a different context. Research suggests all the aforementioned leads to increased student achievement. Simply put, students are then more likely to understand and therefore absorb new material when linkage is being used.

The importance of linking concepts and skills to previously learned material and outside experiences cannot be overstated. Many of our best students probably don't know the equation of a circle, the distance formula, Pythagorean Theorem, and the trig identity $\cos^2 x + \sin^2 x = 1$ are all the same formula, just written differently because they are being used in different contexts. By not introducing these concepts through linking, teachers lose valuable instructional time. They also lose opportunities to introduce the new material using language students are most familiar and comfortable, they lose opportunities to address deficiencies by reviewing and reinforcing previously taught material, students are denied opportunities to increase their understanding by comparing and contrasting those ideas, as well as not seeing the math used in different contexts.

Another example, rather than just having students "flip & multiply" when dividing fractions, the division algorithm might be developed through repeated subtraction – just as was done in fourth grade with division of whole numbers. Solving equations should be connected to the Order of Operations. The standard multiplication algorithm that is taught in fourth grade is the same algorithm that is used in algebra to multiply polynomials. Invariably, student memory, over time, will diminish. An understanding of where theorems, formulas and algorithms (short cuts) originated will enable students to reconstruct concepts and solve problems.

Where possible, linkages should also be made between concepts within the course as well as to student experiences in "real life." Buying candy at a store can be linked to such mathematical concepts as ratios, proportions, slope, ordered pairs, graphing, and functions. Students quickly see that if one candy bar costs fifty cents, then two will cost a dollar. The connection is readily translated to the math they learn in the classroom. As a proportion, 1 candy bar is to \$.50 **as** 2 candy bars is to \$1.00; or written as ordered pairs, (1, .50), (2, 1.00). Linking makes math more relevant and it is very important for students trying to learn the language or students of poverty – by reviewing and reinforcing previously learned concepts and skills in a non-threatening manner.

Buying dirt for a garden or constructing a patio can be linked to volumes of rectangular prisms. The circumference of a circle can be linked to pipe fitting or the odometer and speedometer readings on a car. Systems of equations can be linked to decision-making. The links to make math come alive are too numerous to mention. Teachers must transfer their knowledge, interest and enthusiasm in mathematics to their students. That can't be done by doing rote problems, without seeing the benefit of math in solving everyday problems that too often are not even considered mathematics. Too many algebra students are asked to do the mundane. If they were working with conics, they might be asked to find things like the vertex, directrix and focus of a parabola and that's great. But what would be greater is while they were studying the parabola, that the teacher might make students aware how they use the concept of parabolas in their everyday life. That flashlights, headlights on a car, a satellite dish, and amphitheatres are all examples of how that mathematics, algebra, is used.

Because I believe introducing new concepts and skills through linking is so important to increasing student achievement, especially for students coming from poverty, I have provided a number of examples that can be immediately employed in the classroom.

Example

In middle school, we teach students to add whole numbers by utilizing place value; i.e. tenths to tenths, hundredths to hundredths, etc. In algebra, we teach students addition of polynomials by telling them to combine like terms; i.e., to add constants to constants, x's to x's, x^2's to x^2's, etc. These are the same concepts.

In first, second and third grade, we teach students to add whole numbers by lining up the numerals in columns so that the digits with the same place value are combined; i.e., digits with a place value of one, digits with a place value of ten, etc.

Example

In algebra, we teach students multiplication of binomials by the **FOIL** (**F**irst, **O**uter, **I**nner, **L**ast.) method. In third and fourth grades, students are taught two digit by two digit multiplication by a standard algorithm which is essentially turns into the FOIL method.

Therefore, multiplication of binomials should be introduced by using the standard algorithm before the shortcut is introduced.

To multiply 21×32:

$$\begin{array}{r} 21 \\ \times\ 32 \\ \hline 42 \\ 63\_ \\ \hline 672 \end{array}$$

To multiply $(x+4)(x+3)$:

$$\begin{array}{r} x+4 \\ x+3 \\ \hline 3x+12 \\ x^2+4x\_ \\ \hline x^2+7x+12 \end{array}$$

same algorithm

The next logical step would be to introduce the FOIL Method, linking it to the vertical method of multiplying two polynomials illustrated above.

Besides linking new material to previously learned material, it is helpful to link it to outside experiences as well.

The idea of slope is used quite often in our lives; however outside of school it goes by different names. People involved in home construction might talk about the pitch of a roof. If you were riding in your car, you might have seen a sign on the road indicating a grade of 6% up or down a hill. Both of those cases refer to what we call slope in mathematics.

Kids use slope on a regular basis without realizing it. Let's look at an example, a student buys a cold drink for $0.50, if two cold drinks were purchased, the student would have to pay $1.00.

I could describe that mathematically by using ordered pairs; (1, $0.50), (2, $1.00), (3, $1.50), and so on. The first element in the ordered pair represents the number of cold drinks, the second number represents the cost of those drinks. Easy enough, don't you think?

Now if I asked the student, how much more was charged for each additional cold drink, hopefully the student would answer $0.50. So the difference in cost from one cold drink to adding another is $0.50. The cost would change by $0.50 for each additional cold drink. The change in price for each additional cold drink is $0.50. Another way to say that is the *rate of change* is $.50. In math, we call the rate of change—slope.

In math, the rate of change is called the slope and is often described by the ratio $\frac{rise}{run}$.

The rise represents the change (difference) in the vertical values (the y's), the run represents the change in the horizontal values, (the x's). Mathematically, we write

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Let's look at any two of those ordered pairs from buying cold drinks, (1,$0.50) and (3,$1.50) and find the slope. Substituting in the formula, we have:

$$m = \frac{1.50 - 0.50}{3 - 1} \rightarrow \frac{1.00}{2}$$

Simplifying, we find the slope is $0.50. The rate of change per drink is $0.50

I cannot overstate the importance of using linkages to introduce new or more abstract concepts and skills. Linking allows teachers to introduce new ideas in familiar language in a non-threatening manner, to review and reinforce that knowledge, to compare and contrast, and to see mathematics used in different contexts.

Problem Solving

Mathematics is more than just memorizing rules and procedures, it is a discipline, a way of thinking. Students must be taught and encouraged to think, to imagine, and to be creative in their approaches to solving problems. The National Council of Teachers of Mathematics materials state, "problem solving is not a mystery." It is also not limited to solving traditional story problems or word problems. It is a way of thinking that can be learned. Teachers need to encourage their students to approach learning/problem-solving activities with an open mind and to realize that this kind of thinking takes time and effort to achieve. Students' answers, whether correct or not, should be viewed as opportunities to explore thinking strategies. Open-ended questions that evoke thoughtful responses and require more than one word answers should be presented. Students should also be encouraged to utilize a variety of problem-solving methods. This process requires that teachers provide students with sufficient thought time. While problem solving is difficult to teach, and requires commitment and patience on the part of both teacher and learner, it is an essential experience.

Requiring teachers to have a balance in their delivery of instruction and assessment should come under the heading of *teacher expectancies*. In math, we'd also expect all teachers to employ recognized problem solving/learning strategies, such as:

Go back to the definition
Look for a pattern
Make a table or list
Draw a picture
Guess and check
Examine a simpler case
Examine a related problem
Identify a sub-goal

Write an equation
Work backward

These problem-solving strategies also help students understand mathematical concepts being taught. Its no mistake that *Go back to the definition* is listed first. Too often it is not listed as a problem solving/learning strategy. Without a good definition, without knowledge of vocabulary, students are bound to encounter difficulty in any subject. I don't know how teachers explain to students not to add the denominators when adding fractions without a good definition of a fraction.

Successful teachers are cognizant of the problem solving/learning strategies in their daily instruction. When students encounter difficulty in understanding a concept or mastering a skill, good teachers encourage students to go back to the definition, draw pictures, look for a pattern, examine a simpler or related problem. Students who have been taught and encouraged to use problem solving/learning strategies don't sit idly when they encounter difficulty, they are better prepared to address the problem at hand.

Some may wonder why some students are successful in one subject, but not in another. Let's look at algebra and geometry as an example. We often see students successful in an algebra class have great difficulty in geometry, why is that?

Algebra teachers tend to use *look for a pattern, make a table, examine a simpler case, and write an equation* as strategies that form the basis for most of their instruction. As a result, students grow comfortable learning math with these strategies. Unfortunately, too many students still try to learn algebra by rote memorization. That results in any variation of a problem causing great difficulty and frustration for students. That can be clearly seen on exit exams in mathematics.

Teachers of geometry tend to use: *go back to the definition, draw a picture, examine a related problem, identify a sub goal, and work backward* as their primary strategies. Students who learned algebra by memorizing often run into difficulty in geometry. Students and teachers that use those same strategies to teach or learn geometry that were successfully used in algebra often run into difficulty too – resulting in higher fail rates.

In geometry, students are typically required to use higher order thinking skills that are not being used in a typical algebra class. In my experience, too many geometry students do not have a good visualization of the definitions, postulates, and theorems that are being introduced. As we expect students to learn these, they should also be able to draw a picture that reflects the information being taught as well as to measure the drawings and diagrams to explore, discover, and eventually commit to memory important theorems and procedures.

Students should be required to write their definitions, postulates and theorems on their homework, quizzes or tests, they should also be required to draw a corresponding picture. If they can do that, they will be more successful learning geometry.

Geometry is filled with new terminology and notation, teachers need to be mindful that student success in any subject is dependent upon them learning the language. All too often in math, the difficulties experienced by students has more to do with a lack of understanding of vocabulary and notation than the math concept being taught. Classroom teachers should take the time to ensure students are learning and using that vocabulary and notation and they should also be testing students on it.

And while some problem solving / learning strategies are used more routinely in one subject area vs. another, the fact is all problem solving / learning strategies should be used at appropriate times in all of mathematics.

And finally, let's not forget about "linking" geometry to algebra –referred to as coordinate geometry. Linking allows teachers to introduce new concepts in familiar language, to review and reinforce, to compare and contrast, and to teach in a different context – all of which the research suggests leads to increased student achievement. While coordinate geometry is typically a chapter by itself toward the end of a book in geometry, these links can and should be made all year.

Basic Facts

Mastery of basic facts is an essential part of learning mathematics. When students encounter mathematics concepts they need instant recall of basic facts. Stopping to remember these facts interrupts the flow of thought, which negatively impacts learning.

What constitutes "basic knowledge" depends upon the grade level. Basic facts in elementary school might be arithmetic facts. In middle school, they might be expanded to include the conversions between fractions, decimals, and percents, or the algorithm for adding fractions. In high school, basic facts may also include the Quadratic Formula, the Pythagorean Theorem, knowing what the graph of a 2^{nd} degree polynomial equation looks like or algorithms for solving linear equations.

Since student deficiencies are evident at all levels, teachers should regularly revisit basic facts. Many higher-level thinking processes required for success in high school mathematics courses demand immediate recall of basic facts. The demands of teaching dense curricula and addressing student deficiencies may, at times, overwhelm the teacher. However, if carefully analyzed and incorporated into lesson plans, deficiencies can be addressed successfully.

The most common complaint I hear from teachers is they can't teach their curriculum because their students don't know their basic math facts. The sixth grade teacher will then spend the first three or four weeks of the school year reviewing/re-teaching the arithmetic facts. The students then go on to seventh grade, that teacher has the same complaint, the kids don't know their math facts and they can't teach until they spend three or four weeks having the students memorize them. You guessed it, the kids go into eighth grade, that teacher spends another three or four weeks complaining about how those elementary teachers did not do their job. If you do the math, you can quickly see

that approximately nine weeks of middle school is spent addressing a nonexistent problem. Nonexistent?

My experience has been these teachers, well meaning as they are, review all 100 multiplication facts. If they based their instruction upon data, most teachers would have found out the kids knew their 1's, 2's, 3's, 5's,10's, doubles, and 9's. By incorporating the commutative property, teachers would find there are only about 17 facts in which the kids are experiencing difficulty. Rather than pulling out all the flash cards, students might be better served if their teachers would answer the question; "What do your students know and how do you know they know it?" By answering that question, teachers would work on student deficiencies and not waste time re-teaching facts and procedures the students have already mastered.

If students are struggling with their basic addition and multiplication facts, the following strategies for teaching basic arithmetic facts may be of help. What's important to note in these strategies is the facts are not necessarily taught sequentially. These arithmetic facts, like anything else we teach, should be taught in a manner and order that helps students learn.

Additionally, students in elementary schools should be spending 7 to 12 minutes almost daily reviewing their basic arithmetic facts.

Strategies for Learning Addition Facts

+	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

There are 100 basic addition facts with sums to 18. By using commutativity (3 + 7 = 7 + 3), we can reduce the total number of addition facts students must learn to 55.

1. **Adding Zero** - Students quickly understand that the sum of zero and any number is that number. For example, 0 + 6 = 6. (This leaves 45 addition facts to have students memorize.)

2. **Counting on by 1 and 2** - Students often find sums with addends of 1 or 2 by simply "counting on." (This leaves only 28 facts left to learn.)

3. **Sums to 10** - Students can readily identify sums to 10 by repeated experiences with ten. (This leaves 25 facts.)

4. **Doubles** - For whatever reason, students seem able to remember doubles more easily than other combinations of numbers. For example, 7 + 7 = 14. (Now, 19 facts are left.)

5. **Doubles plus 1** – Adding consecutive numbers. Knowing that 7 + 8 is equivalent to 7 + (7 +1) helps students remember these sums. (This leaves 13 facts.)

6. **Doubles plus 2 –** Adding consecutive odd or even numbers. Knowing that 5 + 7 is equivalent to 5 + (5 + 2)

7. **Adding 9's** - Students can quickly see that when they are adding the units digit in the sum is one less than the number they are adding to 9. For example, **7** + 9 = 1**6** since the 6 in the units place is one less than 7. (Only 8 facts are left.)

8. **Adding 10's**

Thinking Strategies for Learning the Subtraction Facts

1. **Fact families:** This strategy is the most commonly used and works when students understand the relationship between addition and subtraction. When students see 6 – 2 and think 2 + ? =6. However, if this strategy is used with the following strategies, students will find greater success in a shorter period of time.

2. **Counting backwards:** This method is similar to Counting on used in addition. It isn't quite as easy. Some might think if you can count forward, then you can automatically count backward. This is not true –try saying the alphabet backwards. Students should only be allowed to count back **at most** three.

3. **Zeros:** The pattern for subtracting zero is readily recognizable. 5 – 0 = 5

4. **Sames:** This method is used when a number is subtracted from itself; this is another generalization that students can quickly identify. 7 – 7 = 0**.**

5. **Recognizing Doubles:** Recognizing the fact families associated with adding doubles.

6. **Subtracting tens:** This is a pattern that students can pick up on very quickly, seeing that the ones digit remains the same.

7. **Subtracting from ten:** Recognizing the fact families for Sums to 10.

8. **Subtracting nines:** Again, the pattern that develops for subtracting 9 can be easily identified by most students. They can quickly subtract 9 from a minuend by adding 1 to the ones digit in the minuend. 17 – 9 = 8, 16 – 9 = 7.

9. **Subtracting numbers with consecutive ones digits:** This pattern will always result in a difference of 9, 16 – 7 = 9, 13 – 4,= 9 15 – 6 = 9 all have ones digits that are consecutive and the result is always 9.

10. **Subtracting numbers with consecutive even or consecutive odd ones digits:** This pattern will always result in a difference of 8. 14 – 6 = 8, 13 – 5 = 8, 12 – 4 = 8.

These strategies clearly help students to subtract quickly. How you teach these strategies, allowing the students see the patterns develop, will make students more comfortable using these "shortcuts" and get them off their fingers.

Having said that, as with many of the concepts and skills in math, students need to compare and contrast problems to make them more recognizable to them. Without being able to identify the proper strategy by examining the problem, memorizing these strategies may become more burdensome and cause greater confusion than just rote memorization.
So while you might teach one strategy at a time, as you add to the number of strategies students can use for a specific numbers, you will need to review previous strategies and, this is important, combine strategies on the same work sheets asking students to only identify the strategy they would use for each problem and why they are using it. Being able to compare and contrast will lead to increased student understanding, comfort, and achievement using these strategies..

For example,
16 – 9, students are subtracting 9, they add one to the units digit.
15 – 7, students are subtracting numbers with consecutive odd units digits, the difference is 8.
17 – 8, students are subtracting numbers with consecutive units digits, the difference is 9

I have watched students trying to learn their basic math facts by just memorization, re-reading the tables or using flashcards. It's a painful experience for most students and just as agonizing to watch. Memorization and using flash cards are necessary, but how teachers teach those facts can have a significant impact on students' success.

For instance, many teachers use the "families" for students to memorize the subtraction facts. That is, if 5 + 4 = 9, then 9 – 4 = 5. That's alright, those relationships should be developed.

However, students should be taught when they subtract "10", the units digit remains the same in the difference. They should also be taught to recognize that when subtracting "9", the units digit is always one more than the units digit in the minuend.

There are other patterns that might be developed that would help students learn their subtraction facts. For instance, when you subtract consecutive numbers, the answer is always 9. For example, 16 – 7 = 9; 14 – 5 = 9. Notice the units digit in the subtrahend is one larger than the units digit in the minuend.

Continuing this hunt for patterns, when you subtract consecutive odd or consecutive even numbers, the difference is always 8. For example, 13 – 5 = 8; 14 – 6 = 8. Helping students recognize those patterns will accelerate their learning of the math facts and it will also relieve some of the pain often felt by these students.

Algorithms

The use of algorithms – which are systematic, step-by-step procedures used in computation or problem solving – helps to address the difficulty students often have sequencing complex mathematics problems. The National Council of Teachers of Mathematics' (NCTM) *Curriculum and Evaluation Standards* recommends that students use algorithms to compute and solve problems. However, algorithms should not stand alone and usually need to be preceded by concept development.

By developing an understanding of a concept, students will be better able to understand the objective involved. They will then be more willing and able to identify patterns that lead to the shortcuts we call rules, algorithms, formulas, theorems, or conjectures. These shortcuts were developed in many instances because someone recognized a pattern that would give them the desired result without having to do as much work. Teachers should stress to students that the shortcuts, by themselves, often do not make sense. It is vital that students understand the concepts and how and why the shortcuts work. With this knowledge, students are better able to make sense of mathematics and are more likely to use suitable strategies and algorithms. For instance, students that only memorized an algorithm with no understanding might compute 4x13x25 by multiplying from left to right. That exercise would take a few minutes and need some space on their paper. Students who know the algorithm and understand the concept would probably multiply

25 by 4 first, then multiply that result by 13 – in their head in seconds getting a product of 1300.

While there are many different ways to compute and therefore many different algorithms, the teaching of standard algorithms is important because this ensures that students have common frames of reference. This is significant as the standard algorithms developed in elementary grades become the foundation upon which more abstract material in middle school is introduced. For instance, there are many ways of multiplying. The ancient Egyptians used an algorithm known as Repeated Doubling; the fourteenth century Italians used the Lattice Method. In the United States, to maintain consistency, we have identified a standard multiplication algorithm. A variation of this algorithm is later used in algebra when multiplying two binomials (FOIL Method). In another example, the standard division algorithm taught in fourth and fifth grades is used again in algebra when students divide polynomials. It is also used in synthetic division and synthetic substitution when solving higher degree equations using the Rational Root Theorem. The division algorithm is important. Teachers expect students coming into their classes will have had certain learning experiences. If students lack practice with the standard multiplication and division algorithms, or other standard U.S. algorithms, they will probably experience unnecessary difficulty in future mathematics classes. Here are a few examples to demonstrate how standard algorithms are continually used in mathematics.

Example:

The standard algorithm for multiplying two, 2-digit numbers is:

1. Multiply the multiplicand by the digit in the one's column of the multiplier.
2. Indent a space to account for place value and multiply the multiplicand by the multiplier's ten's digit.
3. Add those partial products.

Illustration:

4th Grade		Algebra I
3 2		$x + 4$
$\times$ 2 1	⟵ SAME ALGORITHM ⟶	$x + 5$
3 2		$5x + 20$
6 4		$x^2 + 4x$
6 7 2		$x^2 + 9x + 20$

When young students are asked to model or explain how to determine the number of two's in eight, they might use the repeated subtraction model shown below:

$$8 - 2 = 6, \qquad 6 - 2 = 4, \qquad 4 - 2 = 2, \qquad \text{and} \qquad 2 - 2 = 0$$

Students can clearly see they subtracted 2 a total of four times. Therefore, there are four 2's in 8.

This same concept applies to division of fractions. If asked to divide 3/4 by 1/8, students should be able to use the same repeated subtraction model.

$$\frac{3}{4} - \frac{1}{8} = \frac{5}{8}, \qquad \frac{5}{8} - \frac{1}{8} = \frac{4}{8}, \qquad \frac{4}{8} - \frac{1}{8} = \frac{3}{8},$$

$$\frac{3}{8} - \frac{1}{8} = \frac{2}{8}, \qquad \frac{2}{8} - \frac{1}{8} = \frac{1}{8}, \quad \text{and} \quad \frac{1}{8} - \frac{1}{8} = 0$$

Students can see they subtracted 1/8 a total of six times. Thus, there are six 1/8's in 3/4.

Given opportunities to look for a pattern, for practice and appropriate guidance, students might notice that rather than performing all those repeated subtractions, if they multiplied by the reciprocal of the divisor they would arrive at the desired result. Not only would it give them the correct answer, they would be able to do the problem faster and more efficiently.

$$\frac{3}{4} \div \frac{1}{8} = \frac{3}{4} \times \frac{8}{1}$$

$$= \frac{24}{4}$$

$$= 6$$

When students discover the patterns derived by playing with numbers through teacher guidance, they can be shown that algorithms are nothing more than a faster way to solve problems by applying those patterns. Mathematics then is no longer something magical or mysterious; it becomes a powerful tool to be used in a variety of situations.

By learning an algorithm, students will have a method with which to solve a variety of problems. Likewise, students that know how to solve a problem should be able to verbalize what they have done, verify and defend their solutions, and communicate results. The memorization and utilization of algorithms allows students to do just that.

Technology

"The thoughtful and creative use of technology can greatly improve both the quality of the curriculum and the quality of children's learning. Integrating calculators and computers into school mathematics programs is critical in meeting the goals of a redefined curriculum." (National Council of Teachers of Mathematics. *Curriculum and Evaluation Standards*. Reston, Virginia: National Council of Teachers of Mathematics, Inc., 1989.)

However, the NCTM also says, "Calculators do not replace the need to learn basic facts, to compute mentally, or to do reasonable paper-and-pencil computation." Therefore, appropriate use of technology is dependent upon the age of a student and his/her ability to demonstrate knowledge of basic facts. It is further dependent on the objective of the activity. If the goal is skill attainment, then calculator use is not appropriate. If the goal is exploration or verification, then calculator use may be appropriate.

Modern technology can free students from tedious computations and allow them to concentrate on problem solving and other important mathematics content. Students should be using calculators to strengthen and extend understanding of concepts, explore mathematical functions, investigate problem-solving activities, employ real world applications, and verify results. (In Algebra I and above, the use of graphing calculators is imperative.) *However, it is essential that all teachers maintain a balance between paper-and-pencil computation/drill and the use of technology to enhance problem solving and conceptual learning.* This requires teachers to make a conscious decision as to the appropriateness of calculator use during each and every lesson. Calculators should not be allowed as a substitute for thinking. To increase the likelihood that calculators will be used appropriately, teachers may need additional training. Total dependence on technology is inappropriate, but when combined with an understanding of the underlying concepts and proficiency with basic skills, it becomes an invaluable tool.

Example:

One method to find the zeros of a quadratic function is through factoring. Students should easily be able to find the zeros of the function $f(x) = x^2 - 4x + 12$ without a graphing calculator.

$$x^2 - 4x + 12 = 0$$
$$(x - 6)(x + 2) = 0$$
$$x - 6 = 0 \ \ or \ \ x + 2 = 0$$
$$x = 6 \ \ or \ \ x = -2$$

Students should have learned, however, that the graph of the quadratic function $y = x^2 - 4x + 12$ has x-intercepts at $x = 6$ *and* $x = -2$ indicating the function's zeros.

Example:

A quick method for finding the location maximum/minimum of a parabola is to average the zeros. In the above example, the maximum occurs at $x = \frac{6 + (-2)}{2} = 2$. The y-coordinate of the extremum is $f(2) = 2^2 - 4(2) + 12 = 8$. Again, students should have the ability to do this without the graphing calculator.

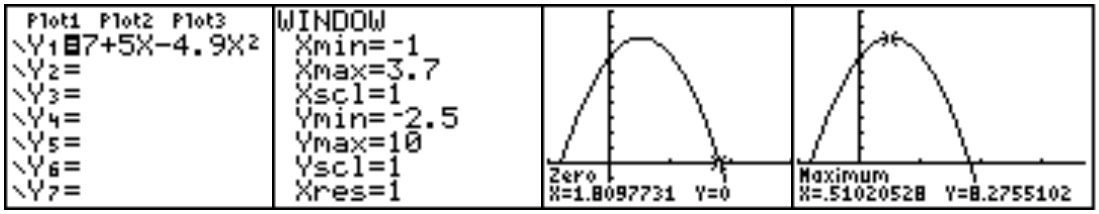
Example:

A graphing calculator is an ideal tool for exploration of the following problem:

A diver stands upon the 7-meter diving platform preparing for a dive. (The 7-meter platform is 7 m above the surface of the water.) The diver jumps vertically upward from the platform with an initial velocity of 5 meters per second. The diver's height above the water can be modeled by the equation $h = 7 + 5t - 4.9t^2$, where t is the elapsed time in seconds since the diver jumped. At what time after the dive does the diver reach the water? At what time does the diver reach a maximum height and how high is it?

The expression $7 + 5t - 4.9t^2$ is not easily factored, nor is completing the square trivial. The quadratic formula leads to solutions, but a graphical exploration yields more information and eliminates tedious calculations.

Students should determine that the diver reaches maximum height of 8.28 m at $t = .51$ s, and enters the water at 1.81 s. Further exploration may prompt discussion on how high the diver jumped above the platform and the meaning of the quadratic's other zero at $t = -0.79$ s.



The Y= Editor Window Screen Finding the Root Finding the Max

To think critically, to problem solve, students need understanding and a body of information to draw from. To do those, they need vocabulary and be able to use technology. Balance in the delivery of instruction and in assessment ensures our students are getting a full, rich curriculum. Balance also ensures students are being taught what the adults say they value in mathematics.

Oral Recitation

In every course, there are certain items that all students should know at the completion of the school year, information upon which understanding and critical thought can be based. Furthermore, the more sophisticated mental operations of analysis, synthesis, and evaluation are impossible without rapid and accurate recall of bodies of specific information. When these items are first introduced, oral recitation should be utilized to help students memorize the information. For example, after the initial development of the Quadratic Formula, an algebra teacher could provide it visually and then orally read the formula several times so the students know how to correctly say it. Following this, the class as a whole could recite it several more times. Finally, individual students could be called upon to recite the formula without the use of a visual aide. This process places the information in students' short-term memory with the expectation that it will then, through repeated and extended exposure, be placed in long-term memory. Oral recitation may need to be used again when important topics are revisited

Oral recitation, is the practice of having the entire class recite important facts, identifications, definitions, and procedures within the instruction and later when they need to be revisited. Concept development generally precedes oral recitation. Whole class recitation (repetition) of this information should be repeated a number of times, however the total time involved should not exceed two and one-half minutes. By having the students first read the information off the board with the teacher, students learn how to read information correctly and how to say it. Oral recitation is a language acquisition strategy that helps all students learn – not just English language learners.

The use of oral recitation should not lead teachers into the "gotcha" practice. While oral repetition will help most students imbed this information into short-term memory, other students learn differently. Incorporating the idea of building success on success might suggest you don't want to call on students that you think might not have it in short term

memory. Use oral recitation from a positive perspective. That is, after the class orally recites for approximately two minutes, call on students that you believe could give the information back to you. Those correct responses create the impression with students that other members of the class were getting the information. As opposed to "nobody is getting it."

Oral recitation is just one method of helping students memorize information. Adults often use it when trying to remember a license plate number or grocery list. This practice anchors information in the brain and helps students absorb and retain information upon which understanding and critical thought is based. The more sophisticated mental operations of analysis, synthesis, and evaluation are impossible without rapid and accurate recall of bodies of specific information.

The process also keeps students engaged in learning, helps them verbalize their knowledge, and suggests that if the information being presented is important enough for the entire class to recite, it is worth remembering.

Memory Aids

"Mnemonics are based on the principle that the brain is a pattern-seeking device, always looking for associations between the information it is receiving and what is already stored. If the brain can find no link or association, it is highly unlikely that the information will be stored in long-term memory. Unfortunately, this scenario is relatively commonplace in the classroom" (Wolfe, 200l). That's another reason linking should be used more often in the classroom.

The brain has trouble storing information that it can not associate to a picture such as letters and numbers. Mnemonics create rhyming links or associations that give the brain an organizational framework on which to hook new information.

In certain circumstances, teachers use mnemonics because they were taught them while they were in school. Many middle school students are introduced to the phrase- Please Excuse My Dear Aunt Sally as a way of remembering the Order of Operations as SOHCAHTOA is familiar to many trig students learning the trigonometric ratios.

Helping students remember is essential, helping them remember over time is important if we are to build credibility in the public education system. As students are first being introduced to definitions, concepts, and skills, their likeness causes them confusion. For instance, if we look at the definitions of complementary angles and supplementary angles, we see how this might occur. Complementary angles are defined as two angles whose sum is 90˚. Supplementary angles are two angles whose sum is 180˚. The definitions are similar and students might mix them up. A teacher might suggest the "c" in complementary might be associated with a corner because it also begins with a "c", hence – 90˚. Likewise, the "s" in supplementary might be associated with the "s" in a straight angle thereby associating that with 180˚.

Memory aids help students study more effectively and efficiently, used in the classroom, they will help students remember, thereby increasing student achievement.

Questioning Strategies

Who's doing the talking in the classroom? For students to learn the language of mathematics they have to read it, write it and speak it. How questions are asked in a math classroom can either promote or hinder dialogue.

Questions can take different forms. Using one form of questioning, directed, a teacher might ask a student for an answer to a particular question and receive a one or two word response. "What's the answer to number 8?" Another form of questioning, regurgitation, students might be asked for formula, procedure or list. A third type of question might be referred to as cueing. In that situation, a teacher asks a question, after the student responds, the teacher repeats the student response, then asks for more information by asking questions like "why?" or "because…". And finally, there are conceptual questions, questions that have students explaining or linking a concept, comparing and contrasting. Questions that elicit more than a one word answer or list, but causes students to communicate their thoughts and understandings of a particular topic.

All of these types of questions should be incorporated into daily lessons. To keep students engaged, teachers need to ask all four kinds of questions. Importantly, teachers need to allow students time to think of an answer before they just give it. Teachers clearly need to understand the value of "wait time" when asking students to engage in discussions.

To help teachers determine the types of questions they asking of their students, administrators observing a lesson might list these four categories and determine the number and types of questions teachers are asking of their students. Knowing the emphasis of their questioning might help teachers get students more engaged in their learning.

Use of Instructional Time

State and local school districts usually determine the classroom time available to teachers and students. Regardless of the quantity of time allocated to classroom instruction, it is the classroom teacher and school administrator who determine the effectiveness of the time allotted.

According to a survey conducted by the American Association of School Administrators, teachers identify student discipline as the single greatest factor that decreases time on task in the classroom. Generally, teachers with well-managed classrooms, have fewer disciplinary problems. These classrooms typically have teachers who have established rules and procedures are in the classroom when the students arrive, and begin class

promptly. They reduce the "wear and tear" on both themselves and students by establishing procedures for make-up work, they arrange their room to accommodate their teaching philosophy and style, and they develop routines that increase overall efficiency. The benefits of establishing these classroom procedures and routines become apparent as the total time on task approaches the allocated time.

The research says: "When teachers begin class immediately, students view them as better prepared, more organized and systematic in instruction, and better able to explain the material. Students also see these teachers as better classroom managers, friendlier, less punitive, more consistent and predictable, and as teachers who value student learning."

That's an awful lot coming from just starting class on time. But, when you think of it and understand our culture, it does shed light on those conclusions. In countries such as Italy or Greece, it is alright for boys who are friends to hug or even kiss each other on the cheek. In America, hugging and kissing is not well received. The way American boys show affection is by hitting. So when two boys like each other, they show it by hitting their friend on the arm. If there was too much "love" being delivered in the punch, the other boy might respond with a little extra affection of his own. Too much of this affection might lead to a fight.

Routines, like beginning class immediately, reviewing recently taught material, orally reciting new material, having students take notes, and ending the class by reviewing important definitions, formulas, algorithms, and the daily objective keep students engaged and on task. Quality time on task is not a "silver bullet" that can cure all the problems facing education; however, it can play an important role in increasing student achievement. Teachers must ensure that the entire class period is used to its full potential. That is, that "An academic focus and on-task behavior are [to be] maintained by the effective use of allocated instructional time." Teachers should begin class as soon as the bell rings with an activity such as a review or a quiz displayed on the overhead. This encourages students to arrive to class on time and it also allows the teacher a few moments to take care of attendance, tardies, new students, etc. This immediate focus on academics sets the tone for the entire class period. Carrying this idea to the end of the period, students should not be allowed to pack up their books and supplies early. (The bell should not be viewed as a signal to students to leave, but instead as a reminder to the teachers that they may dismiss their students.) When students complete their assignment/activity early, they should be encouraged to work on other material or should be provided with an enrichment activity. Students should not be allowed to socialize with those around them as it disturbs the class as a whole. The last few minutes of class should be used to bring closure to the lesson, to review important skills, to work on problem solving, or to further check the level of student understanding of the day's lesson.

Not only is what we teach important, but how we teach it effects student learning. We need to look at proven instructional strategies that help students learn.

To teach math successfully, students must experience a full, rich, and more in-depth curriculum - a balanced delivery of instruction. That balanced delivery must be accompanied by assessments that reflect that balance.

The old axiom is true, testing drives instruction. So as part of the 1st essential, school administrators should ask to see teachers pre-test before instruction begins to ensure the curriculum is being followed, benchmarks are being adhered to, the tests are fair and grades are portable between teachers. By making the test in advance, teachers will be better able to identify areas in which students traditionally experience difficulty and provide resources and strategies that assist students in their learning. There is no substitute for preparation. Today's preparation determines tomorrow's achievements.

Chapter 3
Structures that Support Increased Student Achievement

BACKWARD ASSESSMENT MODEL
Professional Development Through Sharing

Let's first look at a structure that will support ongoing professional development and changes in instructional practices that will result in increased student achievement.

Educational research strongly suggests that professional interaction—at times informal and unstructured—is often far more influential than formally organized professional development, and is more likely to result in changed behavior on the part of the educator.

The *Backward Assessment Model* (*BAM)* changes the way professional development is delivered. Rather than having an outside expert tell teachers what needs to be done, the assessment model uses the expertise of the school's staff. Research suggests professional development should primarily be on-site, on-going, and regularly scheduled. Professional development should be provided by the people who know best, classroom teachers as active participants, and should focus on the discipline teachers' teach, in both content and pedagogy. The *BAM* places the professional development emphasis on *academic standards, assessment,* and *best practices*.

***BAM* is a communication model.** Its strongest attribute is that it provides teachers an opportunity to share their **knowledge, understanding, skills, experiences, resources, and instructional strategies** with each other**.** Experienced teachers generally know where students traditionally experience difficulty and communicate this to less experienced teachers. Likewise, all teachers can communicate knowledge, model successful strategies, and share accommodations and modifications that help students succeed. *BAM* also provides all teachers, experienced and new, opportunities to reexamine and reflect upon their own practices.

There are two basic premises of *BAM*. The first is data from assessments drive instruction and the second is that teachers make a difference, and teachers working together make a greater difference.

To implement the Backward Assessment Model, teachers need to meet together by grade or subject level to discuss what they teach, how they teach it, their student performance, and changes in instructional strategies that will result in increased student achievement. The following is an overview of what those teachers would do using a typical professional development day.

Before instruction on a major unit of study, grade level or subject area teachers should develop a **SPECIFICATION SHEET**. That is, before they begin teaching, each group (3rd grade teachers, algebra teachers, 6th grade science, etc.) should meet together and identify what they expect students to know, recognize, and be able to do and the timelines to accomplish those goals for a specific unit. This piece fits in very nicely to ensure all teachers are familiar with the state's academic standards, school district curriculum guides, or the already established benchmarks.

The second major component in BAM is the **ASSESSMENT BLUEPRINT.** To ensure students are receiving balanced instruction, teachers should determine how they are going to assess their students. The assessment should include the *Teacher Expectancies* for *balance* and some type of agreement on the types of questions that promote *consistency*, *fairness and portability* in the grading system. Portability means that a grade of B earned in one class would transfer and be equivalent to a grade of B in another teacher's classroom. The blueprint does not identify specific questions, but rather the approximate number of questions and type that promote a balanced assessment. In math, for instance, 6th grade math teachers might agree to have approximately 20 questions on a test, then determine how many might be computation, vocabulary/identification, concept/linkage, word problems, modeling, etc.

It is important to note that while grade level or same subject teachers agree on the specifications and the assessment blueprint, it is not necessary for all teachers to give the same assessment or have identical benchmark dates. While it is not necessary, I would recommend that there some common assessments.

If one teacher determined it would take six weeks to cover a fraction unit and another indicated he only needed five weeks, that's okay. But, if one teacher indicated he only needed two weeks and another scheduled twelve weeks, then that is a problem that needs to be resolved. Chances are if teachers are that far off on scheduling, then some are not covering the syllabi and others might not be addressing mastery

With respect to the blueprint, one teacher might decide to have four computation problems, while another might choose to have six. That is okay. The goal of the assessment blueprint is to assess students in similar ways and at approximately the same level of difficulty. This approach will help ensure that a grade of "B" earned in one class would transfer to a grade of "B" in the same class or subject, but taught by a different teacher. It would not be fair to students if one teacher had a question like reduce 6/8 while the teacher across the hall had their students reducing 111/213 on the same unit test. It should be noted, the more the tests are identical, the higher the correlation between students' grades between classes.

On the following pages, examples of a Specification Sheet, Assessment Blueprint, and a sample test reflecting the items listed on the specification sheet and the number and types of questions on the assessment blueprint are provided for a sixth grade test on fractions.

Specification Sheet

Fractions

What students should know, recognize and be able to do.

(The types of questions being asked on a unit test should be base don the material being taught and state and local standards. They should also reflect how that information might be tested on semester exams, CRTs, exit exams, NAEP, and college entrance exams such as the ACT or SAT)

Definitions – fractions, proper, improper, mixed, reciprocal

Identification – numerator and denominator

Equivalent Fractions – converting and simplifying

$+,-,\times$ and $\div$ fractions

Borrowing/Regrouping, whole and mixed numbers

Algorithms for $+,-,\times$ and $\div$

Rules of Divisibility: 2,3,4,5,6,8,9,10

GCF, LCM

Common denominator – methods

Draw models for $=$, $+,-,\times$ and $\div$

Ordering / comparing

Applications (word problems)

Open-ended concept or linkage

ASSESSMENT BLUEPRINT

Fractions

2 Definitions

1 Identification

2 algorithms / information

1 rules of divisibility

2 concept / linkage problems – open ended

1 draw model

1 ordering

1 simplify

4 computation, $+, -, \times$ and $\div$

1 GCF, LCM

3 word problems (applications)

Cumulative questions

Fairness and portability of grades from class to class or between schools should be considered.

MODEL TEST
Fractions

On questions 1-3, write the definition for each.

1. Fraction

2. Proper fraction

3. Reciprocal

4. In the numeral $\frac{3}{8}$, the 8 is called the _____.
5. List two methods for finding a common denominator.

6. Write the steps, as discussed in class, for adding fractions.

On question 7-10, evaluate each expression. Simplify your answers.

7. $\frac{5}{7}+\frac{1}{3}$

8. $12\frac{1}{2}$
$-7\frac{2}{3}$

9. $5\frac{1}{2}\times\frac{2}{3}$

10. $\frac{3}{4}\div\frac{1}{8}$

11. Find the LCM and GCF of 108 and 72.

12. Simplify the following fractions to lowest terms (simplest form.)

a. $\frac{8}{12}$ b. $\frac{27}{63}$ c. $\frac{111}{207}$

13. Write a five-digit numeral divisible by 2, 3, 4, 5, 6, 8, and 10, but not 9.

14. Order the following fractions from least to greatest. Show your work or explain the strategies that you used.

$$\frac{3}{4}, \frac{7}{10}, \frac{5}{7}$$

15. If the numerator of a fraction remains constant and the denominator increases, what happens to the value of the fraction? (Assume the numerator and denominator are positive.)

16. A student added $\frac{1}{7} + \frac{4}{7}$ with a result of $\frac{5}{14}$. The answer is incorrect. What is his error and how would you explain to him the reason behind the correct answer?

17. Draw a model to show that $\frac{1}{2} = \frac{4}{8}$.

18. Bob owns five-ninths of the stock in the family company. His sister Mary owns half as much stock as Bob. Jill owns the rest of the stock. What **part** of the stock does Jill own?

19. Joel worked $9\frac{1}{2}$ hours one week and 11 hours and 40 minutes the next week. How many more hours did he work the second week than the first?

20. A person has $29\frac{1}{2}$ yards of material available to make uniforms. Each uniform requires $\frac{3}{4}$ yard of material. How many uniforms can be made? How much material will be left over?

With experienced classroom teachers involved in this process, it might take 15 or 20 minutes to create a specification sheet that is based on the school district's curriculum documents and state standards.

It generally takes longer to come to consensus on the assessment blueprint. Teachers need to keep in mind the assessment blueprint is a guide and teachers should work toward building a consensus; **it is not a binding agreement**. Classroom teachers continue to make up their own tests unless they want to create common tests by grade level or subject.

A sample test was provided just as an example. A couple of things should be noted. First, the test did not exactly follow the assessment blueprint. It should also be noted that having teachers identify what they want their students to know, recognize, and be able to do is a straightforward process in which teachers readily agree.

The assessment blueprint gets into testing – teachers tend to argue strenuously over this. What we quickly realize is teachers don't want their students tested on what they don't either stress or teach. For example, teachers who are constructivists may not want students to memorize important facts or procedures. Other teachers might not see the value in students understanding what they are being taught and just want kids to memorize the rules. Believe me, a heated argument will develop in these circumstances and hard feelings will follow unless a balanced curriculum is being followed. To settle such disputes, teachers should refer back to their specification sheet, curriculum documents and state standards to determine what should be tested

One area where teachers should take special precautions is in the writing of test questions. Too often classroom teachers use less formal language on their teacher-made unit tests which may result in students not recognizing the same information that is tested on state or national exams. Care should be taken to write test items so students are exposed to the way in which those questions are phrased or tested on standardized exams. For example, in algebra, a direction on a teacher-made test might be to "solve" an equation. On college entrance exams, the same direction would be "to find the solution set over the real numbers such that…." The way the question is asked might cause some students not to connect what they learned in the classroom to what is being tested on high stakes tests.

To summarize, using the *BAM*, teachers determine the unit of study and how long it should take to teach. Next, teachers create the specification sheet and assessment blueprint based on district curriculum documents and/or state standards. Finally, using the *Teacher Expectancies* to ensure balance, teachers create their own unit tests.

That takes care of the paperwork!

Now on to the most important component of the *Backward Assessment Model*—the sharing of ideas, resources, materials, knowledge, skills and teaching strategies.

As states earlier, educational research suggests that professional interaction—at times informal and unstructured—is often far more influential than organized professional development, and is more likely to result in changed behavior.

After the paperwork is completed, experienced teachers should share their knowledge of where students traditionally experience difficulty on a particular unit. Rather than bemoaning the fact that students have performed poorly on those areas historically, teachers should exchange knowledge, resources, experiences, and successful teaching strategies with each other. Modifying instructional strategies and/or resources can result in greater student understanding and increased student achievement.

Teachers could increase their content knowledge by using this time to share their understanding of conceptual knowledge and application of the knowledge and skills taught in class.

Teachers might also examine areas in which the district has not performed up to expectation on state and national tests and address those areas of concern. Teachers might also study their most recently administered test to determine strengths and weaknesses of their instruction. Once that has been accomplished, decisions might be made on how best to address weaknesses during the current school year and how instructional strategies might be changed in future years.

If specific student populations can be identified as doing poorly, the grade or subject level teachers might want to bring into their meetings ELL, special education, reading or instructional strategists to recommend possible changes in instructional methodologies that would be beneficial to identified students.

A lot of work, time and effort are necessary to effectively use the *Backward Assessment Model*. To assist teachers in working together, a professional development agenda is provided on the next page to guide them through this process. You might notice that the school administrator is first on the agenda to discuss issues they need to communicate to effectively run the school. The rest of the day is devoted to what teachers do, how they do it, student performance, and instructional strategies that will increase student achievement.

Professional Development Day Agenda

I. General meeting – discuss items that site administrators need to address
II. Teachers meet by grade level or subject.
    A. Identify the following and discuss using available data:
        1. The next unit of study
        2. The most difficult unit of study as determined by data and teacher experience
        3. The unit of study causing students the most difficulty as identified by local, state, or national test data
    B. Identify what students should know, recognize, and be able to do on the selected unit (Specification Sheet).
    C. Identify how long it should take to teach the selected unit (Benchmarks).
    D. Determine how and what to assess on the selected unit to help ensure consistency (portability) and fairness between classes of the same grade level or same subject (Test Blueprint).
    E. Using data, identify topics within that selected unit in which students traditionally experience difficulty.
    F. Share with each other successful teaching strategies to overcome those difficulties and/or deficiencies.
    G. Share content knowledge, resources, and expertise to address student success on the identified unit.
    H. Using data, discuss way to involve special education or ELL facilitators if specific student populations are not experiencing the same success as the general population.
    I. Examine the results of the last unit test or other testing data to determine strengths and weaknesses of student's understanding of subject matter.
    J. Identify students not meeting proficiency on standards and a plan to remediate those students.
    K. Identify what instructional practices you will change for next year to correct deficiencies and improve student achievement.

An agenda such as this will focus professional development on teaching and learning. This agenda cannot be discussed in a one or two hour meeting; almost the entire day should be set aside for these discussions.

Secondary teachers are subject specific, so determining what unit(s) they want to discuss might be a little easier than elementary teachers who, before they choose a unit, must select a subject.

Administrators can expect to see classroom teachers gripe as they begin this process; that's natural. In fact, when any administrative group gets together, they spend a few minutes sharing their gripes with anyone who will listen too.

Getting people to change the way they do business is tough. People's first concern is how or if the change will affect them. If it does, then they become concerned about how much additional time will be needed to get the job done. There's always plenty of criticism when change is involved, particularly about how if they don't have adequate time now, then when are they expected to find the time to implement the change.

This is a real concern and it's a real problem that needs to be addressed. But before we get there, teachers need to be reminded to focus on results – increased student achievement.

Yes, change affects them and time is an issue, however if teachers and administrators continue what they are doing, they will continue to get what they have been getting. Chances are that's not acceptable. Part of the answer is to focus on the solution and change what needs to be changed even if it changes the way we have been doing business.

Often, the greatest obstacle to this is the school principal who is unwilling to treat his/her teachers as professionals. These principals want to be in control. Teachers need to be given the opportunity to talk in-depth about their job with colleagues. This means a building principal must give up control on the professional development day. This is very difficult for some. The proposed professional development day agenda, with its required paperwork, will keep classroom teachers on task. *But keep in mind, while the paperwork is important, the most important thing teachers will be doing is sharing their knowledge, resources, and instructional strategies with one another.*

In most schools, there are good teachers and not so good teachers in rooms right next to each other. This professional development agenda will have a tendency to level the playing field with all teachers afforded the opportunity to share their knowledge of teaching with each other.

Most new teachers are brought into the profession using the Pier System. That is, new teachers are given a set a keys, a room assignment, information on where to find books and supplies, a roster, and then sent on to the pier where they are thrown into the water to either sink or swim.

BAM is a Peer System. Can you imagine being a brand new teacher where you get to meet your colleagues and have them share with you what you will be teaching, the order in which you teach it, where students traditionally have difficulty, resources and strategies to overcome those difficulties, what and how they test, how they grade, and they actually give you materials to help you get started and learn your profession? That's *BAM*ing.

While *BAM* is a communications model, it is also a structure that supports professional development focused on increasing student achievement.

BAM and the Components of an Effective Lesson guide teachers' preparation and actual instructional delivery through a typical lesson to ensure efficient and effective use of class time and instructional strategies.

This Components of an Effective Lesson is a guide based upon the instructional strategies we have already discussed. Even though we have discussed them before, I'd like to touch on the two reviews scheduled during the class. More often than not, teachers will say they do not have time for two reviews in a typical class period. The typical instructional day will have one review to go over recently taught material or last night's homework. While that's good, teachers need to remember they live in a world of high-stakes testing. Allowing students to forget translates to students not learning. A second review should be scheduled almost daily to address long-term knowledge, mastery, deficiencies, as well as preparation for high stakes tests. The reviews should be based on student performance. These reviews are particularly helpful to English language learners.

The reviews might also involve concepts and skills not being taught this year. The necessity of these reviews becomes readily apparent when students are required to take promotional or exit exams that might contain material that was taught two or three years before.

Since college entrance exams are reported by state, school district, and school, they determine if a student will gain admittance to a particular college or university, and in some cases used to determine financial aid, some might argue these are important exams. Wouldn't it be reasonable to expect teachers take some time to prepare their students for such high stakes tests?

By doing so, test scores will inevitably go up, which is often perceived by the public as increased student achievement.

As can be seen, there is a great deal expected of classroom teachers. BAM and the components provide a structure that helps the teacher incorporate the teacher expectancies.

Chapter 4

Components of an Effective Lesson

Make a good faith effort to teach the assigned curriculum

Before we get into discussion about actual lessons, there must be an understanding that teachers and administrators must do their job. For teachers that means they should plan to make a good faith effort to cover their assigned curriculum. The district must ensure the curriculum can be covered in the time allotted and the school administration needs to ensure that content is being delivered in a way that increases student performance.

Most school districts have established policies similar to the following: "Guides or course syllabi are established for all areas of the curriculum and are to serve as the basis for instruction in district. Members of a professional staff shall utilize these guides as a means of meeting the needs of individual students. Making a good faith effort to teach the curriculum means that teachers plan to cover all the material in the appropriate syllabus."

The development of specific teaching techniques is the responsibility of the individual teacher. It is suggested that these teacher expectancies be incorporated into daily plans. These should be consistent with the district's objectives and proven principles of learning.

In addition, many school districts have also established position statements or guidelines as follows:

STANDARD FOR QUALITY: Adopted secondary course syllabi serve as the basis for classroom instruction.

I. Instructional activities are correlated with stated objectives in adopted secondary course syllabi.

II. Resources are selected to support objectives in course syllabi.

III. Daily, unit, and semester planning includes goals and objectives contained in course syllabi.

IV. Appropriate accommodations and/or modifications are made in alignment with goals and objectives in adopted course syllabi to meet the instructional needs of all students.

Imagine you are the parent of twins, they are enrolled in the same school, but don't have the same teachers. Would you expect them to be learning the same material at about the

same time? Covering the established curriculum with the appropriate benchmarking will create the consistency needed to develop and maintain credibility in the community.

There is no greater factor that affects student achievement than the content of the classes they take. If teachers do not cover the curriculum assigned to them, students will end up with gaping holes in their knowledge. All too often you hear teachers talking about how much time they must spend to review topics that students were supposed to learn the year before. While I sympathize with those teachers and the work they think they must do to remediate those students, the fact is by not covering their assigned curriculum, they will be contributing to the problem as well. To address these deficiencies, it is my belief teachers could better utilize the idea of linking new concepts and skills to those areas of concern and reinforce those basic skills.

The simple fact of the matter is teachers who spend too much time remediating students who have already been remediated will not be able to cover their own curriculum assignment and that will result in the school not making adequate yearly progress. One of the first things I would check if a school is not making AYP is if they were teaching the curriculum assigned to them. How teachers' teach is important, what they teach is of greater importance! Teachers must teach the curriculum assigned to them.

Benchmarks

Is there a need for a consistent, standards-based curriculum? That question may best be answered by asking another question: Have you ever had students transfer into your class who have not acquired the necessary prerequisite skills and knowledge to be successful? If you answered "Yes," then you see the need for a curriculum that is consistent, not only between classrooms within a school, but between schools as well.

Although curriculum guides and syllabi provide classroom teachers with clear goals and expectations, they are often not accompanied by explicit timeframes. Therefore, in order to maintain a consistent mathematics curriculum, benchmarks should be established. (Benchmarks are approximate time lines by which particular concepts and skills are to be taught.) It is suggested that teachers within a school teaching a particular class, work jointly to develop benchmarks. Once developed, benchmarks must be revisited on occasion to allow for necessary revision. Setting and following these benchmarks should ensure adequate coverage of essential course objectives which lead to mastery. By adopting a professional development model such as the Backward Assessment Model, these timeframes would be established for each unit of study.

To further ensure students are meeting academic expectations, common periodic testing during the school year should be scheduled to determine the level at which students are achieving mastery on specific topics.

Frontloading

We discussed the importance of teaching the curriculum and using the benchmarks to assure students are spending the appropriate amount of time on the concepts and skills

being taught to reach mastery. Having said that, there are times when it appears the system is setting students up to fail. While teachers clearly want to teach to mastery, a dense curriculum can get in the way of achieving that goal. Teachers often talk to each other about the issue of *coverage versus mastery* and how time effects student performance. Another factor that impacts student achievement is the sequencing of the material to be taught during the year. If important concepts and skills are left to the end of the school year, teachers might not get to them or if they do, they might have to rush to cover the topic. Not having the time to address mastery will have a negative impact on student performance on high stakes tests.

Frontloading suggests that teachers examine the curriculum assigned to them and determine the most important topics to be taught during the year. Once that determination is made, teachers should ensure they teach that material early enough in the year so they know they will get to it, teach to mastery, and that they will have opportunities to review and reinforce those concepts and skills regularly during the school year.

Naturally, material should not be just arbitrarily moved around, sub-skills still need to be taught as does developing the foundation for what is to be taught. Having said that, let's look at an example.

Fractions are often taught in sixth grade. I would guess there is a consensus that students should be able to compute with fractions. If fractions were introduced late in the year, some teachers might not get to them, others might feel rushed trying to cover them. The seventh grade teachers will be upset if their students come into their classes without the prerequisite knowledge and skills to be successful for them to teach the curriculum assigned to them. That would result in them trying to re-teach the material, giving up the valuable time they need to cover the material their assigned to teach.

Components of an Effective Lesson

Too often in life decisions are made by default – by not making a decision. Many students actually make the decision to not go to college in their freshman year of high school. The decision was made by their class selection – not a conscious decision not to go. The same can be said of some classroom teachers, they do things without consciously making the decision, things that with a little extra thought, they probably would have modified or done differently. Now that we have discussed balance in the delivery of instruction, let's take at how that instruction should be delivered.

The Components of an Effective Lesson is a blueprint for classroom teachers to follow which is easily monitored by school administrators. If fully implemented, the components and teacher expectancies will result in increased student achievement. My experience is that most teachers already loosely employ many of these components in their current lessons. These components are not controversial, but they are not typically being fully implemented.

The "5 + 1" is a subset of the components of an effective lesson and teacher expectancies that leave a paper trail that can be readily seen by an outside observer. The following is an overview of the Components of an Effective Lesson.

INTRODUCTION

DAILY REVIEW

DAILY OBJECTIVE

CONCEPT AND SKILL DEVELOPMENT AND APPLICATION

GUIDED / INDEPENDENT / GROUP PRACTICE

HOMEWORK ASSIGNMENTS

CLOSURE

LONG-TERM MEMORY REVIEW

The Introduction

As part of setting the stage for learning, students should be made aware of each day's objectives and how it might be used in everyday activities. Thus, teachers should explain why students might want to learn what is being taught that day. The introduction provides classroom teachers an opportunity to create interest and enthusiasm in the lesson that will be taught. As an example, if the day's lesson is about the circumference of a circle, the teacher might explain how changing the tire sizes on a car will affect their odometer and speedometer reading, and might reduce the life of their tires. Everyone uses math in their daily life, but too often there is a disconnect between the math they learn in the classroom and the math they use to survive and make decisions. Make those connections.

With some concepts, teachers do a great job of introducing how the math they are teaching can be used. But, we need to do a better job of introducing concepts and skills more often that create the connection between the real world and the math classroom.

First Review

In most cases, teachers are making a good-faith effort to teach the mathematics curriculum. However, there are factors that can hinder their efforts. Virtually all teachers report difficulties stemming from student absenteeism and/or tardies. Also, there seems to be a pattern of students entering middle school and high school with

deficiencies in basic skills. To help correct these deficiencies, daily reviews should be employed. The first review should address recently covered material, including important formulas or facts, algorithms, definitions, and strategies that students will need to recall to be successful in today's lesson. Daily reviews can also provide an introduction to an upcoming lesson. Reviews can be done as written or oral work, and can be done at the beginning or end of a class or as a transitional activity any time during the period. These reviews are important because they provide an opportunity to reinforce information that all students should know at the completion of the school year. A suggestion to help students remember might be to review current material, such as homework, at the beginning of the class so you are better able to monitor student understanding. If as part of the review, the teacher summarized what was taught during the week, then did a couple of problems from the previous night's homework, they would increase student understanding and would find they would be answering fewer questions on the homework, thus preserving instructional time.

Teachers should also teach their students to review using different strategies such as mnemonics, linking, developing relationships, learning in context, and utilizing auditory and visual cues. Knowing "how" to remember is important if we are going to help students learn. Teaching students to recognize that they already use memory skills and helping them to transfer these existing skills will aid them in their efforts to learn. While there is more to learning than just memorization, memorization is an important component of learning.

Stating the Objective

To ensure the day's lesson is focused, teachers should state and write the objective on the board. The objective should be left written on the board for the entire class period to remind the teacher and students what they are expected to learn.

An example of an objective based upon the circumference mentioned in the introduction might be: The student will learn the origin of pi and be able to find the circumference of a circle given the radius or diameter.

While many of us believe students know what we are teaching, based upon the presentation, it is surprising to know many students can not identify the big idea being taught in specific lessons. Stating and writing the objective clearly resolves that problem. The objective should also be copied in the student notebook!

Conceptual Development

Remember, it's not a matter of "if" students are going to forget information they have learned, it's a matter of "when" they will forget it. Without concept development, students will not be able to reconstruct knowledge lost over time.

In mathematics classrooms that lack sufficient concept development, memorization of rules and algorithms is emphasized but little or no attempt is made to help students

understand the "why" of mathematics processes. Concept development should be as important as memorizing basic facts and algorithms. Students understanding of, and comfort level with, new ideas is increased when concept development is done properly.

Sometimes students are able to "get the right answer" even though they don't necessarily understand the "why." Mathematics then becomes an arbitrary set of isolated rules which can often lead to future pitfalls. As mathematics becomes more abstract, "math anxiety" may develop if these rules and algorithms have not been developed with an understanding of why they work. Eventually, students can become frustrated and quit taking math, even though the grade they earned in their last class was average or above.

Developing concepts and linking those ideas to students prior experiences helps to explain the "why" and makes students more comfortable in their knowledge and understanding of mathematics. For example, rather than just having students "flip and multiply" when dividing fractions, the division algorithm might be developed through use of repeated subtraction. Solving equations should be connected to the "Order of Operations." Finding the sum of the interior angles of a triangle might be introduced by having students cut out angles in triangles and piece them together. The Pythagorean Theorem might be explained by using the areas of the squares formed by the sides.

Unfortunately, students all too often "tune out" teachers during concepts development. Since students value what teachers test, concept development must be tested. Students might write a brief explanation of the development of a particular concept as a part of the homework assignment, and then be asked an open-ended question on a test where they must explain the origin of a rule or algorithm.

Presentation Techniques

Nothing ruins a good lesson like a bad example. Teachers must take great care in choosing examples. Teachers need to be careful and pick simple, straightforward examples that clarify what they are teaching that don't bog kids down in arithmetic. Too many teachers think of examples as they are teaching, without much forethought, and wind up picking a variation of the concept or skill which results in confusion for the students. Before variations are discussed, it helps student understanding to first understand the big idea being discussed. Building student confidence and building *success on success* goes a long way to increasing student achievement. Introducing variations of the problems before student understanding is complete often distracts them from learning the objective of the day. Use simple straight forward examples that clarify what is being taught is referred to as "comprehensible input" in the world of teachers working with students whose second language is English

Some teachers have difficulty keeping students focused on the lesson at hand. By "using thorough instructional planning and preparation" and "smooth, rapid transitions between activities," teachers can prevent a great deal of student inattention. Just as important is the recognition of different learning styles. Most secondary school teachers present their

lessons visually and verbally while students sit in rows and work independently. This is in sharp contrast to elementary classrooms where students are customarily seated in groups and the majority of instruction is verbal and tactile. Using a variety of teaching techniques should help develop mathematics concepts in a manner that focuses students' attention, increases their level of engagement, and decreases off-task behavior. When teachers use a combination of verbal and visual instruction, hands-on activities, demonstrations, guided practice, and both independent and group work (cooperative learning), they meet the needs of most students.

Visual Component

When learning new material, many students have a need to see it, or at least visualize it, before it becomes meaningful. This implies that teachers need to prepare lessons and materials well before actual instruction. This might include creating models, charts, slides, graphs, videos, manipulatives, overhead transparencies, and handouts.

Many times the visual component of instruction coincides with the development or presentation of the lesson itself, that is, the teacher gives notes on the board or overhead while they verbally explain the material. This is a powerful instructional technique that many teachers use effectively. When teachers shortcut this process by only "talking through" multi-step problems or having students mentally "visualize" complex situations, students may become confused and disinterested. Many students are primarily visual learners and teachers must be concerned about and creative in meeting their needs.

Research suggests keeping material on the board while you are teaching so students can refer back as needed. If teachers are using an overhead projector, this becomes almost impossible for students to do without interrupting the flow of the instruction. More thought needs to be given in using available technology. I recommend limiting the use of an overhead projector when teaching mathematics – it really limits the ability of students to see patterns develop.

Auditory Component

The auditory component of student learning is important and usually not neglected. While it is important for students to visualize what they are learning, additionally they often must hear an explanation. At times, the auditory learner benefits from group work where he can hear the material being verbalized. New terms and material frequently require repeated explanations. This may become cumbersome if concepts are only presented visually. Finally, if teachers give auditory clues to learning, many students will better remember the material. Teachers should give students time to work in groups, that oral communication reinforces the auditory component.

Oral Component

Oral recitation is the practice of having the entire class recite important facts, identifications, definitions, formulas, algorithms, theorems, and rules during their initial presentation and later when these topics are revisited. To ensure full participation,

individuals are also called upon. The number of times an item needs to be repeated depends upon the difficulty of the material and the ability of the class.

In real life, repetition is used frequently to aid memory or to make a point. For example, adults often use it to remember a license plate number or a grocery list. Advertisers use repetition in presenting their message to the public. (How many times have you found yourself repeating a phrase from a commercial or humming a jingle?)

Often, teachers report that students have performed poorly on assignments or tests because they did not study. While not a substitute for out-of-class studying, in-class oral drill and recitation provides an opportunity for important repetition, a tool in improving students' achievement. When used correctly, they compel students to become active participants in the lesson and teaches them one method for memorizing new information. Oral recitation has the additional benefit of meeting the needs of auditory and English language learners and teaches all students how to say and read mathematics correctly.

Tactile Component

A National Council of Teacher of Mathematics (NCTM) motto states, "Math is not a spectator sport." The tactile learner learns by doing; he needs direct involvement in the mathematics process. Teachers should provide tactile learners the opportunity to explore and understand mathematics through the use of manipulatives, hands-on activities, labs, group work, projects, and paper-and-pencil problem solving. Additionally technology, including calculators, calculator- based labs, and computers should be used to aid the tactile learner. When using these instructional aids and procedures, teachers may need to do the activity as a learner prior to its presentation in order to fully understand the mathematics involved and anticipate students' difficulties.

The integration of visual, auditory, oral, and tactile styles of teaching and learning ensures students of a better opportunity to learn and feel more confident and competent in their abilities to do and understand mathematics.

Practice (Guided/Group/Independent)

When someone takes up a new sport, they do not expect to be proficient immediately. One expects to practice a new activity to get better at it; long, hard extensive practice is almost always necessary to become proficient. Learning mathematics skills can be equated to learning physical skills. Practice, with frequent reinforcement, feedback and evaluation, is essential in order to master abstract concepts. In order to avoid short-changing their students, teachers need to provide practice –guided, group and independent.

As part of developing mathematics concepts, teachers need to give students opportunities to practice new skills with immediate feedback. Initially teachers should include several examples as part of the explanation while giving notes.

Guided and independent practice may be more than paper-and -pencil work. It may include labs, projects, and the use of technology.

Before students are sent home with homework (independent practice), guided practice should be extended to ensure that students are proceeding correctly. Several exercises, similar to the homework assignment, are provided for students to work on in class. Teachers should monitor students carefully, looking for points where they become stuck or confused. If many students stumble on or fail to grasp a given idea or step in an algorithm, the teacher should immediately address the problem on a class-wide basis. If only a few students experience difficulty, these problems can be handled on an individual basis. In either case, a review of the guided practice exercise is recommended before students leave class to begin independent practice.

It must be noted that guided practice should not be "starting homework." Students frequently dawdle during or entirely misuse class time allocated to practice if the time is for "homework". Homework, for the most part, is to be done outside class time; guided practice is done in class with immediate feedback. As an incentive, teachers may add an assessment component to guided practice—essentially a participation grade.

Allowing students to work in groups provides students an opportunity to use the language; that communication has the benefit of having students thinking, reflecting, and organizing their thoughts that not only helps other students in understanding a concept or skill, but it also clarifies a the student's own understanding.

Closure

At the close of the lesson, the daily objective should be repeated by the teacher to emphasize what was taught that day. Teachers might also ask their students to summarize what they learned orally or in writing, how the lesson related to previous knowledge, and how it might be used. By listening to their students, teachers would have a better idea if students understood the lesson. It's just another way on monitoring student learning

Long Term Review

A second review should be employed to address long-term knowledge, address

deficiencies, mastery, and prepare students for high stakes tests. The reviews should also be based upon student performance – not a whim. Can you imagine sending your son or daughter to a tutor who you are paying, only to find out the tutor is working on material your child knows and understands. My guess is you would not feel that was appropriate. You'd expect the tutor to work in areas where additional assistance is needed. No less should be expected of a classroom teacher, assistance should be provided in areas of

deficiency. The long-term review will often address concepts and skills that were learned previously and may not be part of the current year's curriculum.

High stakes tests require students to maintain knowledge over time, without these long term reviews, there is no doubt students will forget information over time. Forgetting in school is often translated to either not being taught or not having been learned. In either case, it's not only a problem for the students, it's a public relations nightmare for the schools. Schools and math departments must have a plan to address the long-term knowledge of their students, allowing students to forget information over time is not a viable option.

If the second review (long term review) is employed during the last five to seven minutes of a class, students will remain on task till the end of the allocated time. That will maximize time on task as well as having the additional benefit of cutting down on potential discipline problems. Implementing a second review period also has the added benefit of providing teachers opportunities to address mastery, deficiencies, and prepare students for college entrance examinations such as the ACT and SAT or high school exit exams.

Components of an Effective Lesson
Summary

Before presenting a lesson, refer to the specification sheet and assessment blueprint for the unit.

INTRODUCTION

- Set the stage for today's lesson (students will take notes, participate in a group activity, etc.)

DAILY REVIEWS

- Provide review for short-term memory over recently taught material
- When correcting homework: provide immediate and meaningful feedback and hold students accountable
- Keep reviews and homework checks brief

DAILY OBJECTIVE

- State and write before introducing the day's main lesson and have students record this in their notebooks

CONCEPT AND SKILL DEVELOPMENT AND APPLICATION

- Teach the big concepts
- Provide the "why" for rules and algorithms
- Link concepts to previously learned material and/or real-world experiences
- Utilize a variety of techniques: students need to see it, hear it, say it, and do it
- Hold students accountable for taking notes and keeping mathematics notebooks

GUIDED / INDEPENDENT / GROUP PRACTICE

- Can be done at different times throughout the lesson to help students process information
- Students need time to think, analyze, work on problems, discuss their solutions and become problem solvers instead of watching the teacher do all the work
- Can be done as an entire lesson that enhances conceptual understanding and/or application of concepts through inquiry, investigation, discovery, lab or problem-solving activities

HOMEWORK ASSIGNMENTS

- Assignments should consist of what teachers value and include a variety of assessment items, including, definitions, computations, explanations, applications, etc. (see the assessment blueprint for the unit)

CLOSURE

- Have students explain in writing what they have learned and apply it
- Restate what was taught

LONG-TERM MEMORY REVIEW

Address mastery, student deficiencies, high stakes tests, and stress important ideas - not necessarily part of this year's curriculum, but based on student knowledge

Chapter 5

2nd Essential - Note-taking

If teachers are not presenting material in an organized fashion, a manner which would help students be more successful learners and if students are left to their own devices to take notes without specific guidance from their teachers, then we are setting many students up to fail.

Note taking is important for a number of reasons. Note taking keeps students engaged in learning. That engagement keeps students busy which results in less discipline problems. The notes students take in the class should reflect what was taught in that lesson, should help students complete their homework assignment, serve as a foundation to prepare for unit or chapter tests, and to review for high stakes exams such as end-of-year tests, college entrance exams, or exit exams.

The notes below reflect what is most often seen in a math classroom. A rule accompanied by a number of examples. While initially learning, students might be able to follow that instruction, record this information and get many of the problems correct on their homework assignment. But, these types of notes will result in student difficulty. Not having the reason behind the rule will result in students thinking mathematics is like mathemagic – particularly when this concept is presented more abstractly in an algebra class.

When you multiply exponentials with the same base, you add the exponents.

Ex. 1 $3^2 \times 3^5 = 3^7$

Ex. 2 $5^4 \times 5^6 = 5^{10}$

Ex. 3 $3^2 \times 3 \times 3^4 = 3^7$

Ex. 4 $5^2 \times 7^3 \times 5^4 \times 7^5 = 5^6 \times 7^8$

Another concern with notes that look like this is that students will fall in very predictable traps. For instance, in Example 3, the second number did not have an exponent. Many students, without conceptual understanding, will tend to believe that since there is no exponent, the exponent is zero.

Notebooks in many math classrooms typically record nothing more than example after example problem. Notes should include a date, title, objective, vocabulary & notation, identifications, conceptual or pattern development that leads to a rule, the rule, example problems using that rule and explanations that would help students understand the concept when something out of the ordinary occurs or when they review at a later date. Teachers should assist students in setting up their notebooks to avoid visual overload when trying to study. Classroom teachers need to remember that at the end of the school year, text books are often collected leaving students with only one resource to review and reinforce what they have learned – their notebook. Student notes are important!

When asked, memory researchers reported the number one "memory aid" which they themselves use is "write it down." Teachers should require students to take notes in all mathematics classes. Notebooks keep students engaged in learning, help them complete their daily homework assignments, enhance their study, and act as a foundation from which to prepare for tests, both unit and high stakes tests. Also, since students are not allowed to keep their textbooks, the student notebook is usually the only mechanism available for review in later years.

Note taking is a process used by students to record important information that they are trying to understand and need to remember. Because of the importance of a student notebook, teachers need to be prescriptive in how notes are taken and accommodating in their instruction so students can take notes.

The notes in a typical student notebook are no more than problem after problem being recorded. At times, a rule or procedure may be written. The problem with at is those recorded problems will not make much sense to students weeks or months after the notes were taken. Saying it simply, the notes most students take in a math class do not help students complete their homework assignment, prepare for a unit test, and review for high stakes tests later in the year.

Notes should typically include a title, the date they were taken, the daily objective, definitions, identifications, pattern or concept development with pictures or diagrams that leads to some conjecture, a formalized rule or algorithm, and a number of example problems used in guided practice. Teachers should also encourage students to write an explanation of what led to the procedure being used to manipulate or solve problems. Explanations in notebooks are especially important when a problem-solving method might be construed as a "trick" and whose rationale would not be immediately obvious to the student when reviewed at some future date. The following "notes" will increase student learning.

Exponentials

March 16, 200X

Objective – To define an exponent and develop rules to multiply exponentials with the same base.

2^{3} – exponent

|

base

In the number, 2^3, read to the third power or two cubed, the three is called the exponent, the two is called the base.

Exponent – the exponent tells you how many times to write the base as a factor.

Ex. 1 $4^2 = 4 \times 4$

Ex. 2 $5^4 = 5 \times 5 \times 5 \times 5$

Ex. 3 $10^6 = 10x\ 10 \times 10 \times 10 \times 10 \times 10$

<u>Evaluating Exponentials</u>

To Evaluate or write an exponential in standard form, you merely compute.

Write in standard form.

Ex. 1 $4^2 = 4 \times 4$
$= 16$

Ex. 2 $5^4 = 5 \times 5 \times 5 \times 5$
$= 625$

Ex. 3 $10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10$
$= 1{,}000{,}000$

Multiplying Exponentials
with the SAME base

Simplify in exponential notation.

Ex. 1 $2^3 \times 2^4$

By definition, $(2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2)$

How many times are we multiplying 2 by itself? Answer – 7
Therefore the exponent is 7 and the answer is 2^7

Ex. 2 $3^2 \times 3^2$

By definition we have, $(3 \times 3 \times 3) \times (3 \times 3)$

How many times are we multiplying 3 by itself? Answer – 5
Therefore the exponent is 5 and the answer is 3^5

Ex. 3 $5^3 \times 5$

By definition we have, $(5 \times 5 \times 5) \times 5$

How many times are we multiplying 5 by itself? Answer – 4
Therefore the answer is 5^4.

N.B. There was not an exponent on the second factor in example 3. Anytime a number does not have an exponent, it is understood to be 1.

Looking for a pattern ~

Ex. 1 $2^3 \times 2^4 = 2^7$

Ex. 2 $3^3 \times 3^2 = 3^5$

Ex. 3 $5^3 \times 5 = 5^4$

Looking at those three problems and those three answers, do you see a pattern that would allow you to simply those problems?

It appears that when you multiply exponentials with the SAME base, you add the exponents. That observation leads to the following rule.

Rule 1. When you multiply exponentials with the same base, you add the exponents.

Ex. 1 $4^5 \times 4^2 = 4^7$

Ex. 2 $7^3 \times 7^6 = 7^9$

Ex. 3 $2^{10} \times 2^{35} = 2^{45}$

***Ex. 4 $11 \times 11^8 = 11^9$

Notice, in Ex. 4, there was no exponent written with the first factor, it is understood to be 1.

These notes are a great deal more extensive than the notes shown with just a rule and sample problems. If students were to revisit these notes over time, the math would make sense.

Finally, while note taking is a student responsibility, teachers need to hold students accountable for taking notes. This need not be complicated or time consuming, but it must be done frequently and consistently to further encourage students to take notes.

It's my opinion that notes are a very important component in increasing student achievement. The notes will help the students complete their homework assignments and should be the primary vehicle used by students to prepare for tests.

Teachers should be very directive, telling the students not only what to write down, but where to write it in their notebook. Spacing is important in student note-taking. If notes are crunched together, it will result in visual overload. That overload won't help students study more effectively and efficiently. A student's notebook sitting in the first row, first seat should be almost identical to the student sitting in the fifth row, fifth seat if they are following teacher directions.

Star System

In addition, I like to use the Star System * when working with students who have not had a great deal of success in mathematics. The star system creates additional emphasis on topics to be tested. For instance, if I tell the students to place three stars (***) by something in their notes, that means that item will be on the homework, practice test, and real test – unchanged. Two stars (**) by information in the notes means that type of problem will be on the test, maybe with numbers being changed or a minor modification in the problem.

As an example of a two star problem (**), the notes could contain a problem involving finding the distance between two points of a line segment. On the homework and practice test, different numbers might be used. Two start problems are typically the types of problems assigned as exercises on the homework.

A one star (*) problem highlighted in the notes suggests this item or a related one could be on the homework or practice test. As an example, I might have students place one star (*) by the procedures for adding, multiplying and dividing fractions. Another example of a one star, I might ask students to find the midpoint of a line segment or I might give students one endpoint and the midpoint and ask them to find the other end point. Those questions would be assigned for homework and one of them might appear on the practice test. But another might be on the real test.

As administrators are monitoring instruction, they should examine student notebooks. If the notes being taken by students do not reflect and reinforce the daily instruction, don't include a title, the date they were taken, the daily objective, definitions, identifications, pattern or concept development with pictures or diagrams that leads to some conjecture, a formalized rule or algorithm, and a number of example problems used in guided practice, then they will not help them complete their daily homework assignment or prepare for a unit test. A school administrator observing that should sit down with the classroom teachers and suggest or recommend changes to enhance the educational experiences of the students in that class. The administrator might suggest the teachers help the students out with having white space by being more directive when giving student notes. The administrator might recommend it may be necessary that more examples be provided by the teacher so the students can see a pattern emerging that leads to a rule. If the teachers were using an overhead, a smart board, or powerpoint, the administrator might recommend that board be used so the students can see the pattern develop in its entirety.

Notes are important for student success, school administrators overseeing math instruction should ensure student notes truly assist students in learning. It's the second essential.

Chapter 6
3rd Essential - Homework

Student achievement rises significantly when teachers regularly assign, and students regularly complete, homework. The additional study that homework provides benefits students at all ability levels. Furthermore, homework gives students experience in following directions, making judgments and comparisons, raising additional questions for study, and developing responsibility and self-discipline. While homework is not the only or most important ingredient for learning, achievement is often diminished in its absence.

In order to maximize the positive benefits of homework, teachers need to give the same care in preparing homework assignments as they give to classroom instruction. Teachers must carefully prepare the assignment, thoroughly explain it, and give timely comments when the work is completed. Homework should reinforce what was taught and learned. Students should not be assigned exercises if they are unsure of how to do the problems.

Homework should reflect what teachers' value – what we have defined as balance. Besides assigning a set of problems for homework, teachers should also assign reading, require students to copy definitions, identifications, algorithms, and write brief explanations of the day's work as part of their homework assignments. As students become accustomed to seeing these items as part of notes, homework assignments and tests, they will also begin to understand their value. And don't forget to assign reading as part of the assignment.

Too often homework assignments in middle school and high school read like this:

Page 89, Problems 1-33 – odd.

A good number of students return to school the next day complaining to their teachers they could not do the homework assignment because they did not know or understand how to do it.

What would better serve their students would be a homework assignment that reads like the following:

> Read Sec. 2.3 – Solving Linear Equations,
> Write the Order of Operations,
> Write the strategy for solving linear equations,
> Explain how the Order of Operations is related to solving linear equations,
> Page 89, Problems 1, 3-5, 7 ,11, 18-21, 27,31 33.

The second example incorporates into the homework assignment what math teachers say they value; vocabulary and notation, conceptual understanding, procedures, reading and writing, and practice problems.

Quite frequently, a middle school math homework assignment appears the same as the 1st example, a page number and exercises.

Students would be better served if the homework reflected what teachers wanted students to know, recognize and be able to do. If the middle school homework assignment was based on a fraction unit, the homework assignment should look more like this.

> Read Sec. 3.3 Add/Subtract Fractions
> Draw a model to add 1/3 + 1/2
> Write the procedure for add/sub fractions.
> Why don't you add the denominators when add/sub fractions?
> Name two ways of finding a CD and under what circumstances would you use each method.
> Page 162, 1, 2 ,4, 5, 12-16, 23, 27 31, 33

While having students practice is important, a homework assignment should not be about just completing an assignment, homework should also be designed to help students learn – to encourage them to study. A homework assignment like this would better reinforce what was being taught in the class and recorded in the notebook than just providing a page number and problem set.

The simple fact of the matter is this, in the more complete and thought out middle school assignment, students who had to write out the procedure for adding fractions would be much more likely to be able to add the fractions that were in the exercises. The same would be true in the algebra assignment. If the students were asked to write the strategy for solving linear equations, the probability of them being successful with the exercises would increase dramatically.

The real good news is that the first four to five questions on a homework assignment came from the instruction that recorded in the student notebooks. That means student can go directly to their notes and answer questions like draw a model, write a procedure, etc because that information is directly answered in their notes. And, if students can answer those questions, they are much more likely to be able to complete the homework assignment.

Homework assignments such as these being recommended have the added advantage of taking the excuse away of "I couldn't do it". Because the first set of questions, before the exercises, can be directly answered by reviewing the daily notes.

Also, if you were working on a concept or skill for multiple days, some of those definitions, procedures or formulas can be asked on multiple homework assignments to ensure the students are learning them. And, if the teacher thinks it might benefit the

students to write that information more than once on a given day, that can be assigned also. Repetition is important to student learning.

Knowing the importance of homework to increase student achievement, school administrators can easily monitor the types of assignments given to students at their schools. If they walk in a classroom and see a homework assignment on the board that contains only a page number and exercises, the supervisor can immediately improve the likelihood of student success by making recommendations that parallel our recommended assignments.

They might suggest or recommend teachers create a homework assignment sheet for a week or a unit that includes reading, writing, definitions, explanations, pattern development, rules, procedures and/or formulas along with exercises to help the students become more successful.

These assignments also have the potential of taking the excuses off the table for students who don't do homework.

Studying

If you want your students to be successful, teach them how to learn. Many students believe the reason some kids are successful in math is because they are smart. They don't equate studying with being successful or being smart. Talk to your students about how they learn; are they visual, audio, kinesthetic learners, or a combination?

Most students don't know how they learn, students who really don't know what it means to study use strategies such as the "Stare & Glare" of method of study. Others use the "Pray to God" method, and others, because of their after school job tend to grasp on to the "Osmosis" method. They placed the open text on their chests, took a nap, thinking when they woke that somehow that print would work its way to their brain. While these methods are not very effective, students continue to use them because they don't have other strategies that work any better for them. Some students need to be taught how to study effectively and efficiently. They need to know methods like saying, writing it and having someone ask questions – like they did in primary school still work. Poor students tend to study until they "almost" have the information, then quit. Students need to be taught that after they study, they should be able to discuss what they studied without having their notes as a prompt – or they need more study. Sometimes teachers inadvertently fail their students by not requiring them to verbalize their knowledge they gained.

Bottom line, teachers need to reinforce to their students that accomplishment is more dependent upon hard work and self-discipline than on innate ability. School administrators also need to be told the same thing; adoption of the latest new program will not lead to success, *work does*. In fact, the only place *success* comes before *work* is in the dictionary.

We also need to talk to students about their concentration times. How long can they study before they start looking around? I know I'm good for about forty-five minutes. While I can talk for days, after forty-five minutes of listening to someone else, I begin to notice how many lights are in the ceiling, a bird flying by the window, and a whole host of other items. Students need to know not only how they study, but how long they can study effectively and efficiently. Students need to be taught to extend their concentration time by studying a few more minutes before they take a short break from studying.

How many times have you heard a frustrated parent tell someone they sent their children to their room for two hours to study to improve their grades? Bottom line, if a student whose concentration time is only thirty minutes is sent to his room for two hours to study, chances are great that an hour and a half will be wasted. Students need to be taught how to study effectively and efficiently. If teachers don't tell them, who will?

For students to be successful, classroom teachers will need to be much more explicit on their expectations of students, telling them to "study" just is not enough.

Studying includes:

Reading	*Thinking*	*Reflecting*	*Organizing*	*Writing*
Analyzing	*Visualizing*	*Reviewing*	*Remembering*	*Recalling*

Studying needs to be explained to them, probably modeled. Students should be told that while they are reading their notes, they should be able to visualize what they are reading. That they should pause and think, reflect, visualize and organize that information in their head. That as they are going over examples, they should be able to go to the next step without looking at their notebook. And after they have read those notes multiple times, that when they closed their notebook, they should recall and remember what is in the notes to help them complete their homework assignment

Notice there is reading and writing in the homework assignment. Most teachers believe that math is a language, language can not be learned effectively without reading, writing and speaking it.

Reading

A common complaint among secondary teachers is their students can't read at grade level or their textbook. The problem is most of today's high stakes testing is composed of quite a bit of reading – problem solving. Secondary subject specialist teachers are often in denial with respect to their role in teaching students to read. The vast majority of students have been taught to read by using fictional texts. The way students read fiction

and the way technical material is read is different. Math students have to read differently to understand their math.

Who's going to tell this to the students if it's not their math teachers? It appears the way most secondary math teachers are handling the reading problem is by ignoring it. In fact, a common strategy seems to stop giving reading assignments because the kids don't understand them. When you say that out loud, it really sounds stupid. Not assigning reading will surely exacerbate the problem and continue the downward spiral.

Understanding a problem is surely part of mathematics, you can't understand problems without having acquired vocabulary and notation and that can't be done without reading. Math teachers have the responsibility to help their students read their texts. Reading math text is different from reading a novel. Students and their parents might not realize that, so classroom teachers have to teach students to read mathematics.

When assigning reading in mathematics, teachers should explicitly introduce new vocabulary and notation before assigning the reading. Teachers should preview the reading and connect the reading to previous knowledge. After the students have read the assignment, teachers should check for student understanding of the reading and correct misunderstandings – just as they do with homework problems.

Students should have a paper and pencil to assist them in their reading of math content. Students reading mathematics don't read by chapter, by section, by page, by paragraph or sentence, they typically read phrase by phrase – every word counts. Students should copy important information, definitions, formulas, examples, and draw pictures to help them comprehend what they are reading.

When students read a novel, their eyes tend to follow the print from left to right at a constant rate. It's not the same when reading a math text. As students read an assignment, their eyes will dart back and forth from their reading, to diagrams, to examples, to diagrams, to examples and back to their reading. They will generally re-read a phrase a number of times before they feel comfortable enough to continue reading.

Students who have experienced success in mathematics don't like or feel comfortable reading their math text. They want it explained to them. Students who have not experienced success in math see reading a math textbook as futile, a waste of their time, and intimidating. Teachers need to teach students how to successfully read their math text. If the math teachers don't, who will?

Teachers cannot increase student achievement in mathematics if the students cannot read mathematics – if they cannot translate English to math and math to English. All of today's high stakes tests are made up of word problems – students have to know how to read mathematics.

Site administrators should ensure that their staff is teaching students to read in the content areas. As administrators observe instruction, they should see evidence of reading

assignments, see the new vocabulary and notation introduced explicitly, they should hear teachers previewing the reading, connecting the reading to previous experiences, checking for understanding and correcting their understanding of what has already been read. If school administrators are not checking for this *teacher expectancy*, they should not expect it to be happening in the classroom – nor should they expect an increase in student achievement.

Writing

Writing helps students clarify and solidify what they have learned and helps them respond to what they have read. Teacher expectancies supported by the Hanlon, vocabulary, reading and writing are seen as important expectancies because of their connection to language acquisition. Educational researchers have identified vocabulary as the single most important factor that leads to comprehension – student understanding.

Classroom teachers should incorporate a number of writing tasks into their daily instruction. Students might be asked to explain a concept, write a word problem, illustrate a concept, give examples or make lists, describe or define, reflect, justify a solution, write a summary, predict what might occur, and compare or contrast what they are learning. The simple fact is that if our students are not required to write, if they are not given feedback on their writing in the content areas, then our students will not acquire the language and will perform well on tests like NAEP – considered to be the nation's report card.

I have long recommended that tests, quizzes, notes, and homework assignments include writing. In mathematics, which is considered a language by many, vocabulary and notation are seldom tested at the upper grades despite what the research suggests.

As part of the homework, teachers might ask students to write a procedure or explain a concept. During note taking, if something appears out of the ordinary – a trick, students should be given time to write an explanation of what occurred so when they study their notes later, the problem makes sense.

Teachers might ask their students to use *concept cards*. On one side of the card the students write the basic concept or procedure, on the other side the students explain how they might address a variation in the concept or procedure.

By asking students to write about what they understood about a lesson or what caused them difficulty or confusion, teachers would gain insight into how they might address their own instruction to increase student achievement.

It is important to realize that when I recommend writing in the content areas, I am doing so to increase student understanding that will result in increased student achievement in that content area. Writing in the content areas will cause students to think, reflect, remember, recall, organize, visualize, and analyze their thoughts. The components of writing are closely associated with the components of studying. Writing in the content

area is different from teaching students to write. Writing in the content area is not intended to be graded based on grammar, spelling, etc. But, if students are careless in spelling or grammar, then the classroom teacher should take corrective action. Writing in the content areas helps teachers understand student thinking – it's another way to monitor student learning and understanding. If students are not using correct vocabulary or notation, then teachers should assist them by writing important words or phrases on the board that students should use in their writing. For example, when working with fractions, if students are using terms like top number and bottom number, the teacher should require the students to use the words numerator and denominator. Feedback is important.

Instructional leaders should encourage their teachers to incorporate writing into their daily lessons by making suggestions, recommendations or giving directions and providing helpful feedback on implementation of writing in the content areas.
The writing process forces the students to reflect, think and to organize their thoughts.

Writing keeps students on task and reinforces concepts, procedures, vocabulary, and notation that teachers say they value. Rather than have students interrupt their instruction, teachers that regularly asked students to summarize the concept being taught would have students who understand that math is more than just memorization of important facts and procedures. Reading, writing, learning notation is extremely important to ELL students as well as students coming from poverty, language acquisition is a necessary component that leads to increased student achievement. That acquisition will not occur without repeated exposure; teachers orally using the language, students orally using the language, and having them read it, write it and speak it.

Star System – Revisited

As we have identified items in the notes using the star system, it might be helpful for students if teachers identified questions on the homework using the star system.

Chapter 7

Essentials 4 & 5 –Test Preparation & Assessment

Practice tests

Classroom tests, unit tests, are criterion referenced tests. They are tests based upon what is taught and learned in the class. As such, the contents of a teacher-made test should not be a secret. As part of the Components of an Effective Lesson, we ask teachers to not only state, but write the daily objective on the board – to be explicit so students know what they are learning.

Testing should not be any different. Students should know exactly what is expected of them. After all, as teachers, many of us had to take exams, we prepared by studying older forms of the ACT, SAT, PPST, or PRAXIS. Not only did teachers study for these exams using practice tests, chances are great those teachers wanted to know the rubric used in grading the tests.

Providing students with practice tests is another way of ensuring students know what to expect on their tests and it also provides the teacher another opportunity to monitor student learning.

Rather than scheduling a day for students to take a practice test in class, the recommendation might be to provide a practice test to all students approximately half way through the unit to be tested. Identify the questions students should be able to answer based upon instruction and the questions that have yet to be covered. At the end of each successive class, be willing to answer questions from the practice test as well as identifying additional questions the students should be able to answer based on the new instruction. This activity keeps the students engaged in test preparation over a period of time and results in students having a clearer picture of what is expected – it also leads to increased student achievement.

Some might argue providing a practice test is teaching to the test. I don't agree. Using the assessment blueprint in BAM, the types of questions asked are based on the specification sheet. That is, what do we expect students to know, recognize and be able to do after instruction. On a practice test, students might be asked to add 5/18 and 7/24, on the real test, different numbers would be used. On the practice test, students might be asked to write the procedure for adding/subtracting fractions, on the real test, the procedure asked for might be for multiplying fractions. A specification sheet/question on a practice test might require the students to find volumes of solids. On the actual test, students could be asked to find the volume of a cylinder, rectangular prism, triangular prism, or pyramids. If that is teaching to the test, that is good. Practice tests don't have to

give the exact test questions away, but the types of questions, the assessment blueprint, should be known by all – especially the students.

The following tests are examples of a practice test and a real test. Take your time and examine Test A that we will use as a Practice Test, then compare that to Test B, the real unit test.

TEST A: Linear Equations and Inequalities Name_____

Date_____

On Exercises 1and 2, give an example of each property:

1. Commutative property of addition:

2. Addition property of equality:

3. Define: Distributive Property

4. List the order of operations:

5. Write out the following phrase in words: $\{x \mid x \in \Re \wedge x > 2\}$

6. Give the mathematical definition of absolute value:

$|x| =$

7. Write the strategy for solving linear equations in $ax + b = c$ format.

Find the following solution sets:

8. $4x - 3 = 17$

9. $5z + 4 = 34$

10. $\frac{c}{3} - 4 = 10$

11. $\frac{2y}{5}+4=2$ 12. $2(3x-4)+5=15$ 13. $6y+2=2y+36$

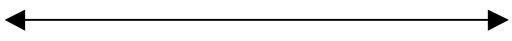
Find the following solution sets:

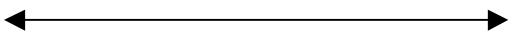
14. $7(2x+1)=4x-13$ 15. $4(2w+1)-3=2w+13$ 16. $\frac{5c}{2}-\frac{c}{3}=9$

17. $\frac{b+3}{4}-\frac{4b-5}{5}=-1$ 18. $|x-3|=4$ 19. $|2y-1|+4=13$

Solve and graph the solution:

20. $3x-5>7$ 21. $4(2z-3)+5\le 6z+7$





22. Solve for l. $A = 2l + 2w$

23. Solve for b. $A = \frac{1}{2}(b + c)h$

24. Which number is *not* a solution of the inequality $2 - 3x \geq -4$?

a. 0 b. -5 c. 4 d. 1

25. Fill in the following reasons using the properties of real numbers and properties of equality:

$2x - 3 = 17$	Given
$2x - 3 + 3 = 17 + 3$	_____
$2x + 0 = 20$	_____
$2x = 20$	_____
$x = 10$	_____

TEST B: Linear Equations and Inequalities Name:_____

Date:_____

On Exercises 1 and 2 give an example of each property:

1. Associative property of multiplication:

2. Multiplication property of equality

3. Define: Distributive Property

4. List the order of operations:

5. Write out the following mathematical phrase in words: $\{x \mid x \in \Re \wedge x \leq 2\}$

6. Give the mathematical definition of absolute value:

$|x| =$

7. Write the strategy for solving linear equations in **not** $ax + b = c$ format.

Find the following solution sets:

8. $5x - 4 = 36$

9. $7z + 4 = 39$

10. $\frac{c}{4} + 5 = 8$

11. $\frac{2y}{3} + 4 = 6$

12. $3(2x - 4) + 5 = 23$

13. $5t + 2 = t + 36$

Find the following solution sets:

14. $5(3x + 2) = 3x - 2$

15. $3(2y + 5) + 2 = 4y + 19$

16. $\frac{3c}{2} - \frac{c}{3} = 7$

17. $\frac{w + 3}{4} - \frac{4w - 5}{5} = 1$

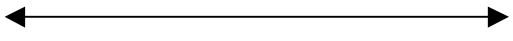
18. $|x + 3| = 7$

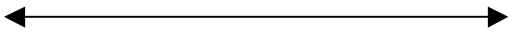
19. $|2x - 1| + 7 = 18$

Solve and graph the solution:

20. $4x - 5 \leq 7$

21. $4(2p - 5) + 2 > 5p - 6$





22. Solve for b. $P = 2a + 3b - c$

23. Solve for c $A = \frac{1}{2}(b + c)h$

24. How many solutions does the equation $-2y + 3(4 - y) = 12 - 5y$ have?

a. None b. One c. Two d. More than two

25. Fill in the following reasons using the properties of real numbers and properties of equality:

$4x + 3 = 15$	Given
$4x + 3 - 3 = 15 - 3$	_____
$4x + 0 = 12$	_____
$4x = 12$	_____
$x = 3$	_____

Notice in these two tests, the format is exactly the same. It's important to build confidence in the struggling students that what you say you will test them on is what you do test. Also notice that while I have changed problems, the students can immediately see that while on the practice test, Test A, problem 1 was asking about a Property of Real #s, the real test, Test B, asked a different question, but it was about the Property of Real #s. Again notice, questions 20 and 21 on both forms of the test are about graphing. The actual problems are different, but they are of the same type. These tests were parallel constructed.

You will also notice there are a couple of problems that did not change from Test A to Test B. Those would have been identified in the notes, homework and practice tests as three star (***) problems. Those are questions that I starred in their notes and explained to students they must absolutely know those and they will be tested on them. The two star problems would have minor changes, such as changes in the numbers used in solving the problem. The one star problems are the ones that vary or have variations.

Transparency => Credibility => Trust

Unit tests are criterion referenced tests, students should know exactly what is expected of them – there should be no surprises.

By constructing the actual test using the same format and using the same type of questions in the same order, students, especially those that struggle, will begin to believe they can be successful in mathematics and they will see their classroom teachers as people who want them to succeed, a person they can trust.

Student confidence, morale and performance will suffer if students lack confidence in their teachers. The solution is clear: Teachers must demonstrate transparency in what is taught and tested. Transparency leads to credibility, which leads to trust. There should be no secrets. Fairness counts.

And, a day or two before the test, teachers should review the practice tests asking students to identify if the problems are one, two or three star problems. If they are two stars, what would they expect to happen on the real test and if they were one star, what else could be asked. Teachers must tell their students they should expect to earn a 100% on the test to build confidence and because they know is on the test, and they are prepared. They are prepared because the instruction, notes, homework, and test preparation prepared them to demonstrate their knowledge.

Now, if classroom instruction is reflected in student notes and homework assignments, the next critical issue is further monitoring of student learning through the use of quizzes and practice tests. We mentioned before that the unit or chapter tests should be constructed before instruction because the classroom instruction should be based on agreed upon standards and objectives – what teachers say they value. These components should all be connected and reinforce instruction.

The expectation is that quizzes should contain items related to vocabulary, notation, identifications, conceptual or pattern development, procedures, as well as problems and applications. Those are the very things that students would have seen and heard during instruction, what they wrote in their notes, and completed in their daily homework assignments.

Practice tests and unit tests should have the same format. Students should easily be able to determine what will be expected of them on their unit test by looking at the practice test and hopefully using the star system.

Making the connection between classroom instruction, student note taking, homework, quizzes and practice tests, and tests are critical, practical things teachers can do to help students learn – to help organize their learning. These Critical 5 that are absolutely necessary for success of struggling students to help them be more effective and efficient in the study of mathematics.

Assessment drives instruction

Monitoring student progress frequently and systematically helps teachers identify strengths and weaknesses in student learning as well as in instruction. Assessing student work comes in many forms, but teachers need to know the answer to this question to improve their instruction and address student needs, "What do my students know and how do I know they know it?"

There is often a disconnect between what teachers say they value in mathematics and what they test. When talking with classroom teachers, they will indicate the importance of vocabulary and notation to be successful in math. But when you look at their own classroom tests, typically there are no questions on vocabulary or notation. Teachers will tell you how important it is to have student understanding, but again, when you look at their unit tests, there won't be questions that require open ended answers that measure student understanding of the concepts being taught.

In fact, many teachers will readily admit that students have raised their hands in the middle of them developing or explaining an important concept only to ask the question; "What's the homework assignment?" Now, why would a student interrupt instruction to ask such a question? The answer is the students know what the teachers' value better than the teachers themselves. Students know teachers' value what's being graded – that is, what's on the homework, quiz, or test. If teachers don't ask questions dealing with conceptual development, linkage, vocabulary, and notation on tests, then chances are students won't spend time studying it.

Teacher-made tests should reflect what is taught and valued in mathematics education. For example, while many teachers say mathematics is a language, this may not be reflected on their tests. If we value students' ability to verbalize their knowledge, then definitions, identifications, and procedures should be part of tests. Many of the rules in math don't make a lot of sense standing alone. For example, when students add fractions,

it seems natural to add both the numerators and denominators, however they are told not to add the denominators. If students learned that a fraction is a part of a unit, made up of a numerator and denominator and the denominators told them how many equal parts makes one whole unit, it would make sense not to add the denominators so they know how many equal pieces makes one unit. Another example, while it is important for students to know they can not divide by zero, they should also be able to give some rationale behind the rule besides saying that's what my teacher said. Having the students write a brief explanation using the definition of division will clarify their understanding. Manipulation of data, vocabulary and notation, open-ended questions, problem solving and appropriate use of technology should be included on tests. Also, to encourage students to review and reinforce previously learned material, teachers should make their tests cumulative.

Teachers using the specification sheet, assessment blueprint, and benchmarks discussed in the Backward Assessment Model would be more likely to have balanced assessments that measure what they say they value in math education. Teachers should not expect of their students what they are not will to inspect.

Teachers should prepare students to succeed. In preparing students for tests, teachers should provide tips on how to study. For instance, students sometimes confuse newly introduced terms such as domain and range. It might be helpful if teachers wrote an ordered pair (x, y) and suggested those are in alphabetical order as is the (domain, range), (abscissa, ordinate) and (horizontal axis, vertical axis). Those connections might help students remember. Teachers should also take the time to help students differentiate between problems that look alike. For example, while students might learn several different methods of factoring polynomials in algebra, they may not be able to determine an appropriate method of factoring when a mixture of problems is presented on factoring. Students have to be taught how to recognize differences and when to apply the appropriate method. Comparing and contrasting leads to increased student achievement. All too often, teachers successfully teach each method to their students on a section-by-section basis, but don't take the time to teach them to compare and contrast these problems so they know which method to use. When the students perform poorly on the test, the conclusion reached by many teachers is the students did not learn how to factor. But the reality is the students did learn what the teacher taught - to factor, but they did not learn what they were not taught – to differentiate between problems that looked similar and factor using the appropriate method.

Tests are formalized vehicles to not only evaluate student learning, but should also act as an assessment tool. As such, tests provide students a blueprint to increase their knowledge. Teachers should use assessment information, particularly questions answered incorrectly, as one way of increasing student performance. Addressing these deficiencies will result in increased student achievement. As will addressing their own instructional practices.

Assessing student work

In order to address student deficiencies, teachers need to know what students know. On many state mandated tests, it would be pretty difficult, if not impossible, to determine deficiencies based on the make up of the test. Often, one test question is meant to measure multiple state standards, including a student's ability to compute. If test items are all made up of word problems, how could a teacher determine if a student missed a specific question because:

a) the student does not speak English,
b) the student has a reading comprehension problem,
c) the student did not understand how to solve the problem,
d) the student did not know how to compute using that number set, or
e) the student made a simple computational mistake.

Without knowing what students know, teachers will frustrate themselves and their students by having to re-teach what the students already know. That boredom often results in students getting off task resulting in classroom management problems. That re-teaching also takes time, time that teachers say they don't have.

Identifying deficiencies and addressing them provide teachers more time to teach the curriculum assigned to them to mastery. In the factoring example given above, if the teacher re-taught the five methods of factoring taught in a first year algebra class, they would spend a lot more time than if they identified the deficiency as being able to distinguish between polynomials.

Teachers complain all the time about their students not knowing their basic arithmetic facts. If teachers looked at each operation, they would find that students actually know most of the facts. For instance, if you asked teachers, can your students multiply by one, by 2, 3, 5, 10, 9, and doubles, the answer to each of those questions is generally yes. That means the students know most of their facts. That information would allow teachers to concentrate on student deficiencies. Knowing that, teachers should spend their time reviewing and reinforcing multiplying by 4, 6, and 8. But not even all those need to be addressed, because the students, using the commutative property, could multiply four by 1, 2, 3, 4, 5, 9, and 10. That means the teacher needs to concentrate on 4x6, 4x7, and 4x8. Continuing, multiplying by 6, students could multiply by 1, 2, 3, 4, 5, 6, 9, and 10 leaving only 6x7 and 6x8 as the facts that need to be addressed. By identifying what the kids don't know, the teacher is able to cut down quite a bit of unnecessary re-teaching. That knowledge of what students know takes a lot of frustration out of teaching and does not bore the students to death.

Another example of assessing what students know might be with adding fractions. On a test, if a student could successfully add 1/4 and 1/3, but could not combine 5/18 and 7/24, some teachers might deduce the student does not know how to add fractions. Another teacher, upon closer examination, might conclude the student has procedural knowledge because of successfully adding the fractions on the first problem, but is getting the second problem wrong because of difficulty finding a common denominator. Rather than re-

teaching the procedure for adding fractions, good teachers will know what their students know and concentrate on finding common denominators. Teachers find out what students already know and what they still need to learn by monitoring and assessing student work – not by mere perception.

Introducing new concepts by linking them to previously learned concepts and outside experiences was emphasized for a number of reasons. One of those was familiarity with language, making the students more comfortable in the concept being introduced. Having said that, the importance of the formality of language was also discussed. On teacher made tests, teachers must be on guard to use the more formal language that students will find on standardized tests on their own classroom tests. Otherwise, students might not recognize the question being asked as one that was taught in school. For example, on an algebra test, a teacher might ask students to "solve" an equation. On a college entrance test, students would be asked to solve an equation by "finding the solution set …". This difference in language might result in a disconnect in what was taught and learned in class and what is being asked on a test. Students not recognizing these different directions mean the same thing might miss a problem they really knew how to do. Unfortunately, this kind of mistake might result in people in the community believing students are not being taught these concepts and skills in school.

If teachers truly understand the importance of vocabulary and notation and its relationship to increasing student achievement, then they would ask students to translate English to math and math to English on their unit tests. For instance, a student might be asked to translate ${}_5P_2$ to English. Students not understanding math or not paying attention to the detail and rigor required in mathematics might say five P two. All students should be taught to read ${}_5P_2$ as a permutation of 5 things taken 2 at a time.

Another example, a word problem might contain the expression, four less than a number, some novice students might translate that very literally from left to right and write $4 - x$ or $4 < x$, because the four was written first. However, if English to math translations were taught and tested explicitly, more students would recognize that as $x - 4$.

If teachers tested what was taught, the tests could be used to explicitly identify deficiencies. For instance, students asked to solve counting problems should be encouraged to use a calculator. A correct answer would suggest the student knew how to solve the word problem. Students not using a calculator who answered incorrectly may not have understood the problem, not have known a particular formula, may not have known how to use the formula, or could have made a simple arithmetic mistake. If teaching students to solve counting problems is important, then test the students on that and allow them to use a calculator. If it is important for students to know a formula, such as ${}_5C_3$, I think it is, then have them write the formula and evaluate it. And make sure they know how to say it – a combination of 5 things being taken 3 at a time.

Experienced teachers can predict common errors students make. All algebra students are taught to solve quadratic equations by the Quadratic Formula. Most students will memorize the formula, find the values of a, b, and c, plug them in the formula correctly,

and evaluate the resulting algebraic expression, then simplify the last step incorrectly. The most common mistake is reducing the fraction without factoring first. When this occurs, some teachers might conclude their students did not learn what they were taught. The fact is, if they memorized the formula, could identify the values for a, b, and c, were able to plug them into the formula correctly, and successfully evaluated the expression, but made a reducing error, then the students did learn what was taught. The teacher needs to address reducing or modify their instruction so students are more successful. For instance, the quadratic formula could be written as a sum of two fractions.

To determine if teacher-made tests are fair, balanced, consistent, cover the curriculum and the grades earned in different classes are portable, site administrators need to compare the content and achievement on grade level or unit tests given by different teachers testing the same topics. This should be a common practice, a practice that almost always necessitates a follow-up discussion by the teacher(s) and administrator.

If the site administrator did not see the balance in the assessment to match the balance on the specification sheet discussed in BAM, consistency should be a point of discussion. If one teacher asked students to reduce a fraction like 4/6 and another asked students to reduce 111/123, a supervisor might ask about the fairness of these questions.

Setting a date for a test

As teachers do their long range planning or use BAM to prepare instruction on a unit, benchmarks are identified. Setting benchmarks is important so teachers can plan to teach their assigned curriculum to mastery. As part of that planning, testing dates are typically identified. Since teachers are in the planning stages, the dates should be flexible. If a test is scheduled for a specific Friday and on the preceding Wednesday the teacher determines the students are not ready for the test, the test should be postponed. There is no sense administering a test to students that are not prepared for the test. Remember, build success on success.

Postponing a test requires the teacher to move forward with new instruction, while giving the students a few more days to successfully prepare for the test. It does not mean stop, re-teach the entire lesson, then give the test. Teachers incorporating the *Components of an Effective Lesson* in BAM could use the last five to seven minutes of each class to address the deficiencies that led to postponing the test. Teachers that stopped all new instruction would run the risk of not adequately covering the curriculum assigned to them.

Teachers would be more inclined to give students additional time, when warranted, if they viewed the test results as a reflection of their own instruction. As we have discussed, teaching is a whole lot more than just presenting material, teachers that do not monitor student learning as they are teaching might not know their students are not understanding the material being taught. A teacher that is surprised by test results probably needs to pay closer attention to questions being asked in class, performance on homework and quizzes, guided practice assignments, as well as discussions taking place in class.

Chapter 8
"+1" Essential - Student-Teacher Relationships

One of the greatest concerns expressed by policymakers has to do with closing the achievement gap while increasing student performance. To accomplish this, a number of elements have to be addressed. One that seems to get lost in the fray is the importance of building positive student-teacher relationships.

Informed Parents

One good way for teachers to begin the school year is by contacting the parents before anything could possibly go wrong to introduce themselves and explain what *they* are going to do to help their child become successful. The phone call is not about what the parents will do it's about what teachers control – that is their actions.

Almost every parent I have come across wants their kids to succeed in school. The problem is they get over their head in their other duties as a parent or work. So, as a parent and grandparent, I understand that.

What I needed for students to be successful was an informed and cooperative parents that believe that I will do all I can to help their child succeed in school. To accomplish this, I introduce myself informally as Bill Hanlon, their child's math teacher. I will ask if this is a good time to speak with them for ten to twelve minutes, and if it is, I follow the outline below.

- Introduction
- Pleasure teaching your son/daughter, nice young man/lady
- Explanation, how I intend to help your child succeed – Instruction
    - Clear instruction, linkage, memory aids
    - Notes, * system, very prescriptive
    - Homework, comes from notes & instruction
    - Oral recitation, procedures & formulas
    - Practice tests, * system
    - Study/flash cards
    - Reviews
- Permission to use those strategies
- Parental help
    - Know when tests are scheduled
    - Examine student notebooks
    - Use flashcards to help study

I quickly explain how I enjoy teaching, how long I have taught and what a pleasure it is to have a nice young man or lady like theirs in your class. I also explain how I want their child to not only succeed, but to excel in math and with their understanding and cooperation, we can make math their favorite subject.

I ask if their kids have ever had trouble in math and what I will do in my instruction by using concept development in linkage to help the students have increased knowledge, better understanding of the math they are learning and how it is applied.

I will explain the star system in the notes and how the instruction and the notes will help the students complete their homework assignments. I will ask if the kids ever complained about not being able to do the homework exercises. When the parents answer in the affirmative, I will discuss how I will do my best never to send students home with an assignment they can not do. I will also discuss the homework format, explaining that the first 4 or five questions on any given homework assignment can be answered by just referring to the notes for definitions, procedures, and explanations. And how if they can answer those questions, the probability of them doing the exercises correctly increases dramatically.

I will explain the star system in the notes, homework, practice tests and assessments. After a brief discussion of this, parents really begin to believe you are a good person who wants their child to succeed.

After gaining this belief, I ask for permissions. That is, I ask a parent, if their child refuses to participate in class-wide oral recitation to embed important formulas, definitions, or procedures in short term memory, would it be okay if they had to write that information twenty to twenty-five times as an alternative?

I explain that the three star questions are in the notes, homework, practice test and real test – unchanged and the tests are parallel constructed. That these items are very important for student success in the class. As such, if their child misses a three star question on a test, you will ask the student, with their support and approval, to write that information so when they are 84 years old talking to their great grandchildren, they will still remember that information. My experience with this is most parents laugh, ask me again about three star questions – making sure their child is not going to be treated unfairly, then agree.

The phone conversation suggests to parents that you care enough to call. It provides classroom teachers an opportunity to come across as a caring educators who will do all they can to help students succeed. It also sends a message to students, a message that indicates that if you will call home for no good reason, if they get out of line in class, that you will surely call home.

By the way, kids don't like the idea that you call. One reason is the parents talk to their kids about how fair you are going to be, how they like you and believe in you. It is going

to be a hard sell for students to tell their parents they aren't doing well because a teacher does not like them.

The bad news, my experience last year was the same as my experience 20 years ago. You only get through to 40% of the parents on the first attempt. So calling home is time consuming, 10 -12 minutes per completed call, plus all the attempts.

That early connection often pays huge dividends on two fronts; classroom management and increased student achievement.

Student-Teacher Relationships

Many teachers have had the experience of having a student not like them and decide not to work, to flunk to teach their teacher a lesson. The fact is research suggests that students will work for teachers for no other reason than loyalty. As the professional, educators need to take advantage of that knowledge and talk to their kids. Teachers also need to watch how they talk to them. They need to be positive. Rather than saying things like "If you don't do your homework, you will fail", they need to say, "If you do your homework, you will be successful."

Remember, treat your students the same way you want your own sons and daughters treated by another teacher. So talk to the kids. While you are not their friend, you can be friendly. Talk to them about sports, their social life, the dance, game, or weekend. Form a bond that suggests to the students if they stopped coming to school, someone would miss them – that you care about them.

If you have ever been in a long-term relationship, your partner may have expressed their frustration about you not expressing your feelings in words. They want you to say you love them even though you think it should be just understood. The same is true in your classroom, you need to tell your students you like them, that you want them to be successful. Reading my evaluations while teaching at the university, many of my students would fill out their anonymous evaluations and comment about how much I liked them, how I liked teaching, and how I wanted them to succeed. A colleague once asked me why my students felt that way about me. The answer was simple, I did my best to tell my students at least twice during the semester that I liked them, I liked teaching, and I wanted them to succeed. They reciprocated by liking me! Remember this Law of Reciprocity, *people you like generally like you, people you don't like generally don't like you either.*

Teachers and administrators must explain expectations explicitly and give examples. Don't have a 12 to 15 year old interpreting those expectations. If you want the students to class on time, does that mean running in the door as the bell rings, at his seat, or in the seat with his book or notebook open and ready for instruction? You need to tell them or you will be frustrated the rest of the school year.

Build trust with your students. Make sure they know you are there for them. For instance. grading papers is not about taking points away from students. It should be about finding out how much they learned and helping them become more successful. Don't get caught up in arguing about points deducted in a test. If a student deserves the points, give them.

And while we are talking about testing, teachers should make testing as much a reflection of their own instruction as student preparation. If students are failing, the first place a teacher should look is in their bathroom mirror at home to find the problem. Remarkably, there is a relationship between not only what students are taught, but how they are taught and what they learn.

Teacher expectations impact student learning

Many students sit in the back of the room for this very reason, they want to be left alone. These unsuccessful learners could be classified, because of their behavior, as reluctant learners. Teachers know them and are often thankful when those students sit in the back - quietly.

And while teachers do feel thankful, we need to refer to the "My Kid" standard and put ourselves in their parent's position. When children are four and five years old, they see their parents as the smartest and strongest people in the world. By the time these same kids reach sixth grade, they begin to view their parents as not very hip, up to date, or sometimes just not smart. As those youngsters reach twenty-five, it seems the parents start to regain some of the intelligence their offspring thought they lost when they were in their teens.

Students in middle school and high school tend to rebel against authority, which more often than not, includes their parents. They would rather listen to the advice of friends, than their own parents. Teachers need to help these parents with their children just the way they would like some other teacher guide their own kids.

Would a classroom teacher want their child to sit in back another teacher's classroom napping during instruction? If they would not want that for their own child, using the "My Kid" standard, they should not let that happen for someone else's children.

Student-teacher relationships are important. Teachers need to earn the trust, confidence and respect of their students by constantly communicating with them, encouraging them to be successful, and showing them how they can be successful.

Students who sit in the back of a room napping or seemingly ignoring instruction appear to be defiant however, upon closer examination teachers might just find out that behavior is a defense mechanism, a defense mechanism that allows them to escape embarrassment or ridicule. Ridicule for not being successful, from being seen as stupid.

Many of these students have not experienced success in the math classroom and have lost hope they ever will succeed in mathematics. Without teacher intervention, these students

will be doomed and underachievement in math may become a way of life. Too many students equate success in mathematics with being smart, not with hard work. That is sometimes unconsciously reinforced by classroom teachers who recognize students that learn quickly as having higher ability than those students that have to work harder or longer to complete a task or assignment. Speed at completing tasks appears to have replaced effort as a sign of ability; high levels of effort might even carry the stigma of low ability – as not being seen as smart by their peers. That results in students not expending the effort needed to achieve to their full potential. Once students begin believing they have failed or are failing because they do not have the ability, they lose hope for future success, they stop trying. Hard work and effort have to be recognized by teachers as critical to student learning.

Teacher expectations can often be interpreted by listening to their own commentary. When teachers talk about "those" students, not "my" students, there appears to be a disconnect between what they do in the classroom and what students learn. You hear these teachers lamenting that these students are absent often, won't come to class on time, won't do homework, won't take notes, etc. What's interesting about those commentaries is that some of those same students can be observed going to other teachers' classes on time, doing their homework, and taking notes. That suggests that teachers do make a difference, teacher expectations and their relationships with students have an affect on what students are willing to do in a classroom. If that is not true, then we should just consider going to video lesson.

Struggling Learners

Student expectations impact learning. People enjoy participating in activities when they are able to participate successfully. Some of us who may be monotonic may be reluctant to sing with a group of friends because we don't feel like our voice is that good. Some people really enjoy a game of chess or putting puzzles together, typically they enjoy those activities because they have experienced a certain amount of success or feel, with additional time, they can be successful. The bottom line is most people will spend time on activities in which they have experienced success or have an expectation of success. We tend to shy away from activities that we don't do well or don't have an expectation of doing well.

The same can be true of students. They shy away from activities in which they have not been successful and feel like they will not succeed. Students who have not experienced success in mathematics might be viewed as reluctant learners. They are typically forced to enroll in a class, a class in a subject in which they have not been successful, and are expected to perform with a certain amount of interest and/or enthusiasm. Their reluctance to participate is the same reluctance adults have when asked to sing, speak before a group, or otherwise participate in an activity they don't enjoy or don't visualize themselves as being successful.

Educators need to better understand the importance of building success on success, students who experience success in math will generally spend more time on math than students who can not see the light at the end of the tunnel.

Classroom teachers must make an effort to build a positive relationship with all students and especially with unsuccessful learners, to build confidence and trust so the students won't feel threatened if they try. To build a relationship by communicating with those students daily that results in those students feeling they can be successful and they would actually be missed if they did not come to school.

To develop such a relationship, teachers need to talk to their students outside the classroom, in the halls, lunchroom, at ball games, dances, and at the store. Since many of these students have confidence problems, teachers might have to schedule a one-on-one conference with individual students during the school day - during another teacher's class, before or after school. Teachers would be wise to conference with these students in a non-threatening atmosphere. For example, teachers should sit with the student at a table, not sitting behind a desk. Teachers should discuss what they are able do to help the student succeed in their class. In other words, the teacher must be part of the plan too. They also need to elicit from the student what they are currently doing and offer constructive suggestions on how to more efficiently and effectively use their time. Once it is determined how students are using their time, the teacher should encourage them to do more. The teachers need to think about what they would like another teacher to say to motivate their own child and make the same kinds of comments in a friendly, helpful manner. Teachers must teach students how to be successful.

Classroom teachers must build success on success. To do that, they must teach students how to be successful. Here are some suggestions to help students succeed in mathematics classroom.

Student Suggestions

1. Neatly copy in your notebook any problems that are put on board. Be sure you understand each step of the problem as it is being explained. Ask questions to clarify any step that you do not understand. Do not wait to have the point explained at a later time.

2. Always try to do as much of the assignment as possible without help. To a great extent, the amount you learn is dependent upon how well you have worked independently. When you practice a skill, it is more likely to become part of your long-term memory. Relying excessively on the teacher, or anyone else, to answer questions and to solve all the problems could result in a lack of understanding. If you are still confused after making your best effort, consider discussing the problem with a classmate.

3. It is necessary to spend time studying at home in order to reinforce what you have learned in class. Do not think that once you have obtained all the answers on an assignment, you are through with the material. After completing an assignment, review the concepts with the idea that you will be expected to know the material on a test. By studying at home, you will discover what you do not understand and will be ready to ask questions in class the next day. A student who has done little studying on his own

frequently knows so little that he is embarrassed to reveal his ignorance. He is often afraid to ask a question that he feels everyone else in class can already answer.

4. When material you have already learned is being discussed, use the opportunity for "over-learning." Try to work a step ahead of the person presenting the problem.

5. DO NOT WASTE TIME IN CLASS! Most of your learning will occur during class time; it is foolish to waste this time. Each class period gives you an opportunity to concentrate on learning a specific concept, to correct your mistakes, and to direct your learning efforts.

6. ALWAYS COME TO CLASS IF AT ALL POSSIBLE! When you are absent, there is no way to fully make up for the class instruction you miss.

7. Always seek to understand rather than simply to "squeak by." The grade you receive is important, but not nearly as important as the mental growth you gain from the process of learning the subject.

8. Memorization will help you absorb and retain factual information upon which understanding and critical thought is based. Knowing and using mathematical vocabulary and notation is key to the understanding of the mathematical sciences.

9. PREPARE FOR TESTS! Your tests are often made up of questions that come directly from homework exercises, class notes, the chapter test or the chapter review. Meet in study groups to discuss items that you think will be on the test. Use the study group for remediation and peer tutoring. Individuals who help others learn, gain a better understanding themselves.

10. If you are making a serious effort and still not doing well, come in after school and talk with the teacher. He/She can probably help you overcome your difficulties.

Unsuccessful students who have disengaged from the classroom must be slowly integrated into the mainstream classroom to avoid possible embarrassment from their friends. The first time the student that plays the role of "sleeper" in the back of the classroom answers a question correctly, well meaning friends of that student might laugh or make some sounds of surprise or astonishment that result in that student feeling ridiculed. That embarrassment could cause that student to revert to the defense mechanism that has served him so well – napping in the back row.

Since reluctant learners often sit in the back of the class, teachers might try moving them toward the front of the class in a very inconspicuous way – like making a new seating chart for the entire class. It would be wise to let that student know what you are doing and why you are doing it so they are not caught off guard making an inappropriate remark. All students should be encouraged to take notes, thereby participating in class.

The teacher should initially check for the reluctant student's understanding by simply asking if the lesson is understood (directive questioning) – not looking for verification by answering an academic conceptual question. As time progresses, the teacher might ask if the reluctant learner agrees with another student's understanding of the lesson. And finally, as the reluctant learner becomes more comfortable and immersed in the class, the classroom teacher should begin to ask the student questions that the student is likely to know and answer correctly – building success on success and self confidence.

Building confidence and trust takes time. Unsuccessful learners have developed defense mechanisms over time to protect themselves from embarrassment and ridicule. Change is not usually embraced quickly or wholeheartedly by everyone – not even by classroom teachers or administrators. Realizing this, teachers need to work patiently with students. If teachers move too quickly, they may inadvertently embarrass the student they are trying to help. That embarrassment could result in a student's trust and confidence being shaken or broken in that teacher.

The reason students generally become reluctant learners is because they were unsuccessful learners. The reason they are unsuccessful is because they probably don't understand what exactly is expected of them.

In the student suggestions for instance, it was suggested the students neatly copy any problems on the board into their notebook. While that is great, what if the student does not know how to take notes, does not title what is in the notes, doesn't date it, doesn't draw pictures, write definitions, look for patterns or procedures? Chances are there are many students in a classroom who could use guidance in the configuration of their notebook. Something simple like leaving white space so there is no visual overload might help students study more effectively and efficiently. Chances are your most unsuccessful students really have nothing to take home to help them with their homework or prepare for tests. Students need to be taught how to take notes.

Another suggestion requires students to memorize information. Is it possible that unsuccessful learners don't know how to memorize material effectively or efficiently? Earlier, we discussed student learning, to tell a student to go home and study does not help that student if they really don't grasp what they need to do at home to study.

So the suggestions are meaningless unless students know what they really mean and know how to apply them. Explaining those suggestions is part of teaching.

Many unsuccessful learners come from homes where the parents might not be able to help their children with schoolwork because they were unsuccessful learners as well. Additionally, a disproportionate number of unsuccessful learners come from poverty, their parents are more likely to work in the evening and not be available to assist their children with homework or make suggestions on how to study effectively.

All to often these students are left to make decisions at very early ages, decisions about what to make for dinner, to watch television, study, or interact with their friends. While many of these students might have good intentions going out the schoolhouse door, when

they get home, those intentions are not carried through. It is a lot easier to watch their favorite television show or talk to their friends than it is to stop having fun and study.

Because of this, it is important that teachers implement the *teacher expectancies* and the *components of an effective lesson – the "5 + 1"*. The components and expectancies provide structure to daily instruction that is helpful to students who have not experienced a great deal of success in a math class because of the long and short term reviews, oral recitation which embeds information into short term memory, note taking that helps students complete their daily homework assignments and prepare for tests, as well as the guided practice to monitor student learning.

While homework and home study are important, teachers need to use their class time effectively so students learn as much as possible in class. Learning difficulties among special populations stem largely from instructional practices that: do not build upon informal knowledge, does not foster learning, or teachers that do not monitor student learning. Special populations will experience difficulty if the instruction begins with the abstract and moves too quickly or if the instruction relies on memorizing mathematics by rote.

Thinking about what causes learning difficulties for special populations, one would realize those are the same factors that cause difficulty for the general student population as well. Good teaching matters!

Belief Systems

Student-teacher relationships is an important component in increasing student achievement, but so are belief systems. Most of us have experienced a college professor that we might have thought was brilliant, but could not teach worth a hoot. Or we may have had the one that may have thought the way to increase student achievement was to separate the men from the boys by being overly demanding. In either situation, that class was not a good place to be. The first example just further illustrates that while teachers' content knowledge is extremely important, there's more to teaching than just knowing information. The second case does certainly does not seem like a very good method of inviting students into more rigorous classes. In fact, it seems like they use this as a method to decrease class size.

As K-12 educators, we don't have the option of trimming the herd like our colleagues in college; we teach the students in our classes. Or, do we? We hear the occasional teacher talk about getting students more appropriately placed – out of his classroom. This trick, often referred to as dumping, is accomplished by giving some sort of pretest. Then, after the test is graded, not so successful students are transferred to another, supposedly more appropriate, class.

Listening to some teachers, one might conclude they don't believe their students can learn. If that's the case, they need to resign and allow someone else to get the job done. Administrators and teachers must believe their students can succeed, that if they do a better job teaching and their students work harder, success will follow.

If you are really interested in your students succeeding, then you should *build success on success*. I have always used the first unit of the year as the unit I shape beliefs, teach students to study effectively and efficiently, as well as teach mathematics.

To *build success on success*, students must first experience success. So, over-teach the first unit, the students over-learn it, all the while teaching them what kind of learners they are, their concentration times, how to take notes, how to study effectively and efficiently. Provide examples of how you remembered important information, allow time at the end of the class for note reviewing, ensuring they have the information they need to successfully complete their homework or prepare for a test.

My belief is that I can successfully teach math to anyone willing to learn. If I can get the students to be successful on the first test and I can show them that success was based on what they did to prepare – not just being smart, I will be on my way to a great school year. Preparing them to learn will help them succeed and make you feel better about your students' accomplishments

If you hear yourself or others talking about "those" students, not *my* or *our* students, then chances are you are not taking ownership in their success. There is a disconnect, a disassociation that acts as a disclaimer to your part in your students' learning.

Elementary administrators have an advantage over secondary principals in that they have typically taught all of the subjects the teachers they supervise teach. They know the subject matter, they are familiar with both the sequencing and benchmarks, and instructional strategies to help students learn. Secondary principals normally come from subject specific areas. They have backgrounds in math, social studies, physical education, or science. All too often, people who don't have a background in the natural sciences feel threatened by their secondary math teachers. Some will acknowledge they didn't understand math, they didn't get the math gene and that's why they were not successful in math. So when they evaluate their math teachers, they are looking at classroom environment, instructional strategies, classroom management, and not really paying close attention to the math content being delivered to the students. That has to change. A lesson's worth should be determined by what students learned – not how well the class seemed to go.

Administrators must also change their belief systems. Many administrators will sit in a math class, evaluate the instruction, knowing full well they did not understand the day's lesson. My guess is if the administrator did not understand the lesson, and they probably took the class in high school, probably graduated high school, earned a bachelors and masters degree, and are mature, then how would they expect the 13, 14, or 15 year old to understand the lesson. If administrators are not understanding the lesson, they need to address that with their teachers, because it is doubtful that the students are getting it.

Contradictory rules – Misunderstood behaviors

As classroom teachers, there are times we just scratch our heads in bewilderment because of student behavior. It just seems that some students find trouble, then continue to dig deeper when confronted about their behavior. That additional digging is often interpreted as a sign of disrespect by many teachers, but it might not be.

For instance, as a sign of respect, many cultures require eye contact when correction in behavior is being discussed. Other cultures, as a sign of respect, discourage eye contact and, in fact, the person on the receiving end of the message is taught to look downward. If classroom teachers are not sensitive to their students' backgrounds, they may interpret a student looking down as a sign of disrespect, which in turn, causes the student an additional problem to deal with.

In more affluent areas, when adults have disagreements, frequently they are handled through litigation. This method to resolve problems has been adopted in the schools using peer mediation. Since a form of peer mediation is used at home to resolve conflicts, some students view this as very consistent to home life. In less affluent areas, disagreements are often handled by fighting. In fact, the people who win those fights are often held in high regard. Students coming to school from those communities are taught at home to take care of themselves by fighting, peer mediation may be construed as a coward's way to resolve a problem. The end result of this might be a student beating up another student to address a grievance – which is seen by educators as bad behavior.

These same types of misunderstandings happen quite often in other circumstances, again the behaviors can be traced back to what is learned at home. For instance, when parents argue at home, children from more affluent homes might be more inclined to listen quietly or take leave. In less affluent areas, children try to diffuse volatile situations at home by using humor to decrease the tension. Learning from that home experience, students might try to apply it at school. For instance, teachers often have to correct the behavior of students. In some cases, rather than being a quick direction, the teacher might spend some time discussing the seriousness of the situation. If students coming from less affluent neighborhoods see this discussion escalating into a more volatile situation, they may say something that to them seems to be funny - not as a sign of disrespect, but a way to calm the situation.

Again, if classroom teachers and administrators are not sensitive to their students' backgrounds, those student might find their way to the office for being disrespectful and possibly being punished for using a strategy they learned at home to successfully diffuse an uncomfortable situation.

The reason that students are not held to adult standards is because they are not adults. They make mistakes in judgment because of their own experiences and/or lack of experiences. As the adult role model, classroom teachers must make every effort to ensure their students know how to act in different situations. Without explicit guidance from teachers and school administrators, students may apply the rules and behaviors they

learned at home to school – not realizing those rules contradict standards of behavior at school.

I cannot overstate the importance of developing positive student-teacher relationships, especially with students who don't care for school, students who have not experienced much success in the classroom. The research strongly suggests that students will work and work harder for teachers out of loyalty.

Chapter 9
Next Steps
Department Improvement Plan

The Department Improvement Plan

As teachers and administrators develop a plan, this should not be perceived as a 5-year plan, the department plan must have immediate and long-term impact. That plan should have the My Kid and Common Sense standards built in as well as implementing the building *success on success* model as a cornerstone described in the Increasing Student Achievement in Mathematics, 5 + 1.

The following topics should be identified as *teacher expectancies* adopted and reinforced by each member of the department and included in the department improvement plan.

a. First test: over teach – over learn; teach content while teaching students how they learn, concentration times, and how to study.
b. Improve student-teacher relationships – talk to students, be positive!
c. Use linkage to introduce new or more abstract concepts and skills – develop concepts, teach the big idea.
d. Use simple straightforward examples to clarify concepts being taught when introducing new material. Don't bog students down in arithmetic.
e. Adopt a balanced approach to instruction, emphasize vocabulary & notation, concept development & linkage, memorization of important facts and procedures, appropriate use of technology, and problem solving.
f. Fully implement the Components of an Effective Lesson, emphasizing the 5 + 1
g. Adopt a homework format to include what teachers' value – not just problem sets.
h. Include reading and writing in the instructional plans.
i. Test what you value, provide practice tests halfway through the unit to help students prepare for the real test. Use the more formal language students will see on high stakes tests.
j. Require students to take notes, use oral recitation to embed information in short-term memory.
k. Use reviews at the end of each class to address mastery, deficiencies, and prepare for high stakes tests.
l. Use time effectively, start the class on time, end the class at the end of the period.
m. Use the Star System on notes, homework, practice tests

Effective Schools

School administrators can not be left out of any school improvement effort. While we are describing a department improvement plan, educational research suggests effective schools generally have strong instructional leaders, a safe and orderly climate, a school-wide emphasis on basic skills, high teacher expectations, and regular assessment of student progress. For that reason, the school administration has to be included in any plan.

According to the U.S. Department of Education, schools with high student achievement and morale show the following traits:

Vigorous instructional leadership
A principal who makes clear, consistent, and fair decisions
An emphasis on discipline and a safe orderly environment
Instructional practices that focus on basic skills and academic achievement
Collegiality among teachers in support of student achievement
Teachers with high expectations of their students
Teachers that believe their students, through hard work, can and will learn
Frequent review of student progress

Effective schools have effective principals, principals willing to observe, supervise, and evaluate their classroom teachers – not just overseers.

Teacher Supervision

Teachers welcome instructional suggestions that result in increased student achievement, but they rarely receive them. According to the U.S. Department of Education, supervision that strengthens instruction and teachers' morale has the following elements:

- agreement between supervisor and teacher on specific skills and practices that characterize effective teaching,
- frequent observation by the supervisor to see if the teacher is using these skills and practices,
- a meeting between the supervisor and teacher to discuss the supervisor's impressions,
- agreement between the supervisor and teacher on areas for improvement, and
- a specific plan for improvement, jointly constructed by the teacher and supervisor.

Many secondary teachers report principal involvement in their classroom does not occur often. Those teachers often indicate when principals do observe classroom instruction, rarely do they receive recommendations that are specific enough to implement in their classrooms that have any impact on instruction or increased student achievement..

For schools to improve performance, building level administrators must be willing to inspect what they expect of their teachers.

The recommendations made in this text are very easy to observe and monitor. Using the Professional Development Day Agenda in the Backward Assessment Model, principals can be involved and monitor the self identified strengths and weaknesses within a department and the changes in instructional practices identified to address student deficiencies. Additionally, the Components of an Effective Lesson can be easily observed and monitored by the principal. Stating and writing the day's objective on the board, closing the lesson by restating the objective and providing a brief over view, having students write about what they learned at the close of the lesson. By providing two review periods, one in the beginning of the period to go over recently taught material and the second to review long-term knowledge and prepare for high stakes tests. Principals can easily determine if homework is more than just a problem set out of a textbook and monitor whether students had an opportunity to practice with guidance from their teachers.

If the components emphasizing the 5 + 1 were adopted within a department improvement plan, principals could focus their recommendations on their implementation. While a checklist could be developed for the components, teachers would be much better served if the principal sat down with the teacher to discuss their observations in greater detail.

While we said this before, it is worth repeating. With respect to school administrators, elementary administrators have typically taught all of the subjects their teachers teach. They know the subject matter, they are familiar with both the sequencing and benchmarks, and instructional strategies to help students learn. Secondary principals normally come from subject specific areas. They have backgrounds in math, social studies, physical education, or science. All too often, people who don't have a background in the natural sciences feel threatened by their secondary math teachers. Some administrators will acknowledge they didn't understand math, they didn't get the math gene and that's why they were not successful in math. So when they evaluate their math teachers, they tend to look more at classroom environment, instructional strategies, a checklist for the components, and classroom management, and not really paying close attention to the math content being delivered to the students. That has to change. A lesson's worth should be determined by what students learned – not how well the class seemed to go.

Administrators must also change their belief systems. Many administrators will sit in a math class, evaluate the instruction, knowing full well they did not understand the day's lesson. If administrators are not understanding the lesson, they need to address that with their teachers, because it is doubtful that the students are getting it.

Improvement plans rarely work unless the school's administration is an integral part of the plan and is actively participating in the process. On the following pages, I have provided an observation sheet that teachers and administrators might discuss and come to

an agreement on items that should be observable on most days during a regular class period.

Administrative Suggestions

1. School administrators should meet with teachers by grade, subject or individually to explicitly go over teacher expectancies and how they will be evaluated using the CEL and teacher expectancies.
2. School administrators should confirm that classroom teachers are teaching their assigned curriculum from the very first day of instruction.
3. If school administrators are not understanding the lesson they are observing, then they need to discuss that with the teacher so they can improve their instruction.
4. School administrators should observe the same classes, meet and discuss their observations, to ensure consistency in the evaluation and supervision of the teaching staff at their respective schools.
5. Classroom observations should begin in September/October with follow-up conferences scheduled within a week that provides specific suggestions, recommendations or directions to improve instruction that will result in increased student achievement.
6. School administrators should examine teacher made tests in common areas to determine if they cover the same material with the same rigor in the same approximate timeframes.
7. School administrators will require teachers to develop expected student grade distributions for each of their classes at the beginning of the school year.
8. School administrators should collect grade distributions on the very first test of the year in September, as an early warning sign, to gain insight on where students or teachers need assistance.
9. School administrators should address the student population yearly to very explicitly go over academic and behavioral expectations
10. School administrators should immediately address students not coming to school prepared with books, notebooks, pencil or paper.
11. School administrators should support teachers. If students are observed off task in a classroom, administrators should go into that class and address students not engaged in learning.
12. School administrators should evaluate the effectiveness of a teacher based on student performance.

SECONDARY MATH OBSERVATION SHEET

The observer will provide a rating on a 4 point scale what was "seen" or "heard" in the classroom. If no evidence was provided to give a rating, then that component or expectancy will be left blank. To give a Level '4' or '1' rating, the observer must give documented evidence of what was "seen" or "heard" in the classroom.

Total Number of Students		# of students	Minutes after the start of class period					
			Start	11	22	33	44	End
Male	Female	On-Task						
		Task						

Level 4	Distinguished		Level 2	Basic
Level 3	Approaching Distinguished		Level 1	Approaching Basic

MATH OBSERVATION NOTES

COMPONENTS OF AN EFFECTIVE LESSON	4	3	2	1
1. Introduction				
2. Daily Review				
3. Daily Objective				
4. Concept/Skill Development				
5. Concept Linkage within Discipline				
6. Concept Linkage outside Discipline				
7. Guided Practice				
8. Group Practice				
9. Independent Practice				
10. Long-Term Memory Review				
11. Closure				
12. Homework Assignment				

TEACHER EXPECTANCIES	4	3	2	1
13. Reinforce Study Skills				
14. Student/Teacher Relationships				
15. Use Simple Examples				
16. Assessment				
17. Student Note-taking				
18. Vocabulary is Stressed				
19. Reading and Writing				
20. Facts and Procedures				
21. Technology Implementation				
22. Problem Solving Process				
23. Memory Aids				
24. Questioning Strategies				

MATH OBSERVATION NOTES

COMPONENTS OF AN EFFECTIVE LESSON	TEACHER EXPECTANCIES
1. Introduction: What will be learned and why it is useful.	13. Reinforce Study Skills: Teach students how they learn (visual, audio, kinesthetic), concentration times, and how to study and how they learned (remembered) information.
2. Daily Review: Provide review for short-term memory over recently taught material. When correcting homework: provide immediate and meaningful feedback and hold students accountable. Keep reviews and homework checks brief.	14. Student/Teacher Relationships: Show your students mutual respect. Talk to your kids, be positive, and use humor to engage them into the lesson or activity!
3. Daily Objective (Specify skills/information that will be learned.) Write this information on the board and have the students record it in their notebooks.	15. Use Simple Examples: The teacher makes a conscience decision to start the development of skills using simple, straight-forward examples that clarify the concept or skill being taught which do not bog students down in arithmetic.
4. Concept/Skill Development: (Give and/or demonstrate necessary information) Teacher	16. Assessment: Continually assesses the progress of students through the lesson

focuses on the big concepts. Utilize a variety of techniques: students need to see it, hear it, say it and do it.	and adapts the lesson according to students' successes and difficulties.
5. Concept Linkage within Discipline: Link concepts and skills to previously learned material and outside experiences.	17. Student Note-taking: Teachers are very directive and prescriptive in how students take notes. Notes should be used to complete homework assignments and prepare for tests. There is accountability for taking notes and keeping mathematics notebooks.
6. Concept Linkage outside Discipline: Link concepts to real-work experiences.	18. Vocabulary is Stressed: Teachers are using direct instruction to teach vocabulary and there is evidence that the teacher is using and requiring the students to use appropriate vocabulary to describe the mathematics.
7. Guided Practice: Partially completed problems are given to students to check for student understanding. Can be done at different times throughout the lesson to help students' process information.	19. Reading and Writing: Teachers introduce new vocabulary, previews reading, connect reading to previous reading and checks student understanding of reading. Teachers also give students time to write about what they have learned at the end of the lesson. Reading and writing (including note-taking) are used during daily lessons.
8. Group Practice: Students need time to think, analyze, work on problems, discuss their solutions and become problem solvers instead of watching the teacher do all the work. Can be done as an entire lesson that enhances conceptual understanding and/or application of concepts through inquiry, investigation, discovery, lab or problem-solving activities.	20. Facts and Procedures: Provide the "why" for rules and algorithms.
9. Independent Practice: The majority of students who are given time to start their homework, worksheet, or seatwork can do so with out disturbing the teacher for clarification.	21. Technology Implementation: The teacher builds on the concept using technology through modeling or presentation.
10. Long-Term Memory Review: Maintain skills, address deficiencies, and stress important ideas for the year. During this review teachers should be review typical questions that are used on the proficiency exam or other high stakes tests.	22. Problem Solving Process: Students are lead to be problem solvers by the teacher modeling or prompting for a variety of ways to solve problems that do not have answers that are automatic
11. Closure: The new material from the lesson is summarized or wrapped-up for the students. An example is having students explain what they have learned and apply it.	23. Memory Aids: Teacher is directive and prescriptive in how students take notes in the class. The use of oral recitation was used to embed information in short-term memory.
12. Homework Assignment: The homework should include vocabulary and notation, procedures, and open-ended questions to check their understanding. The assignment sent home is clear and is manageable for a student to complete in a 20 – 30 minute timeframe.	24. Questioning Strategies: Several different types of questioning strategies were used including, but not limited to front loaded, rear loaded, higher level thinking, and effective use of "wait-time"

The following are suggestions takes from the Components of an Effective lesson and teacher expectancies that administrators might give teachers to improve instruction.

Teacher Suggestions

1. Smile. Be happy. Enjoy your work. You have one of the most rewarding, challenging, and important jobs in the world. Remember, happiness is 99% attitude. Enjoy what you do. If there are some days you think you have it bad as a teacher because students are sometimes not paying attention, think of the plight of flight attendants and how nobody is paying attention to them on any day.

2. Video tape yourself teaching a few classes. View tapes objectively to identify strengths and weaknesses. Ask another teacher to provide constructive criticism. Consider using the Teacher Observation Survey.
3. Be directive in your teaching. Use the Components of an Effective Lesson and the *teacher expectancies*.
    A. Begin class immediately. One approach is to begin class with a 2 to 5 minute review of recently taught material. Another technique is to begin class with a short, timed quiz displayed on the overhead as students walk in the room. These approaches help to discourage tardies.
    B. Be sure to answer questions from the previous night's homework. Students need immediate verification they are understanding and following correct procedures.
    C. Review facts and procedures that students need as background in order to be successful with the concept being taught that day.
    D. To prevent confusion with variation of similar problems, stress the need to recognize appropriate strategies. For example, $2x + 3 = 9$ and $7 = 5 – 2n$ are problems which require a slight variation of the same strategy.
    E. When introducing a new concept, use simple straightforward examples that clarify what you are teaching that don't bog students down in arithmetic.
    F. Introduce topics by linking the new concept to previously learned material or outside experiences. Linking allows teachers to introduce new material in familiar language, to review and reinforce concepts and skills, to compare and contrast, as well as to teach the concept in a different context.
    G. Emphasize vocabulary and notation. There is no more single important factor that impacts student understanding than the acquisition of vocabulary and notation.
    H. Use oral recitation to embed new information into short term memory and learn how to communicate mathematically.
    I. When stressing a point, take extra steps to ensure students are paying attention ("eyes and ears"). Once you have everyone's attention, use voice inflection to provide further emphasis.
    J. Anticipate where students will experience difficulty. Help students avoid frustration by drawing their attention to common mistakes, i.e., when teaching students to simplify an expression such as $3 + 36 \div 4 \times 3$, be sure to warn them about dividing first. Answer is 30.
    K. During note taking, make sure students have more than just problems copied off the board. Student notes should contain vocabulary and notation, conceptual and pattern development (pictures), procedures, explanations, as well as practice problems.
    L. Display an accepting manner that encourages students to ask questions.
    M. Have the students write about what they are learning.

N. Assign homework that reflects your beliefs in what students should know, recognize and be able to do – don't just assign problem sets. Make sure homework assignments are of proper length and students leave the classroom knowing what is expected of them and how to complete the homework. Reading should be part of the homework.

O. Encourage students to allow themselves sufficient "thought time" before asking for assistance. If they still have questions, students should be reminded to refer to their notes. They may also refer to other sources.

P. When a number of students have the same question, review possible strategies at the board so everyone has the benefit of your explanation. Remember, overhead projectors do not allow students to see patterns develop.

Q. Close instruction by restating the day's objective and emphasize the important information students will need to know to understand and complete their homework assignment.

R. Use the last few minutes of the class period for long term reviews, address student deficiencies, or prepare for high stakes tests.

4. Spend sufficient time in class preparation. Most district course syllabi mandate the material to be covered in each course/grade. Use the textbook as a guide, the individual teacher, preferably working with other teachers teaching the same course, determine the sequence to be followed. In preparation, create a unit outline for use during instruction (BAM). Use the format of your notes to guide students in their note-taking, stressing definitions, concept development, pattern development, theorems, procedures, and examples. Anticipate difficulties students that students might experience. Address common errors in your examples so students don't make them. Assign homework according to the arrangements of your notes as opposed to relying completely on the order of the book.

5. Call parents in the beginning of the year to introduce yourself, to explain what you will be doing, discuss student expectations (notes, homework, etc), and tell the parents what a nice young man or lady they have. This should be a very positive call!

6. Develop a positive working relationship with parents. Place a letter on the board each week and make an effort to call as many parents of students whose last name begins with that letter. Discuss their child's behavior, attendance, class participation, homework and finally their grade. If a problem exists in these areas, involve the parent in finding possible solutions. Talk about solving the problem. Early intervention in the form of a phone call home may keep problems from escalating. If a student is displaying inappropriate behavior, ask the parent out of sincere consideration if there are family problems (death, divorce, etc.) that might bring about this behavior. Ask parents to talk to their child about the situation.

7. Do not get caught up in too much paperwork. Keep quizzes to ten questions or less. Also do not feel it is necessary to collect and grade every homework assignment. Visually check to see who has completed the assignment by

walking around the room. Remember to address recurring difficulties so all students' benefit.

8. Be aware of your demeanor. A teacher's knowledge, enthusiasm, energy, and interest of the subject they teach will be very often translated to their students. Read the faces of the students in your classroom. Put yourself in their place. Remember to use positive reinforcement. Many students will work out of loyalty to a teacher they like and respect.
9. Use the entire instructional period – from bell to bell.
10. Talk to your students positively, rather than saying, "If you don't do your homework, you will fail." Say, "If you do your homework, that will clarify your understanding of …." Be specific in your comments, don't just say "good work", say, "I liked your thinking in setting up and labeling the …."
11. Don't blame school rules or policies when addressing a discipline issue. Rather, indicate that you don't like the behavior and it is unacceptable and disrespectful to you.
12. Have all tests, regardless of grade, signed by a parent or guardian and kept in the student notebook.
13. To address student deficiencies, use two strategies, linkage to introduce new material and the second review period to go over deficiencies that won't naturally come up in the regular class.