Why does the ac Method of factoring work?

When factoring trinomials with a leading coefficient of one, we found a pattern that allowed us to factor the trinomial quickly. That is; we found factors of c whose sum was b, then we used those numbers to factor the trinomial.

We see a similar pattern when working with trinomials, $a = 1$, $a \neq 1$, and binomials (difference of two squares).

Given a factorable trinomial $ax^2 + bx + c$, $a \neq 1$, factors into $(dx + f)(ex + g)$

$$ax^2 + bx + c = (dx + f)(ex + g)$$

Expanding $$(dx + f)(ex + g) = dex^2 + dgx + efx + fg$$

By the Transitive Property, $$ax^2 + bx + c = dex^2 + dgx + efx + fg$$

We can see $a = de$, $c = fg$. and $b = dg + ef$

Notice, the coefficient of the linear term, b, is made up of those same factors (d,e,f,g) as the product of $ac = defg$

So by multiplying ac in a trinomial, then using the factors of ac whose sum is b, that allows us to rewrite the linear term of the polynomial as a sum of two linear terms.

<u>Example</u> **Factor $15x^2 + 11x + 2$**

Multiply ac, $15(2) = 30$

Find factors of 30 whose sum is 11, that is 6 and 5

Rewriting $15x^2 + 11x + 2 = 15x^2 + 6x + 5x + 2$

Factor by grouping $= \underline{15x^2 + 6x} + \underline{5x + 2}$

$= 3x(5x + 2) + 1\,(5x + 2)$

$= (5x + 2)(3x + 1)$

This pattern also works when $a = 1$ and when factoring the difference of two squares.

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