Graphing a Rational Function

To graph rational functions, you will need to recall the rules for finding asymptotes and the following guide.

Let $f(x) = \dfrac{p(x)}{q(x)}$ define a function where p9x) and q(x) are polynomials and the rational expression is written in simplest form. To sketch the graph, follow the following steps.

1. **Find any vertical asymptotes.**
2. **Find any horizontal or oblique asymptotes**
3. **Find the y-intercept by evaluating f(0)**
4. **Find the x-intercepts, if any, by solving f(x) = 0. {These are the zeros of of the numerator - p(x)}**
5. **Determine whether the graph will interest its non-vertical asymptotes y = b or**
   y = mx + b by solving f(x) = b or f(x) = mx + b
6. **Plot selected points, as necessary. Choose an x in each domain interval determined by the vertical asymptotes and x-intercepts.**
7. **Complete the sketch.**

Example **Graph** $f(x) = \dfrac{3x^2 - 3x - 6}{x^2 + 8x + 16}$

Going through each step from above.

1. Find the vertical asymptote(s) ~ the denominator is $(x + 4)^2$, so x = – 4 is the only vertical asymptote.
2. Use Rule 2, Case B for finding horizontal asymptotes. y = 3/1 or y = 3
3. The y-intercept is f(0) = –6/16 or –3/8
4. The x-intercepts, f(x) = 0, set the numerator equal to zero and solve; $3x^2 - 3x - 6 = 0$. The x-intercepts are x = 2 or x = –1
5. Setting f(x) = 3 to find where the may intersect the horizontal asymptote
   $\dfrac{3x^2 - 3x - 6}{x^2 + 8x + 16} = 3$, that results in x = –2, the point of intersection is (–2,3)
6. Pick some values of x in each interval to plot some points
7. Sketch the graph