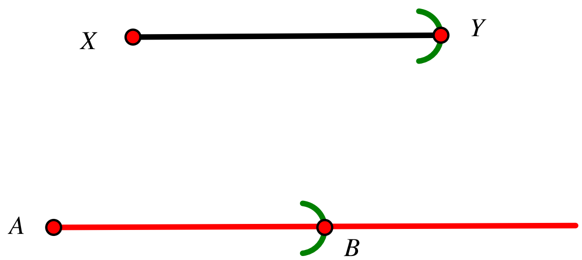
Ch. 0 6 Basic Constructions

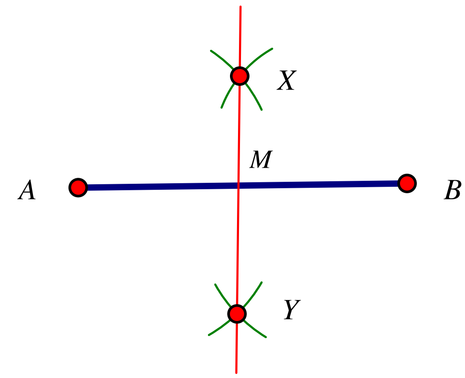
Construction 1 Given a segment, construct a congruent segment.



Steps

1. Draw a line $\overleftrightarrow{AB}$
2. Using a compass with X as the center, construct an arc intersecting $\overleftrightarrow{XY}$ at Y
3. On the line $\overleftrightarrow{AB}$, using the compass with A as the center, construct the same arc and label the point of intersection B.

Construction 2 Given a segment, construct a perpendicular bisector.



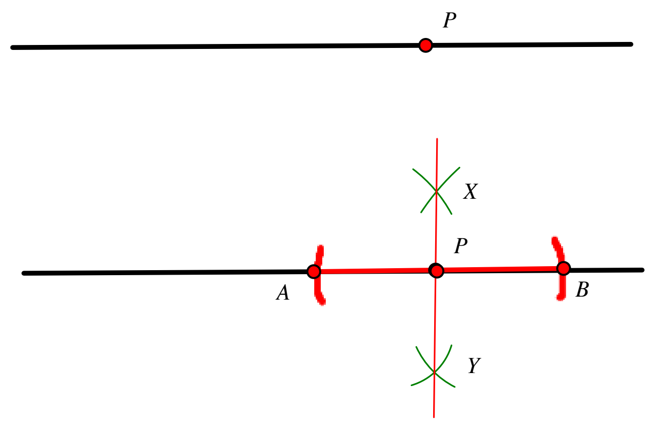
Steps

1. Using a compass with a radius greater than 1/2 $\overleftrightarrow{AB}$ construct two arcs (above & below line segment) having A as center and two arcs having B as center
2. Label the points of intersection X and Y
3. Draw line $\overleftrightarrow{XY}$

Justification

$\Delta AXY \cong \Delta BXY$ and $\Delta AXB \cong \Delta AYB$ by SSS, then $\angle XMB \cong \angle XMA$ and $\overline{AM} \cong \overline{MB}$. $\angle XMB$ and $\angle XMA$ are congruent adjacent angles, therefore they form perpendicular lines and since $\overline{AM} \cong \overline{MB}$, M is the midpoint so by definition a bisector, The next two constructions are based on the previous construction of a perpendicular bisector of a segment. What we do in the following two constructions is make a new line segment using the intersection of the arcs and the given line segment, then construct a perpendicular bisector as we just did.

Construction 3 Given a point on a line, construct a line perpendicular to the line through that point.



Steps

1. Using P as the center, construct two arcs, one on each side of P. Label the points of intersection as A and B.

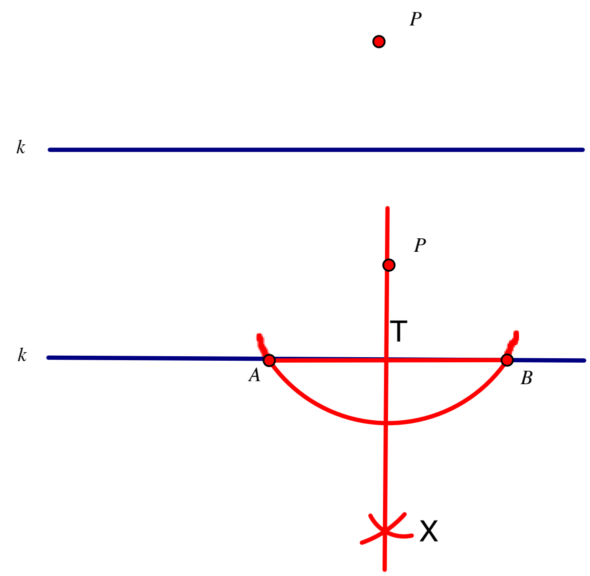
Now use the steps for constructing a ⊥ bisector

2. Using a compass with a radius greater than 1/2 $\overleftrightarrow{AB}$ construct two arcs (above & below line segment) having A as center and two arcs having B as center
3. Label the points of intersection X and Y
4. Draw line $\overleftrightarrow{XY}$

Justification

When $\overline{AX}$ and $\overline{BX}$ are drawn, then ΔAPX ≅ ΔBPX by SSS. By cpctc, ∠APX ≅ ∠BPX, the lines are perpendicular because thet it to form congruent adjacent angles.

Construction 4 Given a point not on the line, construct a perpendicular to the line through the point.



Steps

1. Using P as the center, construct an arc intersecting k at two points. Label the points A and B

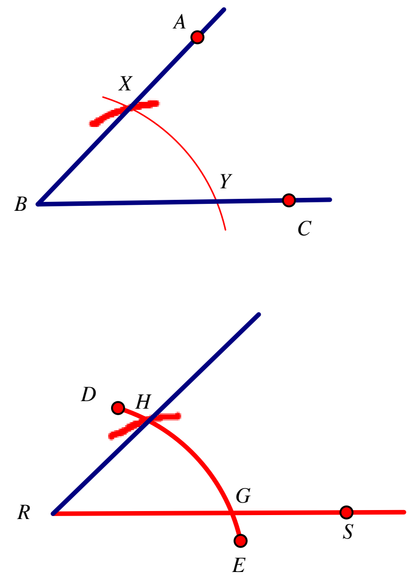
Now use the steps for constructing a ⊥ bisector

2. Using a compass with a radius greater than 1/2 $\overleftrightarrow{AB}$ construct two arcs (below line segment) having A as center and two arcs having B as center
3. Connect the intersection of the arcs X to P

Justification

Draw $\overline{PA}, \overline{PB}, \overline{XA}$, and $\overline{XB}$. $\overline{PA} \cong \overline{PB}$ and $\overline{XA} \cong \overline{XB}$, so ΔPAX ≅ ΔPBX by SSS. ∠PAT ≅ ∠PBT, base angles of isosceles triangle, ∠APT ≅ ∠BPT - cpctc. ΔAPT ≅ BPT by ASA. ∠PTA ≅ ∠PTB, cpctc which are congruent adjacent angles, therefore the lines are perpendicular

Construction 5 Given an angle, construct a congruent angle.



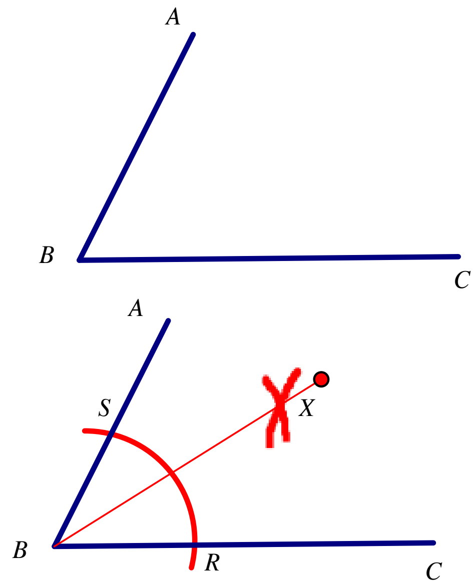
Steps

1. Draw ray RS
2. Using the compass with B as the center, make an arc intersecting the rays and label the intersections X and Y.
3. With the same radius, draw the same arc with R as the center, label the point of intersection G.
4. Using a compass, use Y as the center and measure of the radius YX, construct an arc intersecting ray BA
5. Using the compass with G as the center and radius still YX, construct an arc intersecting arc DE. Label the point of intersection H
6. Draw $\overrightarrow{RH}$

Justification

When $\overline{XY}$ and $\overline{HG}$ are drawn, $\Delta XBY \cong \Delta HRG$ by SSS, By cpctc, $\angle HRG \cong \angle ABC$

Construction 6 Given an angle, construct an angle bisector



Steps

1. Using a compass with B at the center, construct an arc intersecting both rays. Label the intersections as R and S.
2. With points R and S as centers, construct arcs intersecting in the interior of $\angle ABC$. Label the point of intersection as X.
3. Draw ray BX

Justification

When $\overline{RX}$ and $\overline{SX}$ are drawn, $\Delta BRX \cong \Delta BSX$ by SSS, therefore $\angle RBX \cong \angle SBX$ - cpctc and by definition $\overline{BX}$ is an angle bisector.