

## CHAPTER 2 WORD PROBLEMS

### Sec. 1 Word Translations

There is nothing more important in mathematics than to be able to translate English to math and math to English. Vocabulary and notation are very important to understanding and communicating in mathematics. Without knowing what words mean, we'll certainly have trouble answering questions. The research is clear, there is no more single important factor that affects students comprehension than vocabulary.

Listed below are examples of statements translated to algebra. It's very important that you are familiar with these expressions and their translations so you won't later confuse algebraic difficulties with vocabulary deficiencies. – The following should be taught explicitly. Students need to memorize them!

STATEMENT	ALGEBRA
twice as much as a number	$2x$
two less than a number	$x - 2$
five more than an unknown	$x + 5$
three more than twice a number	$2x + 3$
a number decreased by 6	$x - 6$
ten decreased by a number	$10 - x$
Tom's age 4 years from now	$x + 4$
Tom's age ten years ago	$x - 10$
number of cents in $x$ quarters	$25x$
number of cents in $2x$ dimes	$10(2x)$
number of cents in $x + 3$ nickels	$5(x + 3)$
separate 15 into 2 parts	$x, 15 - x$
distance traveled in $x$ hrs at 50 mph	$50x$
two consecutive integers	$x, x + 1$
two consecutive odd integers	$x, x + 2$
sum of a number and 30	$x + 30$
product of a number and 5	$5x$
quotient of a number and 7	$x \div 7$
four times as much	$4x$
two less than 3 times a number	$3x - 2$

By familiarizing yourself with these expressions, you'll look forward to solving word problems. We have already identified and used strategies for solving linear equations in one variable. In word problems, all we do that is different is make our own equations by reading and translating English to math. Piece of cake, don't you think?

The easiest and best way to learn vocabulary is to read your textbook. How you do on standardized tests will often be determined by your understanding of math vocabulary. College entrance exams, the ACT and SAT, use correct terminology so you best get used to it. Where your teacher might ask you to solve an equation, on a standardized test you will be asked to find the solution set, find roots or find zeros. You need to know they mean the same thing.

Write an equation or expression for each of the following statements.

1. John is four years older than Frank.
2. Bob has five times as much money as John.
3. The second angle is thirty degrees more than the first.
4. The sum of the interior angles of a triangle is  $180^\circ$ , The second angle of a triangle is  $45^\circ$  more than the first and the third angle is twice the first.
5. The area of a triangle is half the base times the height.
6. The perimeter of a rectangle is the sum of twice the length and twice the width.
7. Ted is four years older than three times Mary's age.
8. Mark earns a base salary of \$400 per week plus a 6% commission on all his sales.
9. The cell phone bill has a base fee of \$30 per month plus twenty cents per minute.
10. The circumference of a circle is equal to the diameter multiplied by  $\pi$ .

## Sec. 2

## Problem Solving

Now that you know how to translate English to math, it's time to use our knowledge of solving equations with our knowledge of translating English to mathematics. During your first year of algebra, you will learn how to set up different types of problems including, uniform motion, age, coin, mixture, geometry, number and investment. Like everything else in life, the more you do, the more comfortable and confident you will become. These learned formats should give you an idea how to set up and solve problems that you have not encountered in class.

Probably the most important thing to remember is most of us have to read a word problem 4, 5 or 6 times just to get all the information we need to solve the problem and make an equation that describes the relationship.

If there is any one trick to make your work easier, it is to write the smallest quantity as  $x$  and the other unknown in terms of  $x$ . I do that because it often eliminates equations containing fractions.

Study the word translations!

I can not stress enough how important it is for you to give your self a chance to be successful by reading and rereading the word problem in order to get the needed information.

Generally speaking, if you only read the problem once or twice, you won't get the information you need to setup and solve the problem.

Let's look at some word problems and see how to set them up. Remember, after we identify what we are looking for, determine the smallest value and call it  $x$ . The other unknowns will be described in terms of  $x$ .

## Algorithm for Problem Solving

1. Read the problem through to determine the type of problem
2. Reread the problem to identify what you are looking for and label
3. Reread, Let  $x$  be the smallest quantity you are looking for. (just my preference)
4. Reread the problem again and label the other quantities in terms of  $x$
5. Reread the problem to make an equation, use some fact or relationship involving your variables

### **Example 1**

Henry solved a certain number of algebra problems in an hour, his older brother Frank solved four times as many. Together they solved 80. How many were solved by each?

Reading the problem the first time, the problem is about algebra problems.

Reading the problem a second time, I am looking for the number of problems solved by Henry (H) and Frank (F).

Reading the problem a third time, I ask who solved the least number of problems? Hopefully, you said Henry. So let  $H = x$

Reading the problem a fourth time, I see Frank solved four times as many, therefore  $F = 4x$

Read the problem a fifth time to find a relationship. Together they solved 80. That means that

$$\begin{array}{ll} H + F = 80 & \text{Substituting} \\ x + 4x = 80 & \text{Solving for } x \\ 5x = 80 & \\ x = 16 & \end{array}$$

Therefore Henry solved 16 and Frank solved 4 times 16 of 64 problems.

### Example 2

The second of two numbers is two less than three times the first. Find the numbers if their sum is 26.

We are looking for two numbers, I'll call the first one  $x$ .

$$\#1 - x$$

$$\#2 - 3x - 2$$

The sum is 26.

$$\#1 + \#2 = 26$$

Substituting

$$x + 3x - 2 = 26$$

$$4x - 2 = 26$$

$$4x = 28$$

$$x = 7$$

The first number is 7, the second number is two less than three times 7 or 19.

### Example 3

The second angle of a triangle is  $45^\circ$  more than the smallest angle. The third angle is three times the smallest. How many degrees are there in each angle?

We are looking for three angles.

$$1 - x$$

$$2 - x + 45$$

$$3 - 3x$$

$$1 + 2 + 3 = 180^\circ$$

$$x + x + 45 + 3x = 180^\circ$$

$$5x + 45 = 180^\circ$$

$$5x = 135^\circ$$

$$x = 27^\circ$$

You would not be able to solve this problem unless you knew that the sum of the interior angles of a triangle is  $180^\circ$ .

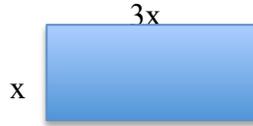
That means the first angle is  $27^\circ$ , the second angle is  $27 + 45$  or  $72^\circ$ , and the third angle is 3 times 27 or  $81^\circ$ .

#### Example 4

The length of a rectangle is three times the width and its perimeter is 48 ft. Find the length and width.

$$L = 3x$$

$$W = x$$



The perimeter is the distance ALL the way around the rectangle.

$$x + 3x + x + 3x = 48$$

$$8x = 48$$

$$x = 6$$

So the width is 6 feet and the length is 18 feet.

Notice, in all of the above problems, I identified what I was looking for and labeled that information. I reread the problem to find the smallest quantity and called that  $x$ . I reread the problem again and labeled the other quantities in terms of  $x$ . And Finally, I reread the problem again and based on the relationships, I made an equation.

Solve the following word problems.

1. Henry solved a certain number of algebra problems in an hour, but his older brother Frank solved 3 times as many, together they solved 60. How many were solved by each?  
Hint: It is often best to let  $x$  stand for the smaller of the quantities. Why?
2. Francis has 4 times as many marbles as George and both together have 90. How many has each?
3. They sum of the edges of a cube is 24 inches. Find the length of each edge.
4. A house and lot are worth \$15,600. If the house is worth 12 times the value of the lot, find the value of each.
5. The part of a bridge pier out of the water is three times the length of the part under water. If the height of the pier above water is 30 feet, what is the length of the part under water?

6. The length of the Panama Canal is about 50 miles. If the length of the rest of the canal is about nine times the length of the Culebra cut, how long is the Culebra cut?
7. Matthew's kite string is three times as long as Joseph's. If the strings were tied together, they would reach a tenth of a mile. How long is each? (1 mile = 5280 feet.)
8. A line segment 40 inches long is divided into two parts, so that one part is four times the other part. How long is each part?
9. The perimeter (distance around) of a rectangle is 108 inches. It is twice as long as it is wide. Find the length; the width. Draw the figure.
11. The Missouri-Mississippi River is 12 times as long as the Hudson River, and their combined length are 4550 miles. How long is the Hudson River?
12. One number exceeds (is more than) another number by 6, and the sum of the two numbers is 26. Find the two numbers.
13. One number exceeds another number by 13 and the sum of the two numbers is 35. What are the two numbers?
14. Divide a line segment 80 inches long so that one part exceeds the other by 20 inches.
15. A baseball team lost 4 more games than it won. If the team played 46 games, how many did it lose? How many did it win?
16. The sum of two numbers is 57. One exceeds the other by 17. Find the numbers.
17. The sum of two numbers is 39. One is 5 more than the other. Find the numbers.
18. A man walks 9 miles, then travels a certain distance by automobile, and twice as far by train. If the whole trip is 108 miles, how far does he go by automobile? How far by train?
19. If it requires 180 feet of fencing to enclose a rectangular lot 30 feet wide, how long is the lot? Draw the figure.
20. Tom wishes to build a pen for his rabbit. He wants it to be three times as long as it is wide, and he wants to use not more than half of his 240-feet of chicken wire. What are the maximum dimensions he can make his pen?

### Sec. 3 Word Problems – Uniform Motion

Solving word problems is what kids in algebra live for. As there are different formats for solving different types of equations, there are different formats for solving different types of word problems.

You should keep in mind there are other methods for solving word problems than the ones I present.

To solve word problems involving uniform motion, we need to know that

$$\text{DISTANCE} = \text{RATE} \times \text{TIME}$$

I will use a distance, rate, time chart, and solve the problems in terms of distance whenever possible. In that way I can avoid fractional equations.

From this perspective there are two types of uniform motion problems. Either

- A. The distances are equal, or**
- B. The sum of the distances equal a number**

**TYPE A.** If the distances are equal, one of two things must occur.

1. You go somewhere and return, or
2. You leave to go somewhere and someone else leaves later and catches up to you

In either case, the distances are equal. Mathematically we write  $D_1 = D_2$

**Type B.** If someone did not catch up to you or if you did not go somewhere and come back, the distances are not equal. That means the sum of the distances must be equal to a number.

Mathematically, we write  $D_1 + D_2 = \#$

Let's see how all this works.

### Example 1

Two trains start from the same station at the same time and travel in opposite directions. One train travels at an average rate of 40 mph, the other at 65 mph. In how many hours will they be 315 miles apart?

First we'll make the  $d=rt$  chart. But we won't fill in the  $d$ .

	<b>d</b>	=	<b>r</b>	x	<b>t</b>
Train 1			40		x
Train 2			65		x

The reason we have an  $x$  in the time column is because they left at the same time and will be 315 at the same time. In other words, their times are equal.

Now, the big question. Are there distances equal? Since they do not meet the criteria in a **TYPE A** problem, the answer is no. That means the sum of the distances must be equal to a number.

$$\begin{aligned}D_1 + D_2 &= \# \\40x + 65 &= 315 \\105x &= 315 \\x &= 3 \quad \text{It will take three hours.}\end{aligned}$$

### Example 2

Bob starts out in his car traveling 30 mph. Four hours later, Mr. Speedster starts out from the same point at 60 mph to overtake Bob. In how many hours will he catch him?

Making the  $d = rt$  chart

	<b>d</b>	=	<b>r</b>	x	<b>t</b>
Bob			30		$x + 4$
Mr. Speedster			60		x

Since Mr. Speedster traveled the least amount of time, we called his time  $x$ . This is a **TYPE A** problem, the distances are equal.

$$D_{\text{Bob}} = D_{\text{Speedster}}$$

$$\begin{aligned}
30(x + 4) &= 60x \\
30x + 120 &= 60x \\
120 &= 30x \\
4 &= x
\end{aligned}$$

It will take 4 hours to catch Bob.

Solve

- Two trains start from the same station and run in opposite directions. One runs at an average rate of 40 miles per hour, and the other at 65 miles per hour. In how many hours will they be 315 miles apart?

First Train	Starting point	Second train
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40 mph x hours  
distance = rate x time  
time

65 mph x hours  
distance = rate x

$$d = 40x$$

$$d = 65x$$

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$$40x + 65x = 315$$

miles \_\_\_\_\_

Or you may like to set up a table like this:

Train	Time	Rate	Distance
First	X	40	40x
Second	X	65	65x

Then Complete.  $40x + 65x = 315$

- Two automobiles start from the same place and travel in opposite directions, one averaging 45 miles per hour and the other 30 miles per hour. In how many hours will they be 900 miles apart?
- Two men, A and B, start toward each other at the same time from points 510 miles apart. If they travel 40 and 45 miles an hour respectively, in how many hours will they meet?

4. Jones and Brown start from two points, which are 375 miles apart and travel toward each other. The latter travels twice as fast as the former. They meet in 5 hours. Find the rates per hour.
5. A man rides out into the country at a uniform rate of 30 miles per hour. He rests 2 hours and then rides back at 20 miles per hour. He is gone 5 hours. How far did he go?
6. A motorboat starts out and travels 9 miles an hour. In 3 hours another motorboat traveling 18 miles an hour starts out to overtake the first one. In how many hours will the second boat overtake the first?
7. Mr. Williams starts out in his auto traveling 30 miles per hour. Four hours later Mr. Speedster starts out from the same point at 60 miles per hour to overtake Mr. Williams. In how many hours will he be overtaken?

Hint: Remember that each will have traveled the same distance when they meet.  $X = \text{Speedster's time}$ .

$$R \times T = D$$

Williams	30	$X + 4$	$30(x - 4)$
Speedster	60	$X$	$60X$

Therefore,  $30(x + 4) = 60x$

8. A freight train is traveling 30 miles per hour. An automobile starts out from the same place 1 hour later and overtakes the train in 3 hours. What was the rate of the automobile?
9. C and D start from two points 480 miles apart and travel toward each other. They meet in 8 hours. If C travels 6 miles per hour faster than D, find their rates.

## Sec. 4 Word Problems - Mixture

One method of solving mixture problems is to do the problem in terms of what is being added.

That means if you have a problem involving a mixture of antifreeze and you are going to add water to it to dilute it, then do the problem in terms of water. An iodine problem that has you adding alcohol to dilute it should be done in terms of alcohol.

Just remember, when solving mixture problems, we **DON'T** start off with water and add salt to get salt water.

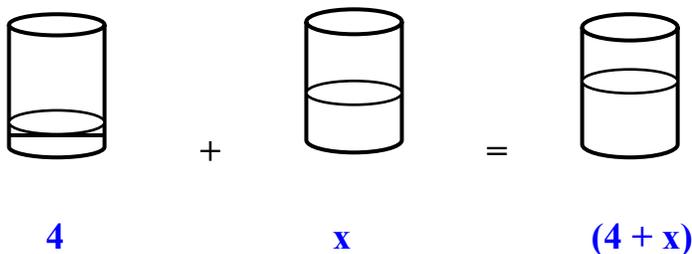
### Example 1

A pharmacist has 4 quarts of a 15% solution of iodine. How much alcohol must be added to reduce it to a 10% solution of iodine?

What's being added? Hopefully, you said alcohol. Therefore our equation will look like this:

$$\begin{array}{rcccl} \text{ALCOHOL} & + & \text{ALCOHOL} & = & \text{ALCOHOL} \\ 4 \text{ qts} & + & x \text{ qts} & = & (4 + x) \text{ qts} \end{array}$$

We start off with 4 quarts, add x quarts, then end up with (4 + x) quarts.



Notice that you started with 4 quarts and added x quarts on the left side of the equation and you ended up with (4 + x) on the other side. The parentheses indicate that it's one container,

Of the original 4 quarts, 15% is iodine. Since we are doing the problem in terms of what we are adding – alcohol, we must change 15% iodine solution to an 85% solution of alcohol. Let's write the equation.

$$.85(4) + x = .90(4 + x)$$

Where'd the .90 come from? Well, since I wanted to end up with a 10% solution of iodine, that meant it must be a 90% solution of alcohol.

Multiplying both sides of the equation by the common denominator – 100, we have

$$\begin{aligned} 85(4) + 100x &= 90(4 + x) \\ 340 + 100x &= 360 + 90x \\ 10x &= 20 \\ x &= 2 \end{aligned}$$

You would have to add 2 quarts of alcohol to reduce the mixture to 10% iodine.

### Example 2

How much water must be added to a 30 quarts of a 75% acid solution to reduce it to a 15% solution of acid?

I'm adding water, so we have

$$\mathbf{WATER} + \mathbf{WATER} = \mathbf{WATER}$$

Starting off with 30 quarts and adding x quarts, we should end up with (30 + x) quarts

$$30 + x = (30 + x)$$

The problem describes the solution in terms of acid, we have set it up in terms of water. So, we have to change the percentages and put them in the problem.

$$.25(30) + x = .85(30 + x)$$

Again, we multiply both sides of the equation by 100 to get rid of the decimal point.

$$\begin{array}{rclcl} 25(30) & + & 100x & = & 85(30 + x) \\ 750 & + & 100x & = & 2550 + 85x \\ & & 15x & = & 1800 \\ & & x & = & 120 \text{ qts} \end{array}$$

Remember to do the problem in terms of what's being added, then make sure your percents describe the solution in your equation.

### Example 3

A club received \$270.00 for tickets for their initiation. If the price of the tickets are \$10 for members and \$6.00 for non-members and the total number of tickets sold was 35, how many tickets of each kind were sold?

$$\begin{array}{ll} \$10 \text{ tickets} & x \\ \$6 \text{ tickets} & 35 - x \end{array}$$

Value of Tickets + Value of Tickets = Value of Tickets

$$\begin{array}{rclcl} 10x & + & 6(35 - x) & = & 270 \\ 10x & + & 210 - 6x & = & 270 \\ & & 4x + 210 & = & 270 \\ & & 4x & = & 60 \\ & & x & = & 15 \end{array}$$

There were 15 \$10 tickets and 20 \$6 tickets sold.

## Sec. 5 Work Problems

When solving work problems, I typically set the equations up in a chart where work = rate x time - very much like the formatted distance rate problems.

There are basically two things we need to understand to do these quickly. First, when we set up the chart, we are most frequent doing ONE job. So work is often equal to ONE.

Second, we need to recall unit rates. That is, if it takes you 5 hours to do 1 job, then your rate is  $1/5$  of the job per hour.

### Example 1

Joe can cut his lawn in 4 hours with a hand mower. His friend Tom can cut the lawn with a power mower in 3 hours. How long will it take for the boys to cut the lawn if they work together?

So we have a work problems, that means  $w = rt$  and they want to complete ONE job working together.

	W	=		r	t
J	1			$1/4$	x
T	1			$1/3$	x

Since they work together, we have

$$1/4x + 1/3 x = 1$$

$$12\{1/4x + 1/3 x\} = 12 (1)$$

$$3x + 4x = 12$$

$$7x = 12$$

$$x = 12/7$$

So together it will take them  $1 \frac{5}{7}$  hours to cut the lawn.

Now keep in mind, the unit rate is the reciprocal of the time it takes to do a job. That means if it takes  $2 \frac{1}{2}$  hours to do a job, which is  $5/2$  hours, then the unit rate is  $2/5$ . In other words  $2/5$  of the job is done per hour.

### Example 2

One pipe can fill a tank in 5 hours. Another pipe can fill the tank in 8 hours. How long will take the two pipes to fill the tank together?

Again, the work is filling ONE tank. The unit rates are  $1/5$  and  $1/8$  respectively.

$$\begin{array}{r} W \\ 1 \end{array} = \begin{array}{r} r \\ 1/5 \\ 1/8 \end{array} \begin{array}{r} t \\ x \\ x \end{array}$$

$$\begin{aligned} 1/5 x + 1/8 x &= 1 \\ 40\{1/5 x + 1/8 x\} &= 40(1) \\ 8x + 5x &= 40 \\ 13 x &= 40 \text{ or } x = 3 \frac{1}{13} \text{ hours} \end{aligned}$$

### Example 3

A printing press can do a job in 4 hours. After a second press was purchased, the same job can be completed by both machine is  $2 \frac{1}{2}$  hours. How long would it take the second machine alone to complete the job.

This problem is a little different in that it already tells you how long it takes for the machines to complete the job working together. So, let's set up our chart and reason our way through. We are still completing ONE job, the rate of the first machine is  $1/4$ . If it takes  $x$  hours for the second machine, the second machine's rate is  $1/x$ . The time is given as  $2 \frac{1}{2}$  hours which we will write as  $5/2$ .

$$\begin{array}{r} W \\ 1 \end{array} = \begin{array}{r} r \\ 1/4 \\ 1/x \end{array} \begin{array}{r} t \\ 5/2 \\ 5/2 \end{array}$$

$$\begin{aligned} 1/4 (5/2) + 1/x (5/2) &= 1 \\ 8x(5/8 + 5/2x) &= 8x (1) \\ 5x + 20 &= 8x \\ 20 &= 3x \\ 20/3 &= x \text{ or } x = 6 \frac{2}{3} \text{ hours.} \end{aligned}$$

Solve

1. A grocer wishes to mix one-dollar coffee with 80-cent coffee to produce a mixture of 200 pounds to sell for 84 cents a pound. How many pounds of each kind should he use?

Hint: Try making a table like this:

Kind	Number of Pounds	Value in Cents
\$1 coffee	$n$	$100n$
80-cent coffee	$200 - n$	$80(200 - n)$
84-cent coffee	200	200 84

Why do we use  $200 - n$  to represent the number of pounds of 80-cent coffee?

Hence,  $100n + 80(200 - n) = 16,800$

Complete the solution.

2. A grocer wishes to mix \$1.20 tea with \$1.50 tea to make a mixture of 60 pounds worth \$1.30 a pound. How many pounds of each kind must he mix?
3. A merchant wishes to mix walnuts selling at \$2.25 a pound with almonds selling at \$2.40 a pound so as to make a mixture of 120 pounds worth \$2.30 a pound. How many pounds of each kind of nuts must he use?
4. How much water must be added to 12 quarts of a 10% solution of salt and water to reduce to a 6% solution?
5. A pharmacist has 4 quarts of a 15% solution of iodine. How much alcohol must he add to reduce it to a 10% solution?

6. How much water must be added to 30 quarts of a 75% solution of acid to reduce it to a 15% solution?
7. How much pure disinfectant must be added to 30 gallons of an 8% solution to increase its strength of 25%?
8. The radiator of an automobile already contains 12 quarts of a 10% solution of alcohol. How much alcohol must be added to make a mixture of 20% alcohol?
9. How much alcohol must be added in exercise 8 to make a mixture containing 25% alcohol?
10. How many quarts of milk containing 4% butter fat and how many quarts of cream containing 29% butter fat must be mixed to make 40 quarts of cream containing 20% butter fat?
11. How many quarts of a solution half of which is acid must be added to 10 quarts of a solution one-fifth of which is acid to form a solution which is three-tenths acid?

## MISCELLANEOUS PROBLEMS

1. Kenneth solved a certain number of problems and Harold solved 2 more than twice as many. Together they solved 38. How many did each solve?
2. If 2 is added to a certain number, the result is the same as would be obtained if twice that number were subtracted from 32. What is the number?
3. Harry wished one summer to earn enough money for his next years expense in college, which would amount to \$1560. His father said, "For every dollar that you earn I will give you four dollars." How much must Harry earn?
4. During the summer Frank earned \$60 less than three times as much as his brother Fred. Together they earned \$1220. How much did each earn?
5. A grocer wishes to combine 75-cent candy with 50-cent candy to make 40 pounds of a mixture he can sell for 65 cents a pound. How much of each kind should he use?
6. A ship leaves a harbor sailing at 28 mph. A plane leaves  $6\frac{1}{4}$  hours later. At what rate must it fly to overtake the ship in an hour and 15 minutes?
7. A man flew to another city for a meeting at the rate of 260 mph. Later he returned by train at the rate of 60 mph. If his total traveling time was 4 hours, what was his flying time?
8. Divide an estate of \$35,000 among three sons so that the second son gets \$5,000 more than the youngest, and the eldest twice as much as the youngest. How much does each get?
9. A certain kind of concrete contains twice as much sand as cement and 5 times as much gravel as cement. How many cubic feet of each of these materials will there be in 2000 cubic yard of the concrete?  
Hint: How many cubic feet in a cubic yard?

10. Find each man's share of the profits of a business if A receives twice as much as B, and B receives twice as much as C and the profits for the year are \$35,000.
11. A's city tax was \$43 more than his state tax. His income tax was \$560. His total tax for these three amounted to \$815. How much was his state tax?
12. Seven boys and girls went into the corner drug store. Each boy got an ice cream soda at 30 cents, and each girl took a chocolate nut sundae at 35 cents. The bill was \$2.25. How many boys were there? How many girls?
13. A boy made 40 quarts of lemonade, which contained 20% lemon. How much water would he have to add to reduce it to 10% lemon?
14. For a certain football game the coach wants to start one sophomore and at least twice as many seniors as sophomores and juniors combined. What is the maximum number of juniors he can start?