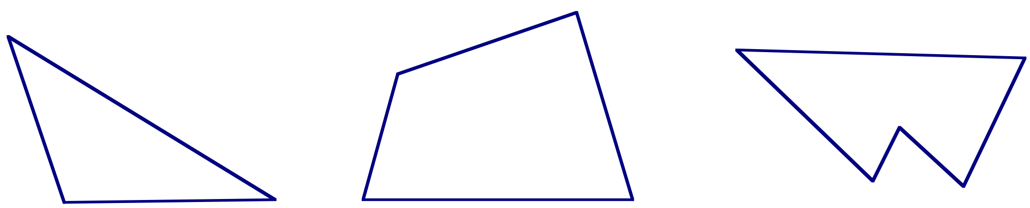
Chapter 7

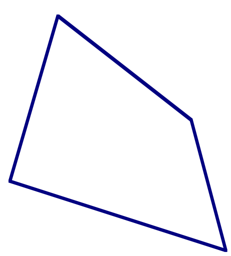
Polygons

A **polygon** can be described by two conditions:

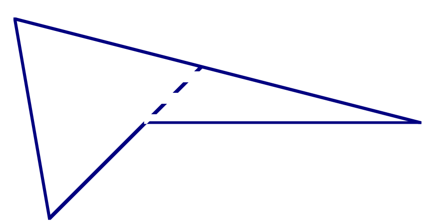
1. No two segments with a common endpoint are collinear.
2. Each segment intersects exactly two other segments, but only on the endpoints.



Convex polygon - a polygon such that no line containing a side of the polygon will contain a point in the interior of the polygon.



convex polygon

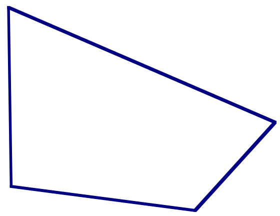


concave polygon

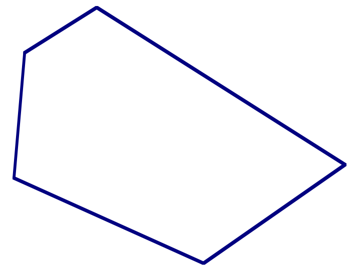
Notice, if I extend a side (dashed line) on the right polygon, the line contains interior points. Therefore it is not a convex polygon, it is called a **concave** polygon.

What's the sum of the interior angles of a quadrilateral? a pentagon? an octagon?

The answer is, I just don't know. But …, if I draw some pictures, that might help me discover the answer.

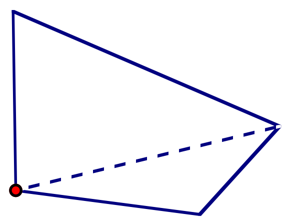


quadrilateral

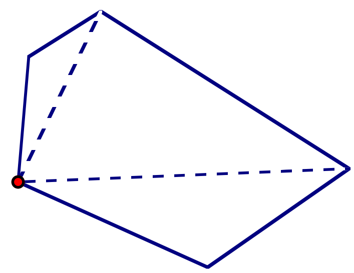


pentagon

By drawing diagonals from a single vertex, I can form triangles.



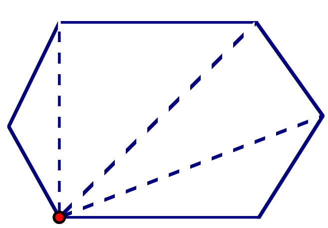
4 sides - 2 triangles



5 sides - 3 triangles

Those observations might lead me to believe the sum of the interior angles of a quadrilateral is 360° because there are two triangles formed. In a five-sided figure, a **pentagon**, three triangles are being formed. Since the sum of the interior angles of each triangle is 180°, then the sum of the interior angles of a pentagon is 540°.

The number of triangles being formed seems to be two less than the number of sides in the polygon. Try drawing a hexagon and see if the number of triangles formed is two less than the number of sides.

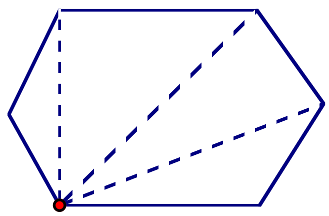


6 sides - 4 triangles
formed from one vertex

These examples suggest a polygon with $\boldsymbol{n}$ sides would have **($\boldsymbol{n}$ – 2)** triangles formed. So, by multiplying the number of triangles formed by 180º should give me the sum of the interior angles. Sounds like a theorem to me.

Theorem **The sum of the interior angles of a convex polygon is given by (n – 2) 180º**

Example 1 Find the sum of the interior angles of a hexagon.



From the last theorem, we know the sum of the interior $\angle$s of a polygon is $(n–2)\ 180^\circ$

A hexagon has 6 sides, so $n = 6$ and the number of triangles formed is two less than the number of sides.

$$
\begin{aligned}
\text{Sum int } \angle\text{s} &= (6–2)180^\circ \\
&= (4)180^\circ \\
&= 720^\circ
\end{aligned}
$$

Regular polygon **- a convex polygon with all angles and segments congruent.**

Example 2 Find the measure of each angle of a regular octagon.

Knowing that an octagon has eight sides, I don't need to draw the picture. All I need to do is find the sum of the interior angles and divide that result by the number of angles – 8.

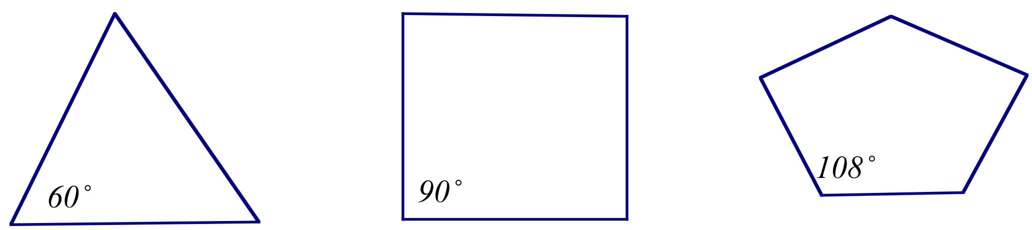
Sum int ∠s octagon = $(n–2)180^\circ$
$= (8–2)180^\circ$
$= (6)180^\circ$
$= 1080^\circ$

Now that I know the sum of the interior angles, I divide that by the number of sides/angles.

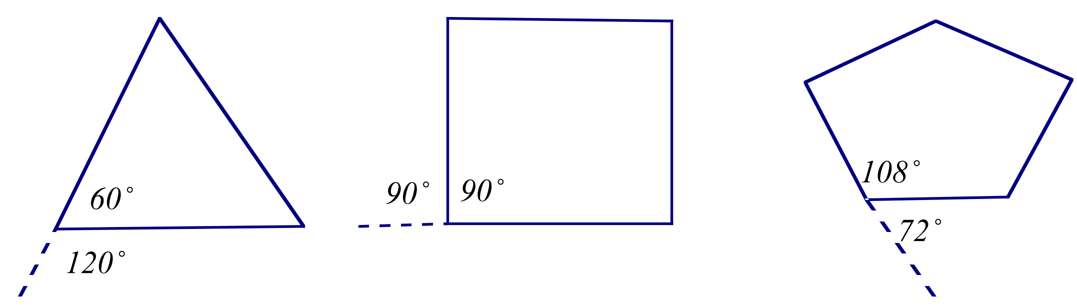
Each ∠ of a regular octagon = 1080/8 = 135˚

Exterior Angles – Polygons

If we played with these pictures of regular polygons longer, we'd find more good news that would lead to another theorem.



Let's say we drew a number of regular polygons, polygons whose sides and angles are congruent as we just did. We could find the measure of each exterior angle of the triangle, one angle at each vertex because we have two adjacent angles whose exteriors sides lie in a line – they are supplementary.



If each interior angle of a triangle measures 60˚, then each exterior angle would have measure 120˚. And the sum of the exterior angles would be 360˚.

Looking at the square, each interior angle would measure 90˚, each exterior angle would also measure 90˚. And the sum of the exterior angles would be 360˚.

And finally, in the pentagon, each interior angle measures 108˚, then each exterior would measure 72˚. And the sum of the exterior angles would be 360˚.

Is there a pattern developing there?

Let's summarize, in the triangle there are 3 exterior angles each measuring 120˚, their sum is 360˚

In the square, there are four exterior angles each measuring 90˚, their sum is 360˚.

In the pentagon, there are five exterior angles each measuring 72˚, their sum is 360˚.

It would appear the sum of the exterior angles of the examples we used seem to be 360˚. That might lead us to the following theorem.

Theorem **The sum of the exterior angles of a convex polygon, one angle at each vertex, is 360^o.**

The sum of the interior angles of a polygon changes with the number of sides. But the sum of the exterior angles always measure 360˚. That's important to know.

Assume I have a regular polygon whose exterior angle measured 40˚ and I wanted to know how many sides the polygon had, could I make that determination?

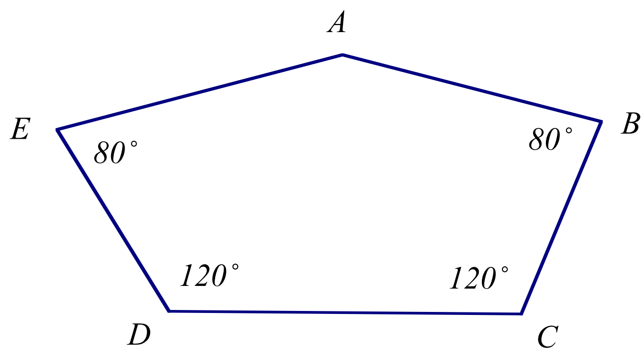
Since the polygon is regular, we know all angles must be congruent. If each exterior angle measured 40˚ and the sum of the exterior angles must be 360˚, then 360˚ ÷ 40˚ = 9. The polygon must have 9 sides!

Let's make a minor change, let's say the interior angle of a regular polygon measured 150˚, how many sides would it have? To make this determination, I would have to know the measure of the exterior angle. Since the interior angle measures 150˚, the exterior angle must measure 30˚since they would form a straight angle.

If each exterior angle measured 30˚, the 360˚ ÷ 30˚ = 12.

The polygon would have 12 sides.

Example 1 Given the diagram, find m∠A



Since this is a pentagon, the sum of the interior angles will be given by the formula (n–2)180˚, where n = 5. Substituting, we have (3)180˚ = 540˚

The sum of the given angles is:

$$m\angle B + m\angle C + m\angle D + m\angle E = 400^\circ$$

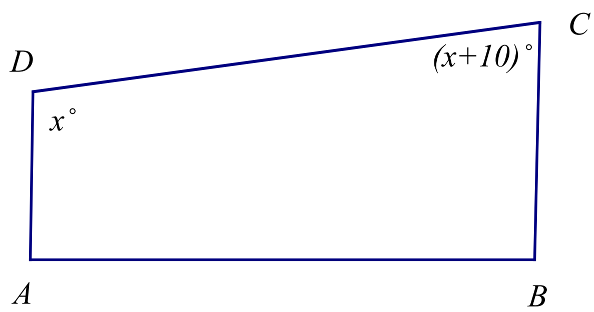
Since the sum of the 5 ∠s must be 540˚, and the 4 ∠s = 400,

$$m\angle A + [m\angle B + m\angle C + m\angle D + m\angle E] = 540^\circ$$

$$m\angle A + 400^\circ = 540^\circ$$

$$m\angle A = 140^\circ$$

Example 2 Given $\overline{AD} \parallel \overline{BC}$, find the value of x.



In this example, two lines were given as parallel. That should make you take note. What do I know about angles when two parallel lines are cut by a transversal?

One of the theorems we learned was *if two parallel lines are cut by a transversal, the same side interior angles are supplementary.*

$\angle$D and $\angle$C are same side interior angles.

That tells me that $m\angle D + m\angle C = 180^\circ$

$$x + (x+10) = 180$$
$$2x + 10 = 180$$
$$2x = 170$$
$$x = 85^\circ$$

Example 3 If the interior angle of a regular polygon measures 120˚, how many sides does it have?

An interior and exterior angle of a polygon form a straight line, thy are supplementary. If the interior angles measures120˚, then the exterior measures 60˚.

Since the sum of the exterior angles of a polygon measure 360˚ and one exterior angles measures 60˚, then finding the quotient; 360/60 = 6. The polygon has 6 sides.

Angles of a Polygon

The sum of the interior angles of a triangle is 180°

The exterior ∠ of a triangle is equal to the sum of the 2 remote int ∠'s

The sum of the interior angles of a convex polygon is given by $(n - 2)180°$

The sum of the exterior angles of a convex polygon is 360°

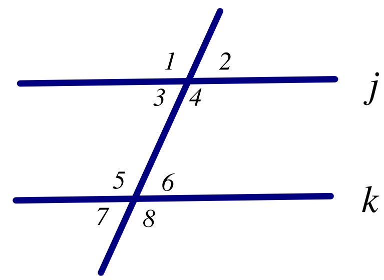
Polygon classification

Polygons are often classified by the number of sides.

Triangles –	3 sides
Quadrilaterals –	4 sides
Pentagons–	5 sides
Hexagons–	6 sides
Heptagons–	7 sides
Octagons–	8 sides
Nonagons–	9 sides
Decagons–	10 sides

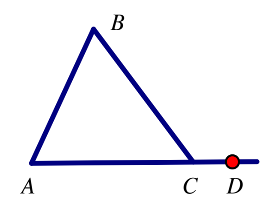
Angles; Parallel lines & Polygons

1. If j || k, then ∠ 5 is congruent to what other angles? Give a reason for each.



2. If *j* || *k* and *m*∠ 1 = 100˚, then ∠ 6 =

3. The number of obtuse angles in an obtuse triangle is



4. If *m*∠ A = 50˚ and *m*∠ B = 80˚, then ∠ BCD equals

5. The sum of the measures of the interior angles of a convex octagon is

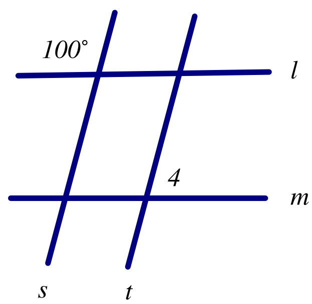
6. The sum of the measures of the exterior angles of a convex pentagon is

7. The total number of diagonals that can be drawn from one vertex of a hexagon is

8. In Δ ABC, $\overrightarrow{BX}$ bisects ∠ ABC, *m*∠ A = 110˚ and *m*∠ C = 40˚, then *m*∠ ABX is

9. If the measure of the interior angle of a regular polygon is 140˚, how many sides does the polygon have?

10. Given the figure; $l \parallel m$ and $s \parallel t$ find the $m\angle 4$.



11. Explain why it is not possible for the measure of an exterior angle of regular convex polygon to be 22˚