Teaching Struggling Students Mathematics

"6 + 1" Strategies

Changing the beliefs of struggling students by *Building Success on Success*

A relatively simple, no nonsense and proven approach to address the needs of struggling students that requires hard work.

Bill Hanlon

Bill Hanlon, former Director of the Southern Nevada Regional Professional Development Program (RPDP), has been an educator for over thirty years. His educational experiences include teaching at the junior high, senior high, and college levels. He was also the coordinator of the Clark County School District's Math/Science Institute and was responsible for K-12 math audits.

He served as vice president of the Nevada State Board of Education, Regional Director of the National Association of State Boards of Education (NASBE) and as a member of the National Council for Accreditation of Teacher Education (NCATE) States Partnership Board. Bill was also a member of Nevada's standards writing team in mathematics, served on the LearningFirst Alliance Review Team of the NCTM's standards, and was a member of a national panel to review TIMSS research. He hosted a television series, "Algebra, You Can Do It!" and taught math at the University of Nevada, Las Vegas, to prospective classroom teachers.

Bill also provides professional development for teachers and administrators around the country that focuses on the content teachers teach, how they teach and assess it, student performance, and changes in instructional strategies that result in increased student achievement. He stresses the importance of linking the concepts and skills being taught to previous learning and outside experiences.

Bill is a noted speaker, an author, educator, consultant and coach for schools as well as a national presenter for organizations such as AASA, ASCD, ALAS, NMSA, NASSP, NSBA, and NCTM.

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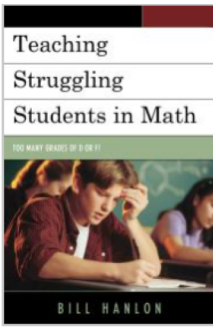

Overview "6 + 1" Strategies

~ Organizing Student Learning by Linking Instructional Practices ~

Based on the "Building Success on Success" model, we use the "6+1". That is, we recommend very powerful, but common sense, researched based instructional and assessment strategies to help struggling students learn and believe they can be successful in mathematics. We serve central office and school administrators, as well as classroom teachers in implementing a platform that organizes student learning by linking: ***planning***, ***instruction***, ***student notes***, ***homework assignments***, ***test prep*** and ***assessments*** that focus student learning. The "+1" is the importance we place on student-teacher relationships and how those relationships impact student achievement.

1. Preparation

1. Teachers identify what they expect students to know, recognize, understand, communicate and be able to do. Based on that, teachers would create a parallel constructed practice test using recommended template.
2. Teachers would also identify how they will introduce new material; through conceptual development, linkage or directed discovery and identify simple straightforward examples that work, with no variations, that clarify instruction and don't bog students down in needless arithmetic.
3. Teachers have a clear visual of what they expect student notes to look like based on their instruction.

2. Instruction

1. Teachers would begin class immediately with a quick, crisp, purposeful review (QCPR) of recently taught material and topics students will need to remember from previous learning to be successful in the day's lesson. During this time, students are not doing problems or answering questions, they are listening to the teacher. Teachers might also do a representative HW problem or two during this time. This QCPR should not extend beyond 10 minutes.
2. After the QCPR, check for proficiency (CFP) by having students do a couple of problems.
3. Teachers begin instruction by having the students write the date, title, and objective in their **note**books. The teachers then begin instruction introducing any new definitions and identifications and how to say them, then by using concept development, linkage or directed discovery and pattern development find strategies, rules, formulas, or algorithms. All of that information should be contained in the notes along with guided practice problems. Students should be paced through notes and guided practice problems. When a new formula, procedure, definition is introduced, the class will orally recite it for approximately 90 to 120 seconds to embed it in short term memory and teach students how to read, say and write math correctly.

3. Notes

Student notes support and reflect the day's instruction. Teachers should help students organize their notebook so they can study effectively and efficiently and have white space so there is no visual overload. Teachers should pace the students' note-taking as well as pace them through guided practice problems. Students should be able to review from their notes how concepts and skills were developed, as well as seeing definitions, formulas, procedures and practice problems. Parents should be able to follow student notes so they can become more engaged their child's educational experience and help with the material. Teachers should also use the highlighting system in their notes based on what is going to be tested.

4. Homework

Homework assignments are based on what teachers expect their students to know, recognize, understand, communicate and be able to do. In other words, HW, like notes will support and reflect instruction. That is HW assignments will include definitions, formulas, procedures, explanations, etc. and problem sets as well as exercises.

5. Test Prep

Two or three days before the unit/chapter test, teachers will pace their students through the parallel constructed practice test, one problem at a time. That is, the students do #1, then the teacher does #1, this process continues to the last question. This is the last time teachers will be able to monitor student understanding before the exam. If students show hesitancy or cannot do a problem, the teacher can do that problem in class and provided further clarification and practice the following day – before the real exam.

6. Assessment

The assessment should be parallel constructed to the practice exams. The test template will be employed on both the practice test and actual assessment so our expectations are very clear to students with respect to what we expect them to know, recognize, understand, communicate and be able to do.

+1. Student-teacher Relationships

Treating students in your own classes the same way you would like your own child treated is the foundation of student-teacher relationships. This can be enhanced by the transparency used in the "6+1" model that leads to credibility and trust.

Implementing the "6+1" helps organize student learning; focusing their learning so they can study and prepare for assessments more effectively and efficiently resulting in increased student achievement.

Sample

Homework

Linking homework assignments to instruction, notes, test preparation and assessments helps focus student learning.

A typical secondary math assignment in the United States looks like this:

Page 165, 1 – 30 odd

A more appropriate homework assignment that would encourage studying would look more like this:

Read Sec 4.2 Solving Linear Equations – 1 variable
Define Linear Equation
Write a strategy for solving linear equations in ax + b = c format
How do you solve an equation that is not in ax + b = c format
Explain the relationship between solving linear equations and the order of operations
Copy and redo examples 2 & 5 from the notes
Page 165; 1, 3, 4, 6, 11, 13, 14, 15, 19, 22, 29

N.B. - fewer exercises assigned.

Students who are not able to do the exercises are to copy all the examples and guided practice problems from their notes.

~ Linking Content ~

Linking new concepts and skills to previously learned math and outside experiences increases student understanding and achievement.

Linking allows teachers to introduce new concepts and skills using familiar ideas and language, which results in students being more comfortable in the in their knowledge, understanding and application of math.

Linking allows teachers to remediate as they teach their curriculum by reviewing and reinforcing previously learned mathematics. It also provides opportunities for interventions by allowing teachers to address student deficiencies.

Linking allows teachers and students to see how the math learned in the classroom can be used in different contexts which enhances their understanding of those concepts and skills

Linking encourages vertical articulation between grade level teachers. Elementary teachers see how the foundation they are laying leads to the success of their students. Secondary teachers see how they can better introduce new concepts and skills by scaffolding on what students already have learned.

For example, rather than introducing the topic of polynomials by name and using new terms such as monomial, binomial, and trinomial, students would be better served if the teachers introduced polynomials by reviewing place value and expanded notation (topics from 1st and 2nd grades), then showing the students how to add numbers from left to right using place value. By reviewing and reinforcing these topics first, the transition to adding/subtracting polynomials will be much easier for the students in algebra.

Teachers could relate functions to buying cold drinks; one cold drink is $.50, two is a dollar, three is a dollar fifty, etc. Writing these in ordered pairs, we'd have (1, .50), (2, 1.00), (3, 1.50), etc. By introducing functions by using concrete examples, it would make a great deal more sense to students. Having a teacher introduce a function by giving the following definition often results in

eyes glazing over. *A function is a special relation in which no two ordered pairs have the same first coordinate.*

Teachers should take the time to link concepts and skills to previously learned mathematics and outside experiences. The Pythagorean Theorem, for instance, can be linked to areas of squares, similar triangles, the distance formula, equation of a circle, and the trig identity ($\cos^2 x + \sin^2 x = 1$). By introducing these concepts and skills using linkage, teachers can save instructional time by not completely re-teaching the Pythagorean Theorem in its different forms and they would be reviewing and reinforcing knowledge and skills teachers indicate the students do not possess.

In order to create interest and enthusiasm in mathematics, teachers should also relate how concepts taught in the classroom are used in the real world, whether it be how the size (circumference) of tire effects the speedometer and odometer readings in a car or how it is used in pipefitting. Be able to find the vertex, focus, and directrix of a parabola might be better received if students understood how those are related to flashlights, lasers, satellite dishes or acoustics. Students might take a greater interest in math if they understood the topics they are learning can be used in decision-making, such as which phone company to use based on their rates when solving systems of equations.

Sample

Linking the Standard Multiplication Algorithm with Multiplication of Polynomials

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$$\begin{array}{r} 3\ 2 \\ \times\quad 2\ 1 \\ \hline 3\ 2 \\ 6\ 4 \\ \hline 6\ 7\ 2 \end{array} \qquad\qquad \begin{array}{r} 3x + 2 \\ \times\quad 2x + 1 \\ \hline 3x + 2 \\ 6x^2 + 4x \\ \hline 6x^2 + 7x + 2 \end{array}$$

The 32 corresponds to the 3x + 2, the 21 corresponds the 2x + 1. Both the two 2-digit numbers and the polynomials are multiplied using the same standard algorithm. Note, the partial products 32 and 3x + 2 correspond as do 64 and $6x^2 + 4x$. The final products have the same coefficients 672.

Choosing simple straightforward examples that work, that clarify and don't bog students down in needless arithmetic allows students to more readily see how math is linked.

By using this linkage, teachers can introduce "new" concepts and skills in more familiar language, which makes students feel more comfortable in their knowledge, understanding and application of math.

Linkage also provides teachers an opportunity to review and reinforce previously learned math or address student deficiencies as they follow their assigned curriculum – remediating along the way.

Scaffolding

Linking the Standard Multiplication Algorithm with Multiplication of Polynomials using the Distributive Property

$$\begin{array}{r} 3x + 2 \\ \times \quad 2x + 1 \\ \hline 3x + 2 \\ 6x^2 + 4x \\ \hline 6x^2 + 7x + 2 \end{array}$$

$$(2x + 1)(3x + 2)$$

$$6x^2 + 4x + 3x + 2$$

$$6x^2 + 7x + 2$$

The polynomials on the left, (2x + 1) (3x + 2), is multiplied using the same standard algorithm taught in elementary. The same polynomials on the right are being multiplied using the Distributive Property.

Notice the partial products are the same (color coded) as is the final product. 2x is distributed over the (3x + 2), then the +1 is distributed over the (3x + 2)

So, we can see that multiplying polynomials using the Distributive Property is directly linked to multiplication of polynomials using the standard algorithm as that is linked to the standard multiplication algorithm in arithmetic.

By using these linkages and using simple straight forward examples that work and clarify concepts and skills, teachers can introduce "new" concepts and skills in more familiar language, which makes students feel more comfortable in their experiences, knowledge and understanding.

Linkage also provides teachers an opportunity to review, reinforce and scaffold previously learned math or address student deficiencies as they follow their assigned curriculum – remediating along the way

Linking Addition with Place Value
With Polynomial Addition

$$3\,4\,1 + 2\,5\,6 = (3+2)100 + (4+5)10 + (1+6)1$$
$$= 5(100) + 9(10) + 7(1)$$
$$= 597$$

$$(3x^2 + 4x + 1) + (2x^2 + 5x + 6)$$
$$= (3+2)x^2 + (4+5)x + 1 + 6$$
$$= 5x^2 + 9x + 7$$

The 341 corresponds to the $3x^2 + 4x + 1$, the 256 corresponds the $2x^2 + 5x + 6$. Notice the coefficients of the sums - 597 corresponds to the $5x^2 + 9x + 7$

When adding using the standard algorithm, we ask students to add the hundreds column to the hundreds, the tens to the tens, etc. In adding or subtracting polynomials, we ask students to use the same concepts but describe it by saying "combine like terms". Notice the notation AND language changes to describe the sum, but the math is the same.

Choosing simple straight-forward examples that work, that clarify and don't bog students down in arithmetic allows students to more readily understand and succeed in math.

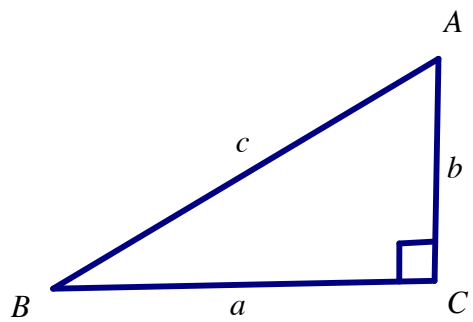
By using this linkage, teachers can introduce "new" concepts and skills in more familiar language, which makes students feel more comfortable in their knowledge, understanding and application of mathematics.

Importantly, linkage also provides teachers an opportunity to review and reinforce previously learned math or address student deficiencies as they follow their assigned curriculum – remediating along the way.

Linking the
Pythagorean Theorem and the Distance Formula

Pythagorean Theorem

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the two legs.



$$c^2 = a^2 + b^2$$

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

When using the Pythagorean Theorem, typically the lengths of two sides are given and students are asked to find the third side.

The Distance Formula is the Pythagorean Theorem, but rather than having two of the three sides given explicitly, students are asked to find the lengths of two of the sides using ordered pairs. Notice, rather than saying the horizontal side is *a*, we indicate that $a = x_2 - x_1$ by subtracting the x –coordinates. The same for the vertical side, rather than saying it is *b*, we indicate the length of *b* can be found by subtracting the y–coordinates; $b = y_2 - y_1$.

Finally, rather than calling the side opposite the right angle the hypotenuse, we call that *d*, for distance.

As you can see from above, making those substitutions and taking the square root results in the Distance Formula.