A
LOOK at algebra

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In a nutshell, you have learned how to:

1. solve linear equations and inequalities
2. solve quadratic equations and inequalities
3. solve systems of linear equations and inequalities
4. solve equations containing absolute value
5. solve equations containing radicals

Along the way, you learned many manipulative skills using the Properties of Real Numbers.

We also looked at graphs in one and two variables and learned we could graph them by inspection, if we took the time to see some patterns develop.

And, because we enjoy problem solving so much, we studied formatted word problems so we might relate them to other problems we may encounter later.

Please review the Algebra, *you can do it* sheets along with this packet.

Linear equations

The general strategy for solving linear equations and inequalities was to use the Gift Wrapping analogy. That is, to use the Order of Operations in reverse, using the opposite operation to isolate the variable.

We learned to solve equations in the $ax + b = c$ form. Equations that did not look like that, we used the Properties of Real Numbers, to change it into that form.

Example

Solve:	$3x – 2 = 10$	
	$3x – 2 + 2 = 10 + 2$	APE
	$3x + 0 = 12$	AI/Arithmetic fact
	$3x = 12$	Prop of Zero
	$x = 4$	DPE

Example

Solve: $5(2x – 3) + 8 = 13$

This problem looked different from the $ax + b = c$ problems because of the parentheses. Using the Distributive Property, we get rid of the parentheses, combine terms, and make the problem look like one we solved before.

$10x – 15 + 8 = 13$
$10x – 7 = 13$
$10x – 7 + 7 = 13 + 7$
$10x = 20$
$x = 2$

Example

Solve: $x/2 – x/3 = 10$

This problem looks different because it has fractions. Using the Multiplication Property of Equality, I can multiply both sides of the equation by the common denominator, that eliminates the fractions and makes the problem look like one I have solved before..

$6\{ x/2 – x/3 \} = 6(10)$
$3x – 2x = 60$
$x = 60$

Equations containing absolute value

When solving equations containing absolute value, you must remember the expression on the inside of the absolute value signs could be positive or negative. When solving equations, we are looking for ALL the values of the variable that will make the open sentence true, we must solve the equation for both conditions, when the expression is positive and when the expression is negative. That means we should get two answers!

Example

Solve: $|2x + 3| = 11$

If $2x + 3$ is a positive number, then $2x + 3 = 11$. Solving for x, we have $2x = 8$, or $x = 4$.

But, $2x + 3$ could have been a negative number. If that were the case, then $2x + 3 = -11$. Solving for x, we have $2x = -14$, or $x = -7$.

Notice, if $x = 4$, then $2x + 3$ is 11 and the absolute value of 11 is 11. If $x = -7$, then $2x + 3$ is negative 11, and the absolute value of a negative 11 is also 11.

Example

Solve: $|5x - 3| > 27$

Translated that means that

$-27 > 5x - 3$	OR	$5x - 3 > 27$
$-24 > 5x$		$5x > 30$
$-24/5 > x$		$x > 6$

Example

Solve: $|2x - 5| < 9$

Translated that means that

$-9 < 2x - 5$	AND	$2x - 5 < 9$
$-4 < 2x$		$2x < 14$
$-2 < x$		$x < 7$

$$-2 < x < 7$$

It might be helpful to keep in mind when the absolute value is less then, you have a conjunctive statement, when it's greater than, you have a disjunctive statement.

Quadratic equations

Zero Product Property

We looked at three ways of solving quadratic equations, the first using the Zero Product Property. That is, if AB = 0, then either A or B has to be zero.

To solve quadratics using the Zero Product Property,

1. we put everything on one side of the equation, zero on the other side,
2. factored, and
3. set each factor equal to zero.

Example

Solve: $x^2 + x - 20 = 0$

$(x + 5)(x - 4) = 0$ factoring

In order for that product to be equal to zero, one of the factors has to be zero. Setting each factor equal to zero, we have

$x + 5 = 0$ or $x - 4 = 0$

$x = -5$ or $x = 4$

Completing the Square

We learned how to complete the square by noticing the relationship (pattern) between the linear and constant terms in perfect squares when the coefficient of the quadratic term was ONE.

To solve quadratics by completing the square,

1. put the constant on the other side of the equation
2. made sure the leading coefficient was ONE
3. took half the linear term and squared it
4. added that number to both sides of the equation
5. factored
6. solved the resulting equation

Example

Solve: $x^2 + 6x - 1 = 0$

$x^2 + 6x = +1$

$x^2 + 6x + \_ = 1 + \_$ Take ½ the linear term (6) and square

$x^2 + 6x + 9 = 1 + 9$ Adding 9 to both sides

$(x + 3)^2 = 10$ Factoring

$x + 3 = \pm\sqrt{10}$

$x = -3 \pm\sqrt{10}$

Quadratic Formula

The Quadratic Formula was derived from completing the square using $ax^2 + bx + c = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example

Solve: $x^2 + 5x - 4 = 0$

Using the formula, a = 1, b = 5, and c = -4. Plugging those values into the Quadratic Formula, we have

$$x = \frac{-(5) \pm \sqrt{25 - 4(1)(-4)}}{2(1)}$$

$$x = \frac{-5 \pm \sqrt{41}}{2}$$

Systems of linear equations

We used two methods to solve equations simultaneously, the addition method and substitution.

To solve a system of equations of two equations with two variables, we will reduce it to one equation with one variable – a problem we already know how to solve.

Addition Method

To do that by using the Addition Method, we will make the coefficients of one of the variables the same, but opposite in sign. Then, by adding the equations together, one of the variables will fall out and the resulting equation will be linear in one variable – an equation we can solve.

Example

Solve:

$$3x + 2y = 6$$
$$6x - 2y = 12$$

By adding these equations together by the APE, we have equation that we know how to solve.

$$9x = 18$$
$$x = 2$$

If x =2, then by plugging that value back into one of the equations, we have

$$3(2) + 2y = 6$$
$$6 + 2y = 6$$
$$2y = 0$$
$$y = 0$$

These two lines intersect at the point (2,0).

Example

Solve:

$$3x + 2y = 5$$
$$4x + 3y = 6$$

In this example, none of the coefficients are the same. Using the Properties of Real Numbers, I make that happen by multiplying both sides of an equation by the same number.

$3x + 2y = 5$	$\longrightarrow$	(x3)	$9x + 6y = 15$
$4x + 3y = 6$	$\longrightarrow$	(-2)	$-8x - 6y = -12$

adding the equations $x = 3$, and $y = -2$, the solution $(3, -2)$

Substitution

To solve equations simultaneously using substitution, you solve for one variable in one equation and plug that into the other equation. Generally, the only time you would use the substitution method is when one of the coefficients on one the variables in 1.

Example

Solve: $y = 2x + 1$
$2x + 3y = 19$

y was already solved for in the first equation, so plug $(2x + 1)$ into the second equation, we have:

$2x + 3(2x + 1) = 19$

$2x + 6x + 3 = 19$
$8x + 3 = 19$
$8x = 16$
$x = 2$ If $x = 2$, then $y = 5$ so the point of intersection is $(2, 5)$

Equations containing radicals

To solve equations containing radicals, isolate the radical, then get rid of it. Solve the resulting equation. Be careful, when you are raising powers, you might introduce extraneous solutions, so you MUST check your answers.

Example

Solve:

$$\sqrt{x-2}-4=6$$
$$\sqrt{x-2}=10$$
$$\left(\sqrt{x-2}\right)^2=10^2$$ Squaring both sides to get rid of the radical
$$x-2=100$$
$$x=102$$

Example

Solve:

$$2-y-2\sqrt{y+1}=0$$
$$2-y=2\sqrt{y+1}$$
$$(2-y)^2=[2\sqrt{y+1}]^2$$
$$4-4y+y^2=4(y+1)$$ Squaring both sides to get rid of the radical
$$4-4y+y^2=4y+4$$
$$y^2-8y=0$$
$$y(y-8)=0$$

y = 0 or y = 8

IMPORTANT – check your answers, 8 does not work when you plug it back into the original equation, so y = 0 is the only answer.

You could have an equation that contains more than one radical, if so, isolate one at a time and get rid of that radical.

Example

$$\sqrt{x-5} = \sqrt{x} - 1$$
$$x - 5 = x - 2\sqrt{x} + 1$$
$$-6 = -2\sqrt{x}$$
$$36 = 4x$$
$$9 = x$$

Note well – when you squared the right side, the radical did NOT disappear. The radical needs to be isolated again!