VENN DIAGRAMS

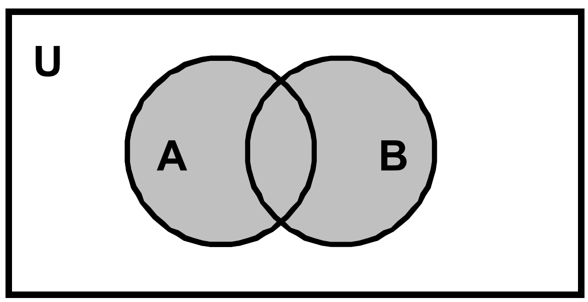
In the last section we represented sets using set notation, listing the elements in brackets. With Venn Diagrams, we define everything exactly the same way, but the definitions are in the form of pictures.

For instance, the set union of A and B, written $A \cup B$ was defined as:

$$A \cup B = \{x / x \in A \text{ or } x \in B\}$$

With Venn Diagrams, the definition again is all the members that belong to A or B, but we show that by shading in the circles A and B.

A U B



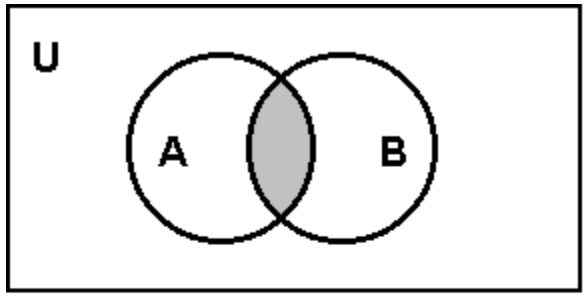
That's important, the set union is defined by shading in both circles of the Venn Diagram, all the members belong.

Let's look at the set intersection of A and B, that was defined as the members that belonged to both sets.

$$A \cap B = \{x / x \in A \text{ and } x \in B\}$$

That is illustrated by shading only the portion of the circles that overlap. Those elements belong to A and also to B.

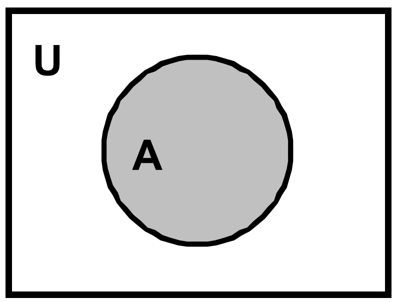
A ∩ B



Notice, there were rectangles around those examples. Those rectangles represented the universal set. All the elements under discussion.

To represent a single set, such as A. I would draw one circle and shade it in.

A

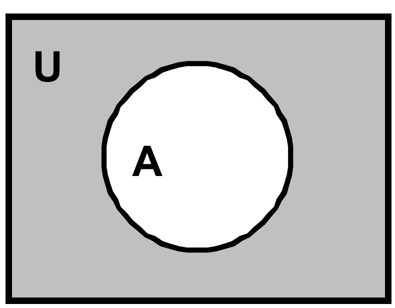


How would the complement of A, -A be illustrated? I'm glad you asked. Just like before, the complement of A are all the members of the universal set not in A.

$$\sim A = \{x \,/\, x \in \cup \text{ and } x \not\subset A\}$$

That's illustrated by shading in the rectangle, but not any part of the circle.

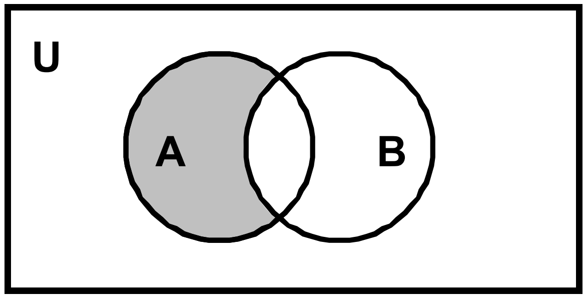
$\sim$**A**



I know, you love coloring, you want to do more. Let's look at the set difference. Remember A – B was defined as all the elements in A, but couldn't belong to B.

$$A - B = \{x / \in A \text{ and } x \notin B\}$$

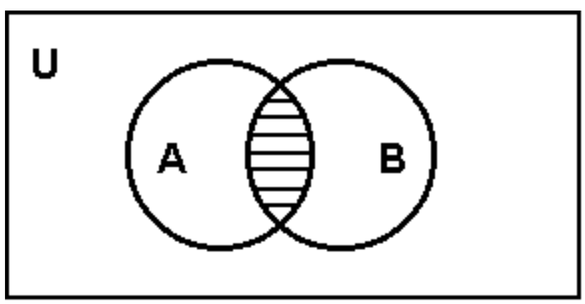
A – B



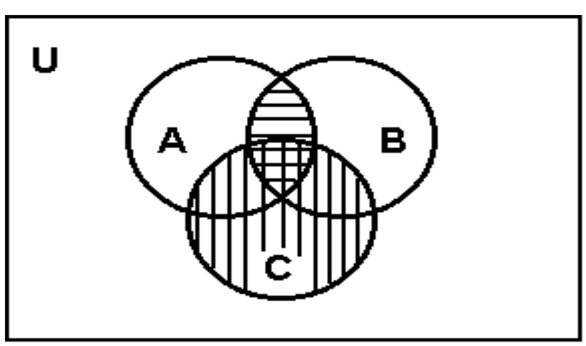
Having those operations defined through Venn Diagrams, we can now play with more sets. Again, the illustrations are important, introducing a third set using a circle does not in any way change what we have already defined. In fact, if there is a third set, we work with just two at a time, just like we did with sets.

Example Shade $A \cap B \cap C$

First we'll find the intersection of A and B, completely ignoring C. After that, we take that result and intersect it with C. We already know be definition, that $A \cap B$ looks like this

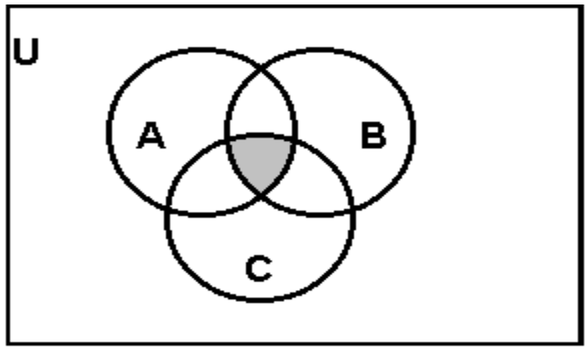


Now, we'll take that shading and intersect that with circle C.



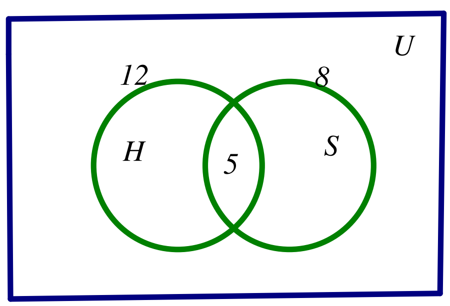
I shaded $A \cap B$ with horizontal lines and C with vertical so you can tell the difference. Where the shading overlaps is what these sets have in common. That almost makes sense, for an element to belong to A, B and C, there is only one region within the three circles that satisfies that.

$A \cap B \cap C$



Example 1 There are 19 boys who belong to the Breakfast Club. 12 like ham, 8 like sausage and 5 like both ham and sausage. How many in the club like ham only? Only like sausage?

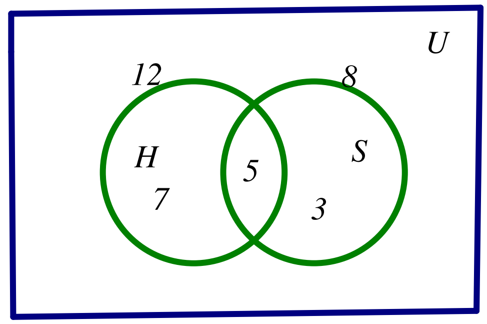
The best way to start these types of problems is to create a Venn Diagram labeling the circles as H for ham and S for sausage. The intersection of the circles represents the members of the club that like both ham and sausage.



Now if there are 12 club members who like ham and 5 are already accounted for because they like ham and sausage, how many like only ham? 12 – 5 = 7

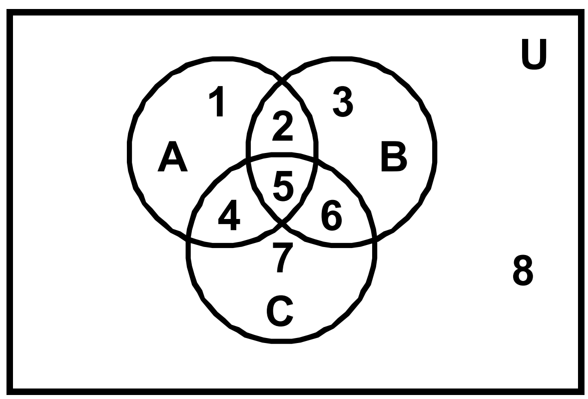
Using the same argument, how many like just sausage? 8 – 5 = 3

Let's fill in the Venn Diagram.



When you add the numbers of who likes ham only, sausage only and ham and sausage, that totals 15. There are 19 members in the club, where are the other 4?? They would be located outside the circle because they don't like ham or sausage.

Let's look at a Venn Diagram made up of three sets in which the regions are labeled. Now, we'll describe each region.



Region 5 is in all three circles. So any elements in region 5 would belong to all three sets. In other words; $A \cap B \cap C$.

What about Region 2? Those are the elements in A and B, but not C. How might you describe Region 6? Those are elements in B and C, but not in A. Try Region 4. The elements in A and C, but not B.

This is fun, let's look at some more regions. Region 1 describes the elements in A only. What about region 3? Those elements are only in B. Region 7 then would be the elements in C only. Region 8 would describe elements that are not members of any of the sets, but belong to the universal set.

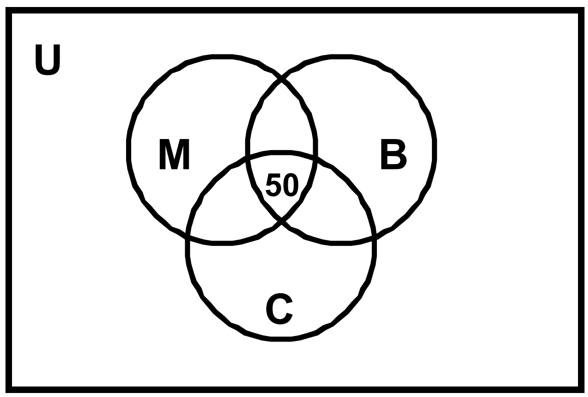
It's important that you become familiar with how each of those regions might be described. Being able to describe those regions would allow you to solve some problems.

Example 2

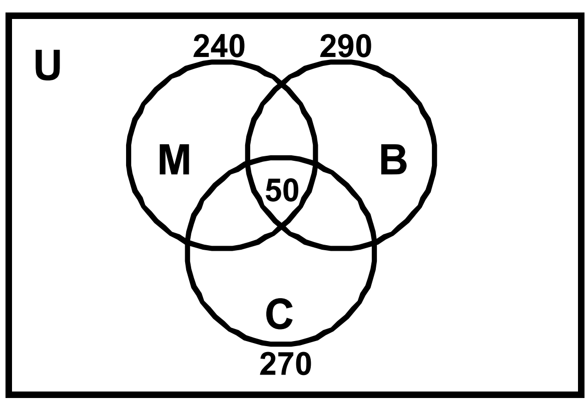
A survey was taken of 650 university students. It was reported that 240 were taking math, 290 were taking biology, and 270 were enrolled in chemistry. Of those students, 80 were taking biology and math, 70 were taking math and chemistry, 60 were taking biology and chemistry, and 50 were taking all three classes. How many students took math only?

At first glance, you might not think this is possible because the numbers add up to more than 650. But if you are familiar with how the regions are described, we can determine how many were in each region.

In going about this problem, I would tell you to draw a Venn Diagram and begin by filling in Region 5, the students that took all three courses.



After doing that, we'll place the number of students taking each course on the circle because we don't know where those students should be located within the circles.



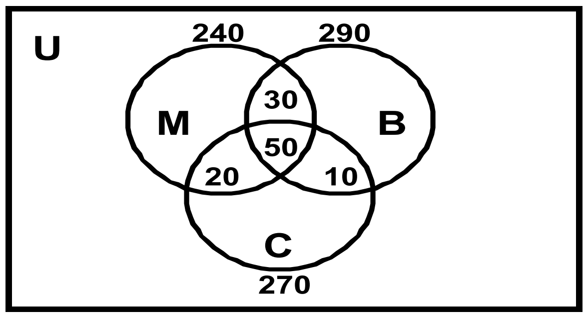
Okay, now we can have some fun by determining what regions the students should be located.

For instance, it says that 80 students are taking math and biology. We have 50 of those accounted for in Region 5, how many does that leave to be in Region 2? 80 – 50 = 30

That's easy enough. Using that same reasoning, 70 students are taking math and chemistry, how many students would then be in Region 4? Well, 70 – 50 = 20. That's pretty easy, don't you think?

Ok, how many students should be in Region 6? Since there are 60 students enrolled in biology and chemistry, and 50 of them are accounted for in Region 5, that leaves 10 students for Region 6.

Let's fill in those numbers:

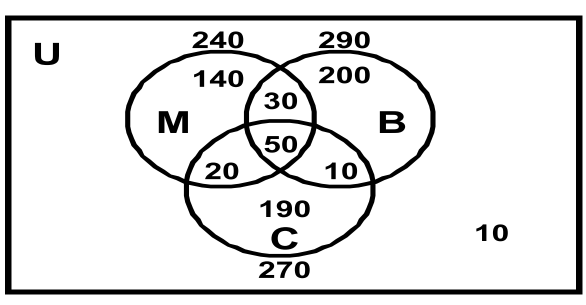


Now how many students would be in Region 1? Now remember, there are supposed to be 240 students taking math, we have 100 accounted for in Regions 2, 4, and 5. That leaves 140 students in region 1, taking math only.

How many students are taking biology only? Well, we were told that 290 students were taking biology, we have 90 of them accounted for in Regions 2, 5, and 6, that leaves 200 students in Region 3.

How many are taking only chemistry? We know there are 270 students taking chemistry, we have 90 accounted for in Regions 4, 5, and 6, that leaves 190 in Region 7.

Filling in those numbers and taking the numbers off the circle, we have the following information.



We have one slight problem, if we add those regions within the circles, the total is 640 students. The problem stated 650 were surveyed, we're missing ten students. Where are they? That's right, they would be in Region 8, not taking any of those courses.

Now tell me, was that fun?

How many students took math and biology, but not chemistry?

How many students took math and chemistry, but not biology?

How many students took biology and chemistry, but not math?

How many students took only math?

How many students took exactly two of the courses?

Now, let's make a point. Solving these problems require you to understand the language and translate that to math and to be able to add and subtract. My point is the math is not hard.

VENN DIAGRAMS

1a. A survey of 70 high school students revealed that 35 like folk music, 15 like classical music, and 5 like both. How many of the students surveyed do not like either folk or classical music?

2a. Out of 35 students in a finite math class, 22 are male, 19 are business majors, 27 are freshmen, 14 are male business students, 17 are male freshmen, 15 are freshmen business majors, and 11 are male freshmen business majors. How many upperclass women non-business majors are in the class? How many women business majors are in the class?

3a. A survey of 100 college faculty who exercise regularly found that 45 jog, 30 swim, 20 cycle, 6 jog and swim. 1 jogs and cycles, 5 swim and cycle, and 1 does all three. How many of the faculty members do not do any of these three activities? How many just jog?

4a. After a genetics experiment, the number of pea plants having certain characteristics were tallied, with the results as follows.

22	were tall
25	produce green peas
39	produce smooth peas
9	are tall and produce green peas
17	are tall and produce smooth peas.
20	produce green peas and smooth peas
6	have all three characteristics
4	have none of the characteristics.

(a) Find the total number of plants counted.
(b) How many plants are tall, but produce peas which are neither smooth nor green?
(c) How many plants are not tall, but produce peas which are smooth and green?

5a. A survey of 80 business executives found the following recommendations on college majors for business students.

36	recommended liberal arts courses
32	recommended business courses.
32	recommended technical courses.
16	recommended technical and business courses.
16	recommended business and liberal arts courses.
14	recommended liberal arts and technical courses
6	recommended all three.

(a) How many executives recommend liberal arts, but neither of the other two?
(b) How many recommend none of these three types of courses?

VENN DIAGRAMS

6a. The musical tastes of a number of college students were surveyed, and it was found that:

22	like Johnny Cash
25	like Elvis Presley
39	like The Carpenters
9	like Johnny Cash and Elvis Presley
17	like Johnny Cash and The Carpenters
20	like Elvis Presley and The Carpenters
6	like all three
4	like none of these performers

(a) How many students were surveyed?
(b) How many like exactly two of these three performers?
(c) How many like Johnny Cash only?
(d) How many do not like Johnny Cash?

7a. The following data shows the preferences of 110 people at a wine-tasting party.

99	like Spanada
96	like Ripple
99	like Boone's Fair Apple Wine
95	like Spanada and Ripple
94	like Ripple and Boone's
96	like Spanada and Boone's
93	like all three.

How many of the people:

(a) like none of the three beverages?
(b) like Spanada, but not Ripple?
(c) do not like Boone's Farm Apple Wine?
(d) like only Ripple?
(e) like exactly two wines?

8a. Routine physical examinations of 500 pre-school children revealed that 40 had dental problems, 45 had vision problems, 55 had hearing problems, 15 had dental and vision problems, 15 had dental and hearing problems, 20 had vision and hearing problems, and 10 had dental, vision, and hearing problems. How many of the children had none of the three kinds of problems?

9a. Human blood can contain either no antigens, the A antigen, the B antigen, or both the A and B antigens. A third antigen, called the Rh antigen, is significant in human reproduction, and again may or may not be present in an individual. Blood is called type A-positive if the subject has the A and Rh, but not the B antigen. Subjects having only the A and B antigens are said to have type AB-negative blood. Subjects having only the Rh antigen have type O-positive blood, etc. In a certain hospital the following data on patients were recorded:

25	patients had the A antigen,
17	had the A and B antigens,
27	had the B antigen,
22	had the B and Rh antigens,
30	had the Rh antigen,
12	had none of the antigens,
16	had the A and Rh antigens,
15	had all three antigens.

(a) How many patients are represented here?
(b) How many patients have exactly one antigen?
(c) How many patients have exactly two antigens?

10a. A local merchant uses television, radio, and newspaper advertising. To determine the effectiveness of advertising, he questions 200 customers during a special after-hours sale to see how they knew about the sale. He found that 115 had seen television ads, 75 had heard radio ads, and 125 had read newspaper ads. He also found that 30 received information from television and radio, 70 from television and newspapers, 25 from radio and newspapers, and 10 from all three. If everyone else said they heard it from a friend, how many heard it from a friend?

11a. In order to prepare a report on agricultural prospects for his county, the county farm advisor questions 100 farmers about their crop plans for the following year. He finds that 75 intend to plant corn, 55 will plant soybeans, 35 will plant wheat, 35 will plant corn and soybeans, 25 will plant corn and wheat, 15 will plant soybeans and wheat, and 10 will plant all three. How many of the farmers will plant only one crop? How many will plant at least two crops?

12a. In a group of 150 primary students, 100 watch “Sesame Street,” 55 watch “Electric Company,” and 65 watch “Mr. Rogers’ Neighborhood.” If, in addition, 35 watch “Sesame Street” and “Electric Company,” 45 watch “Sesame Street” and “Mr. Rogers,” 30 watch “Electric company” and “Mr. Rogers,” and 20 watch all three, how many watch none of the three? How many watch only “Sesame Street”?

VENN DIAGRAMS

13a. At a pow-wow in Arizona, Native Americans from all over the Southwest came to participate in the ceremonies. A coordinator of the pow-wow took a survey and found that:

15	families brought food, costumes, and crafts:
25	families brought food and crafts:
42	families brought food:
20	families brought costumes and food:
6	families brought costumes and crafts, but not food:
4	families brought crafts, but neither food nor costumes:
10	families brought none of the three items:
18	families brought costumes, but not crafts:

(a) How many families were surveyed?

(b) How many families brought costumes?

(c) How many families brought crafts, but not costumes?

(d) How many families did not bring crafts?

(e) How many families brought food or costumes?

14a. A survey of people attending a Lunar New Year celebration in Chinatown yielded the following results:

120	were women:
150	spoke Cantonese:
170	lit firecrackers:
108	of the men spoke Cantonese:
100	of the men did not light firecrackers:
18	of the non-Cantonese-speaking women lit firecrackers:
78	non-Cantonese-speaking men did not light firecrackers:
30	of the women who spoke Cantonese lit firecrackers.

(a) How many attended?

(b) How many of those who attended did not speak Cantonese?

(c) How many women did not light firecrackers?

(d) How many of those who lit firecrackers were Cantonese-speaking men?

15a. A chicken farmer surveyed his flock with the following results. The farmer had:

9	fat red roosters:
2	fat red hens:
37	fat chickens:
26	fat roosters
7	thin brown hens:
18	thin brown roosters:
6	thin red roosters:
6	thin red hens:

Answer the following questions about the flock. [Hint: You need a Venn Diagram with regions for fat, for male (a rooster is a male, a hen is a female), and for red (assume that brown and red are opposites in the chicken world).] How many chickens were:

(a) fat? (b) red?

(c) male? (d) fat, but not male?

(e) brown, but not fat? (f) red and fat?

16. Country-Western songs seem to emphasize three basic themes: love, prison, and trucks. A survey of the local country-western radio station produced the following data:

12	songs were about a truck driver who was in love while in prison:
13	were about a prisoner in love:
28	were about a person in love:
18	were about a truck driver in love.
3	were about a truck driver in prison who was not in love:
2	were about a prisoner who was not in love and did not drive a truck:
8	were about a person who was not in prison, not in love, and did not drive a truck:
16	were about truck drivers who were not in prison.

(a) How many songs were surveyed?

Find the number of songs about:

(b) truck drivers: (c) prisoners:

(d) truck drivers in prison: (e) people not in prison:

(f) people not in love:

17A. A survey of 80 sophomores at a western college showed that:

36	took English:
32	took history:
32	took political science:
16	took political science and history:
16	took history and English:
14	took political science and English:
6	took all three

How many students:

(a) took English and neither of the other two?

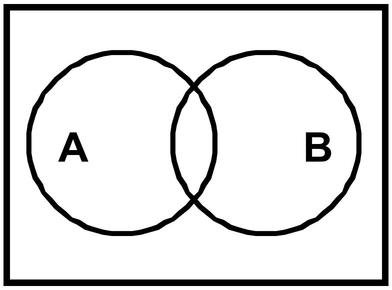
(b) took none of the three courses?

(c) took history, but neither of the other two?

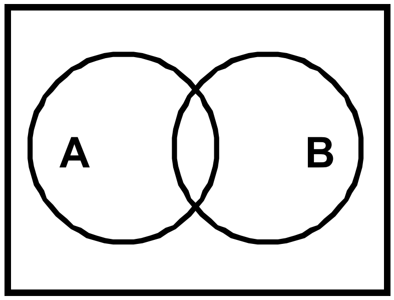
(d) took political science and history, but not English?

(e) did not take political science?

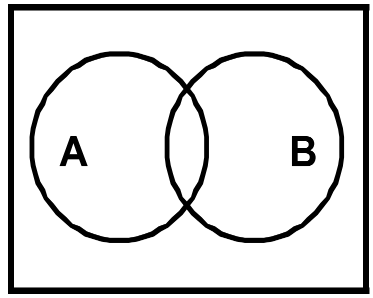
A



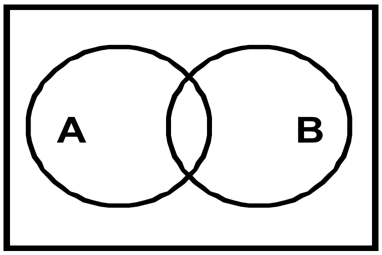
B



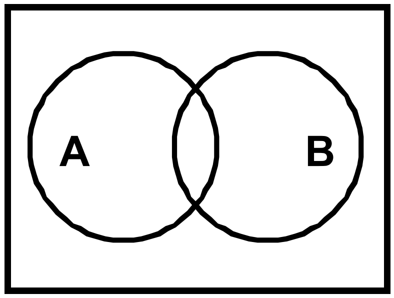
$\overline{A}$



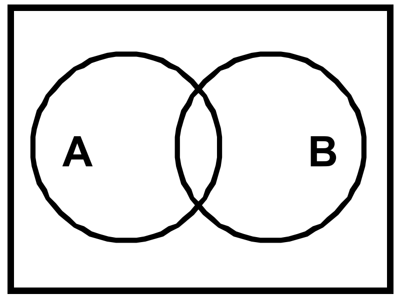
$\overline{B}$



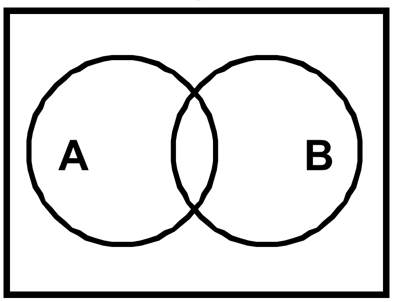
$A \cup B$



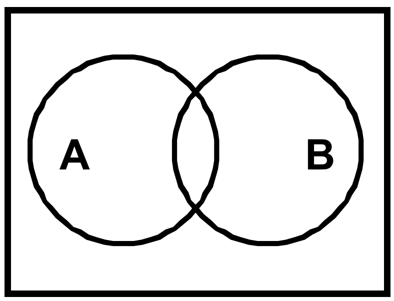
$A \cap B$



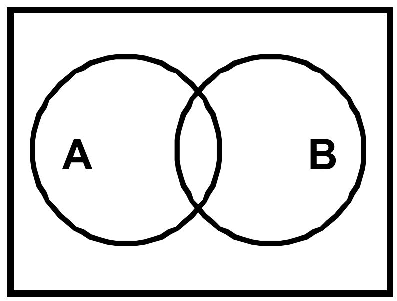
$\overline{A} \cup B$



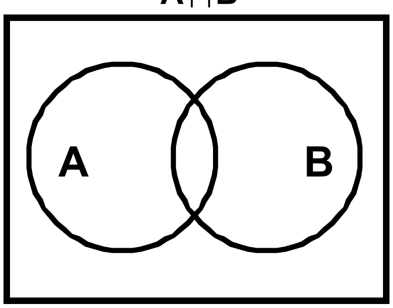
$A \cap \overline{B}$



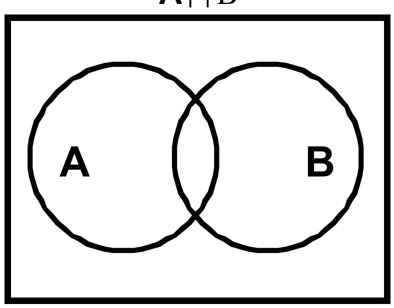
$\overline{A \cup B}$



$\overline{A \cap B}$



$\overline{A} \cap \overline{B}$



$\overline{A} \cup \overline{B}$