CHAPTER 13 Equations with Radicals

A radical equation is an equation in which there is a variable inside the radical sign (radicand).

Example 1 $\sqrt{x} - 2 = 4$

The strategy we'll use to solve equations containing radicals will be based on

$$\sqrt[n]{a} = b, \text{ then } \boldsymbol{a = b^n}$$

In other words, to get rid of a square root, we square both sides of the equation. To get rid of a third root, we cube each side of the equation. To get rid of any root, we raise both sides of the equation to the same power as the index. Remember, when an index is not written. it is understood to be 2.

Algorithm for Solving Equations Containing Radicals

1. Isolate the radical
2. Raise both sides to a power equal to the index
3. Solve the resulting equation
4. Always, always check your answer!

Example 2 **Solve for x;** $\sqrt{x} - 4 = 3$

Isolate the radical $\sqrt{x} = 7$

Square both sides $x = 49$

Check the answer.

Example 3 **Solve for x;** $\sqrt{x} - 2 = 5$

$\sqrt{x} = 7$

$x = 49$

Check your answer!

Example 4 **Solve for x;** $\sqrt{x+4} + x = 8$

Isolate the radical $\sqrt{x+4} = 8 - x$

Square both sides $x + 4 = 64 - 16x + x^2$

Squaring both sides results a quadratic equation, put everything on one side, zero on the other side. Factor set each factor equal to zero.

$$0 = x^2 - 17x + 60$$

$$0 = (x - 5)(x - 12)$$

$$x = 5 \quad \text{or} \quad x = 12$$

Check the answer. Notice that 12 does NOT work and is called an **extraneous root**. The only answer is 5.

When we squared both sides of an equation, the degree is raised. There is a corollary to the **Fundamental Theorem of Algebra** that states an equation of degree *n* will have *n* roots. So when we solved the quadratic equation, degree 2, that corollary suggested we would have two roots. However, the original equation was of degree 1. In other words, by squaring both sides of an equation, we may introduce solutions to the higher degree equation that do not work for the original equation. Therefore, it is always necessary to check your solutions in the original equation to ensure we did not introduce extraneous solutions.

You'll notice the "solving" part can sometimes be LINEAR or QUADRATIC. You need to be able to identify different types of equations and the strategies for solving them to be able to solve radical equations.

To solve radical equations, all you do is isolate the radical and raise to the power of the index. The rest is determined by the type of equation you have.

Example 5 **Solve** $\sqrt[3]{3x-9}-2=-4$

$\sqrt[3]{3x-9}=-2$ Isolate radical

$3x-9=-8$ Cube both sides

$3x=1$

$x=1/3$

Example 6 Solve $\sqrt{x^2-8}=2-x$

$x^2-8=4-4x+x^2$

$-12=-4x$

$3=x$

Check your answer. 3 does not work

What that means is there is no real value of x that will satisfy the original equation. We say the answer is the **null set** or **empty set**. We write it this way ∅

Up to this point, we have only examined equations that had only one radical. If there are two radicals in an equation, isolate one of them and get rid of that radical, then isolate the other radical.

That does not make the problem more difficult to solve, it only makes it longer.

Example 7 **Solve**: $\sqrt{x+7}-1=\sqrt{x}$

1. Isolate one of the radicals $\sqrt{x+7}=\sqrt{x}+1$
2. Square both sides $x+7=x+2\sqrt{x}+1$
3. Isolate the other radicals $6=2\sqrt{x}$
4. Simplify $3=\sqrt{x}$
5. Square both sides $9=x$
6. Check your answer

Notice, if there are two radicals, you must isolate one at a time. The problem is longer, not more difficult and the same strategy is used.

The biggest difficulty experienced by many students is squaring a binomial containing a radical. That brings us back to the special products learned earlier.

$$(a + b)^2 = a^2 + 2ab + b^2$$

Remember, square both the first and last terms, then find the product of the two terms and double.

Examples

$$(\sqrt{5} + 3)^2 = 5 + 2\sqrt{5} \cdot 3 + 9 = 14 + 6\sqrt{5}$$

$$(\sqrt{3} - 7)^2 = 3 - 2\sqrt{3} \cdot 7 + 49 = 52 - 14\sqrt{3}$$

$$(\sqrt{x} + 4)^2 = x + 2\sqrt{x} \cdot 4 + 16 = x + 8\sqrt{x} + 16$$

You should do enough of these so you can do the multiplication in your head. Otherwise the multiplication becomes the problem rather than solving the radical equation. I know, you want to do another problem.

Example 8 Solve for x; $\sqrt{x-5} - \sqrt{x} = -1$

Isolate one of the radicals first. My preference is to isolate the radical with the longest expression in the radicand

$$\sqrt{x-5} = \sqrt{x} - 1$$
$$x - 5 = x - 2\sqrt{x} + 1$$
$$-6 = -2\sqrt{x}$$
$$3 = \sqrt{x}$$
$$9 = x$$

Check you answer!

To solve radical equations:

1. Transform the equation so that the radical is alone in one member.
2. Raise both members to a power equal to the index of the root.
3. Solve the resulting equation by any of the standard procedures.
4. Check carefully each apparent root in the original equation, rejecting any which extraneous.

Solve the following, check your answers!

1. $\sqrt{x} = 2$
2. $\sqrt{x} = 3$
3. $\sqrt{x-1} = 5$
4. $\sqrt{x+2} = 10$
5. $2\sqrt{x+3} = 8$
6. $\sqrt{x} - 4 = 3$
7. $\sqrt{x} + 3 = 5$
8. $\sqrt{x-4} - 2 = 8$
9. $3\sqrt{x} - 1 = \sqrt{x} + 1$
10. $\sqrt{x+8} + \sqrt{x} = 8$
11. $\sqrt{x+7} = 2 - \sqrt{x-5}$
12. $\sqrt{4x+3} = 2\sqrt{x-1} + 1$