Why does the *Reducing Method* work for finding <u>Common Denominators?</u>

Finding a common denominator is finding a number that is a multiple of all the numbers given to you – called the Least Common Multiple (LCM). Another way of saying that is we are finding a number that all the other numbers divide into evenly.

There are a number of ways of finding the LCM or LCD; multiplying, writing multiples, prime factorization, and the Reducing Method. The Reducing Method follows directly from the prime factorization method.

Using the Prime Factorization Method, we would use a factor tree and write each number as a product of prime numbers.

So, to find the LCM of 60 and 18,

1. Write each number as a product of primes using a factor tree.
2. Write all the factors that appear in each number (without repetition)
3. Use the largest exponent that appears on each specific factor

Step 1. $60 = 2^2 \text{ x } 3 \text{ x } 5$

$18 = 2 \text{ x } 3^2$

Step 2. 2 x 3 x 5

Step 3. $2^2 \text{ x } 3^2 \text{ x } 5 = 180$. The LCM is 180

Now, let's say we want to add 5/18 + 7/60. To accomplish that, we first have to find a common denominator, often times, we want the ***least common denominator*** (LCD). The LCD and LCM are the same. Based on what we just did, we know the LCD is 180.

Using the ***Reducing Method***,

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1. Write the two denominators as a fraction and reduce it.
2. Cross multiply
3. The product is the LCM or LCD

$$\frac{18}{60} = \frac{3}{10}; \ \mathbf{18 \text{ x } 10 = 60 \text{ x } 3 = 180}$$

Writing the prime factors of those numbers in fractional form and reducing, we have

$$\frac{18}{60} \rightarrow \frac{2x3^2}{2^2x3x5} = \frac{3}{2x5}$$

Cross multiply, $2^2 \text{ x } 3^2 \text{ x } 5$. Reducing divides out the repeated factors while cross multiplying ensures all the factors are being used.