INTEGERS

Integers are positive and negative whole numbers, that is they are;
$\{\ldots -3,-2,-1,0,1,2,3\ldots\}$. The dots mean they continue in that pattern to both positive and negative infinity.

Before starting our study of integers, we will look at absolute values of numbers. The notation we use is parallel lines, with a number between them. That is, | 8 |, that is read *the absolute value of 8*.

In intermediate grades, the absolute value is often defined as the distance from zero – a positive number. Later, in algebra, it will be defined by a piecewise function.

So, whether the number inside the absolute value signs is positive or negative, when you take the absolute value, the answer will be positive.

Examples $|7| = 7$; $|12| = 12$; $|-3| = 3$ $|18| = 18$

Notice all those answers are positive

We can also perform operations within absolute values, using the Order of Operations, we do inside grouping symbols first.

Examples $|4 - 10| = |-6| = 6$; $|8 - 10 + 4| = |2| = 2$

Examples $-|8| = -8$ $-|-5| = -5$,

In the last two examples, we take the absolute value first, then take the opposite because the negative sign was outside the absolute value signs.

Back to integers. Like all number sets, integers were invented to describe things that happen in our environment.

When we work with signed numbers, we are often working with two different signs that look exactly alike. They are signs of value and signs of operation. A sign of value tells you if the number you are working with is greater than zero (positive) or less than zero (negative). Signs of operation tell you to add, subtract, multiply or divide.

Example

Sign of
Operation

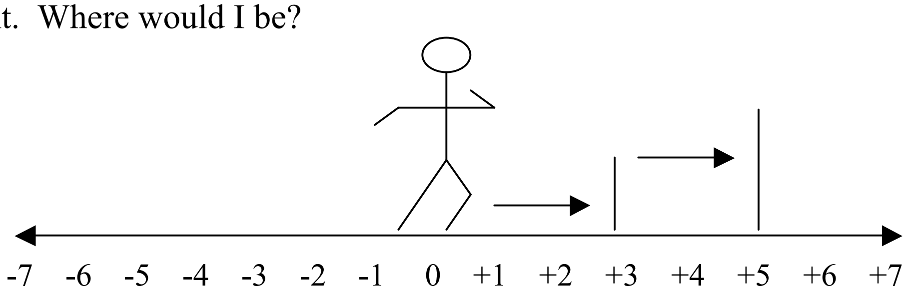
(+3) + (+2)

Sign of **Value** — Sign of **Value**

Notice that the signs of value and the sign of operation are identical.

Adding Integers

One way of explaining integers is with a number line. Let’s say I was standing on zero and I walked three spaces to the right, and then walked two more spaces to the right. Where would I be?



You’ve got it. You’d be 5 spaces to the right. Piece of cake, you’re thinking.

Now let’s see that same example incorporating mathematical notation. Let’s agree that walking to the right is positive, walking to the left will be negative. Easy enough.

Example 1 So 3 spaces to the right could be labeled, 3R or +3
2 spaces to the right could be labeled, 2R or + 2.
Now we said we’d end up 5 spaces to the right, 5R, or +5.

Now let’s define walking mathematically, we’ll agree to define that as addition.

Translating that problem of walking to the right, then walking further to the right mathematically, we have

$$3R \;+\; 2R \;=\; 5R$$
$$(+3) + (+2) = +5$$

The sign of operation tells you to walk, the sign of value tells you which direction.

Guess what happens next, that's right we make a rule that will allow us to do problems like this without drawing a picture.

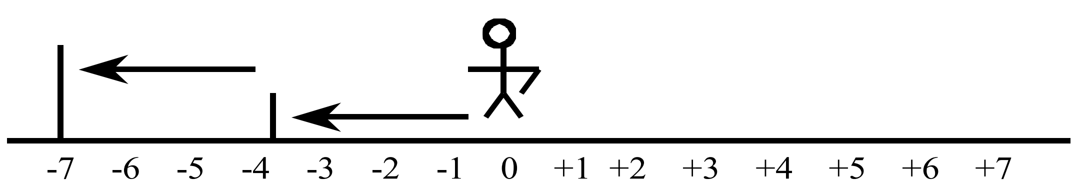
Rule 1. ***When adding two positive numbers, find the sum of their absolute values, the answer is positive.***

Example 2 $8 + (+9) = +17$

From the above example, notice that the 8 does not have a sign of value. We will now agree that when a number does not have a sign of value, it is understood to be positive.

Using those same agreements, walking is still defined by addition, going right is positive, going left is negative.

Example 3 Let's see what happens when we walk 4 steps to the left from zero, then 3 more steps to the left. Where will we end up? If you said we'd up 7 spaces to the left, we're in good shape.



Mathematically, we'd express that like this

$$4L \;+\; 3L \;=\; 7L$$
$$(-4) + (-3) = -7$$

This idea of walking around the number line is pretty cooool! Of course, now what we do is generalize this and make that into a rule so we don't have to always draw a picture.

Rule 2. ***When adding two negative numbers, find the sum of their absolute values, the answer is negative.***

Example 4 $(-5) + (-6) = -11$

Try a couple for yourself

1. $(-12) + (-8)$

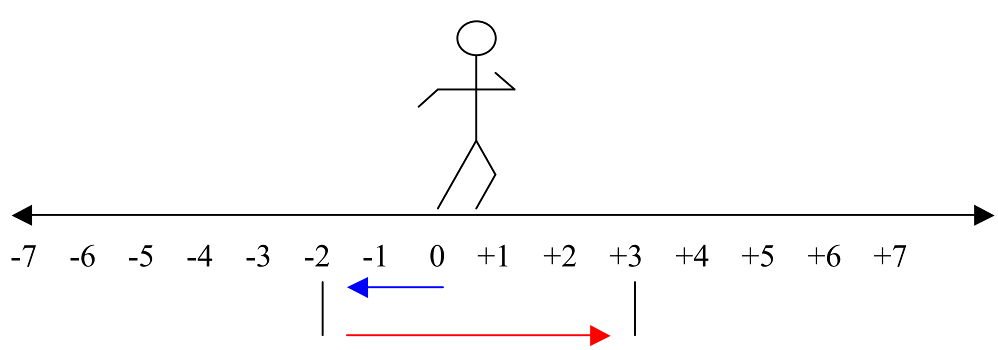
2. $(-9) + (-6)$

Some real good news, the rules we are developing also work for other number sets like fractions or decimals.

3. $\left(-\frac{1}{4}\right) + \left(-\frac{3}{5}\right)$

Yes, I know you love to walk, so let's take another walk on the number line. This time we are going to walk in different directions.

Example 5 Again, starting from zero, let's walk two steps to the left, then 5 steps to the right. Where will I end up?



Hopefully, by using the number line, you see that we'll end up 3 spaces to the right.

Mathematically, changing $2L + 5R = 3R$

to $(-2) + (+5) = +3$

Example 6 this time starting out walking 4 to the right, then going 9 to the left.

Again, using the number line, where should we end up? If you said 5 to the left, you are making my life too easy.

Mathematically, we'll change $4R + 9L = 5L$

to $(+4) + (-9) = -5$

The good news was the first 2 rules appealed to our common sense. The first rule, walking to the right, then walking further to the right, we end up on the right the total number of steps we took.

The second rule, walking to the left, then walking further to the left, we end up on the left so the same number of steps we have taken.

Now, starting off at zero and going in different directions, can you determine if you will end up on the right or left side of zero by just looking?

Example A As an example, 4R, then 6L, which side of zero will you end up on, the left or the right?

Example B 7R, then 2L, which side of zero will you end up on, the left or the right?

You might begin to notice after doing a few of these, you will end up on the side with the larger number.

Now that we know which side of zero we would be on, left or right, which translates to positive or negative, what coordinate will we be standing on?

How can you determine that without drawing a picture? Yeah, that's right, when adding (walking) a positive number and a negative number, you subtract.

That's the issue many have with math, the rules sometimes don't make sense unless you develop the ideas as we just did.

Now if we play with the two preceding examples long enough, we'll come up with a rule (shortcut) that will allow us to do these problems without drawing a number line.

Rule 3. ***When adding one positive and negative number, find the difference between their absolute values and use the sign of the Integer with the greater absolute value***

Example 7 $(-12)+(+8)=-4$

Example 8 $7+(-5)=+2$

Simplify

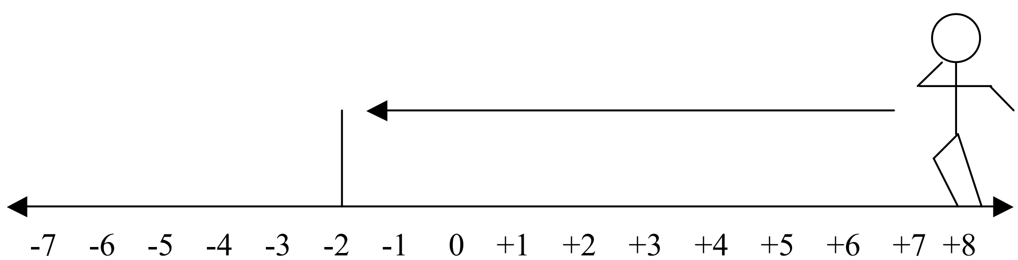
$(+7)+(+3)$	$(+8)+(+5)$	$(+2)+(+4)$
$(-7)+(-3)$	$(-8)+(-5)$	$(-2)+(-4)$
$(-7)+(+9)$	$(+8)+(-11)$	$(-5)+(+2)$
$+8+6$	$+7-5$	$-6-4$
$+8+(-2)+(-7)$	$-7-2-5$	$6-4-5$
$-8+3-5+4$	$5-3+6-7$	$7-8+5-4$

Subtracting Integers

Now that we have learned to add positive and negative numbers, I'll bet you know what's coming next. Yes indeed, its subtraction.

Remember, we defined addition as walking the number line from zero. Well, we are going to define subtraction as finding the distance between two locations on the number line and the way you have to travel to get to the first address. Going right is still positive, going left is negative.

EXAMPLE 1 Let's say I want to know how far you must travel if you were standing on $(+8)$ and you wanted to go to the location marked as (-2).



Looking at the number line, you are standing on +8. Which direction will you have to go to get to –2? If you said left, that's good news and mathematically it translates to a negative number. Now, how far away from –2 are we? Using the number line we see we would have to walk 10 spaces to the left or –10.

Mathematically, that would look like this

$(-2)-(+8) \rightarrow$ walking 10 spaces to the left, (-10)

Example 2 This time you are standing on –5 and want to go to –1. Draw a diagram. How far and what direction would you have to move? 4 spaces to the right would be the correct answer.

Mathematically, we have $(-1)-(-5)=+4$

Rule 4. **When subtracting signed numbers, change the sign of the subtrahend (second number) and add using rule 1, 2 or 3, whichever applies.**

Example 3 $6-(+13)$

$=6+(-13)$ change sign & add

$=-7$

Simplify

1. $8-(+2)$	2. $(-3)-(-4)$	3. $(2)-(+5)$	4. $(-5)-(3)$
5. $(-10)-(-2)$	6. $(9)-(5)$	7. $9-5$	8. $(+8)-(-4)$
9. $10-6$	10. $6-10$	11. $6-(-10)$	12. $8-(-4)$

Up to this point, we have developed rules for adding and subtracting signed numbers. Those rules came from observations that we made that allowed us to do problems without drawing pictures or using manipulatives. Using pictures and/or manipulatives is important so the kids have an understanding of the concepts being introduced. The rules, standing alone, don't make sense, but allow us to compute much faster.

Multiplying/Dividing Integers

Here's a new agreement for multiplication and division models. Traveling east (right) is positive, traveling west (left) is negative. Sounds familiar, doesn't it? Now, future time will be defined as positive, past time as a negative number. And you'll be at your lovely home which will be designated as zero.

ILLUSTRATION 1

If you were at home (at zero) and a plane heading east at 400 mph passed directly overhead, where will it be in 2 hours?
If you don't know what distance equals rate x time, now you do.

Translating English to math, going 400 mph East is +400, and since we are looking at future time, 2 hours will be +2.

Now, standing at zero and the plane heading east for 2 hours at 400 mph, it will be 800 miles east in 2 hours.

Mathematically, we have:

400 mph east x 2 hrs future = 80 miles east
(+400) x (+2) = +800

Makes sense.

<u>ILLUSTRATION 2</u>

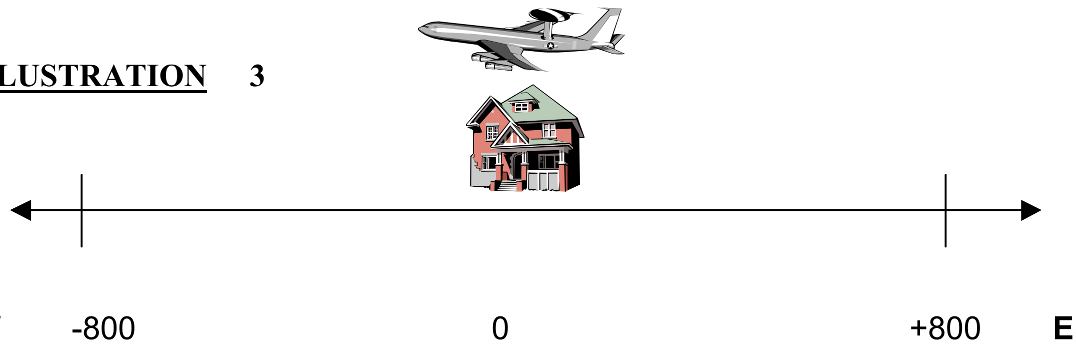
The plane is directly over your house heading east at 400 mph. Where was it 2 hours ago? Going east at 400 mph is written as +400, we are using past time, so that's –2.
He'd be 800 miles west.

Translating English to math we have

400 mph east x 2 hrs past = 800 miles west
(+400) x (- 2) = -800

Oh, yes, this is a piece of cake. Don't you just love math?

<u>ILLUSTRATION</u>   3



The plane is heading west at 400 mph, where will it be in 2 hours if it is directly over your head now?

He'd be 800 miles west.

Translating English to math we have

400 mph west x 2 hours future = 800 miles west
(-400)   x   (+2)   =   -800

<u>ILLUSTRATION</u>   4   Using the last illustration

The plane is heading west at 400 mph, where was it 2 hours ago if it is directly over your home now?

Translating English to math we have

400 mph west x 2 hours past = 800 miles east
(-400)   x   (-2)   =   +800

1. $(+400)\text{ x }(+2) = +800$

2. $(+400)\text{ x }(-2) = -800$

3. $(-400)\text{ x }(+2) = -800$

4. $(-400)\text{ x }(-2) = +800$

That might lead us to believe multiplying numbers with like signs results in a positive answer, while a negative answer appears when you multiply numbers with unlike signs.

Those observations leads us to a couple more rules.

Rule 5. ***When multiplying or dividing numbers with the same sign, the answer is positive.***

***Example*:** $(+5)\text{ x }(+4) = +20$ $(-6)\text{ x }(-7) = +42$

Rule 6*.** ***When multiplying or dividing numbers with different signs, the answer is negative.

***Example*:** $(-5)\text{ x }(+8) = -40$ $(+9)\text{ x }(-3) = -27$

Don't you just love it when things work out?

Multiply or Divide

1)

$-7(+6)$ $\frac{-48}{8}$ $\frac{+54}{-9}$

2)

$-6(+10)$ $-8(3)$ $\frac{32}{-4}$

3)

$\frac{-8}{+2}$ $\qquad -6(3) \qquad 6(-12)$

4)

$8(-7) \qquad -6(3) \qquad 8(-5)$

5)

$\frac{-40}{4} \qquad \frac{+40}{-8} \qquad -5(9)$

6)

$\frac{-60}{+5} \qquad 4(-9) \qquad -8(+6)$

7)

$-7(+9) \qquad \frac{-63}{+7} \qquad -3(12)$

Let's look at the rules we developed by looking at those patterns on the number line. We have three rules for addition, one for subtraction and two for multiplication/division.

Addition

Rule 1: Two positive numbers, take the sum of their absolute values, the answer is positive.

Rule 2: Two negative numbers, take the sum of their absolute values, the answer is negative.

Rule 3: One positive, one negative, take the difference between their absolute values, use the sign of the number with the greater absolute value.

Subtraction

Rule 4: Change the sign of the subtrahend and add using rule 1, 2, or 3,. whichever applies.

Multiplication/Division

Rule 5: Two numbers with the same sign are positive.
Rule 6: Two numbers with different signs are negative.

When working with these rules, like all rules in math, work with binary operations. That is, the rules work for only two numbers at a time. In other words, if I asked you to simplify $(-3)(-4)(-5)$, the answer would be – 60.

The reason is $(-3)(-4)=+12$, then a $(+12)(-5)=-60$

In math, when we have two parentheses coming together without a sign of operation, it is understood to be a multiplication problem. We leave out the "X" sign because in algebra it might be confused with the variable x.

Stay with me on this, often times, for the sake of convenience, we also leave out the "+" sign when adding integers.

Example: $(+8)+(+5)$ can be written without the sign of operation $\rightarrow$ $+8+5$, it still equals $+13$ or $8+5=13$.

Example: $(-8)+(-5)$ can be written without the sign of operation $\rightarrow$ $8-5$, it still equals –13 or –8 – 5 = –13

Example: $(-8)+(+5)$, can be written without the sign of operation $\rightarrow$ $8+5$, it still equals. –3 or –8 + –5 = –3

For ease, we have eliminated the "X" sign for multiplication and the "+" sign for addition. That can be confusing.

Now the question is: How do I know what operation to use if we eliminate the signs of operation?

The answer:: If you have two parentheses coming together as we do here, $(-5)(+3)$, you need to recognize that as a multiplication problem.

A subtraction problem will always have an additional sign, the sign of operation. For example, $12-(-5)$, you need to recognize the negative sign inside the parentheses is a sign of value, the extra sign outside the parentheses is a sign of operation. It tells you to subtract.

Now, if a problem does not have two parentheses coming together and it does not have an extra sign of operation, then it's an addition problem. For example, $8-4, -12+5$ and $9-12$ are all samples of addition problems. Naturally, you would have to use the rule that applies.

Simplify and name the appropriate operation

1. $(-4)+(-9)$
2. $(-5)(6)$
3. $-7-(+3)$
4. $-10\ -4$

Answers: 1. add, –13, 2. mult, – –30 3. sub, –10

4. add –14.

Simplify. First determine if the problem is $+, -, X$, or $\div$, then write the rule that applies to the problem.

1. $(+8)+(-3)$

2. $(-5)+(-4)$

3. $(-5)(+6)$

4. $-8-5$

5. $24 \div (-6)$

6. $5-(-3)$

7. $(-5)-(+8)$

8. $-3+9$

9. $(-6)(-5)$

10. $(-5)(-4)(-2)$

11. $-5-4-2$

12. $(-4)^3$

13. $-2+8-10+4$

14. -4^2

Problem Solving

1) In a certain game one couple made a score of 320, while another couple made a score of –30, what was the difference in scores?

2) At noon the thermometer stood at $+12^0$, at 5 pm it was –8. How many degrees had the temperature fallen?

3) The height of Mt. Everest is 29,000, the greatest known depth of the ocean is 32,000 ft. Find their difference.

4) On 6 examination questions, Bob received the follow deductions for errors $-4, -2, 0, -5, -0, -8$. What was his mark based on 100 pts?

5) A team lost 4 yards on the 1st play and gained 12 yards on the 2nd play. What was the net result?

6) The average temperature of Mars is –60. The average temperature of Venus is 68^0. What is the difference in temperature?

7) How long did a man live who was born in 73 B.C. and died in 25 B.C.?

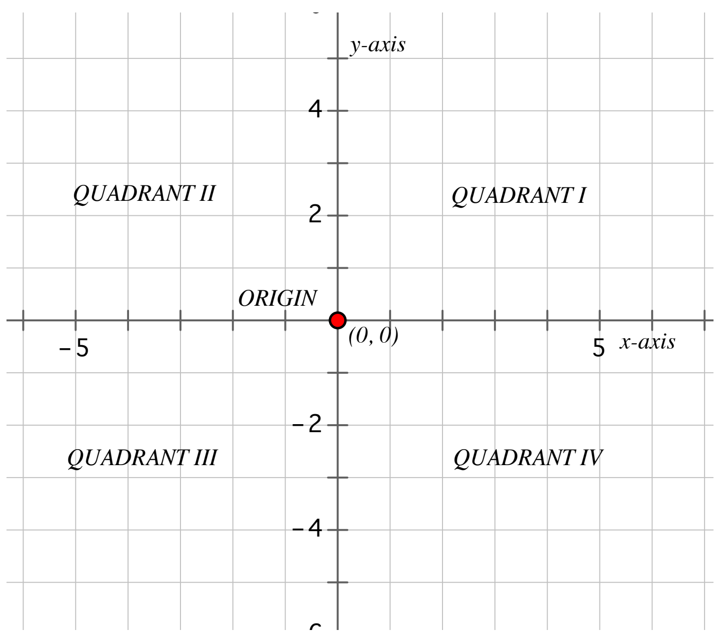
8) Roberto traveled from an altitude of 113 ft. below sea level to an altitude of 200 ft below sea level. What was the change in altitude?

Rectangular Coordinate System

Cartesian Coordinate System

The Rectangular Coordinate System, also called the Cartesian Coordinate System because it was named after DesCartes, divides a plane (flat surface) into four sections called QUADRANTS. The quadrants are numbered counterclockwise from 1 to 4, written as Roman Numerals; I, II, III, IV.

The dividing lines are called axes. The horizontal axis is called the x-axis, the vertical axis is called the y-axis. The axes are just number lines drawn horizontally and vertically. Their point of intersection is called the ORIGIN (beginning point).



To find points on a plane we use ORDERED PAIRS. Ordered pairs are made up of 2 coordinates, an x and a y. They are called ordered because the x-coordinate is always listed first, the y-coordinate is written second. Ordered pairs are written in parentheses, (x, y). So an ordered pair (5, 2) means you go over 5 on the x-axis, then up 2 on the y-axis.

An easy way to remember how to write ordered pairs is to recognize they are in alphabetical order. The x-axis corresponds to horizontal axis, the y-coordinate corresponds to the vertical axis. Notice horizontal and vertical are in alphabetical order too. (x, y) --≥ (h, v)

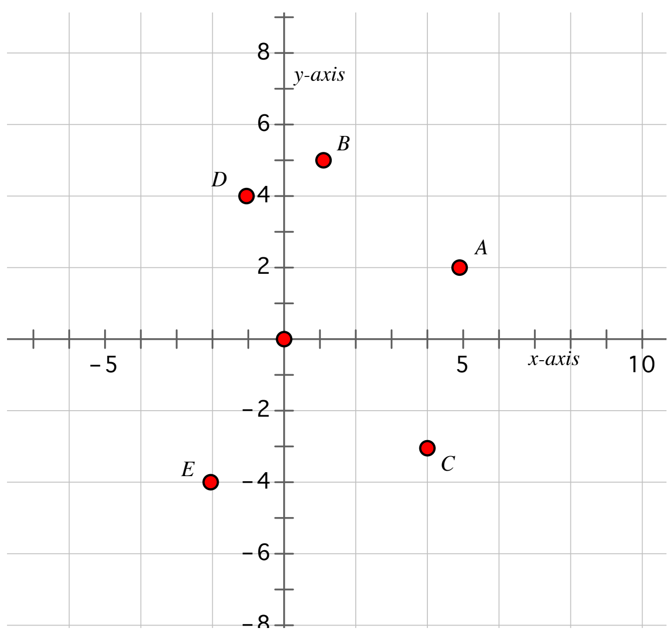
Ordered pairs represent points on a plane, a location. Maps are used in a similar manner using an alpha-numeric graph. That is a graph with one axis being made up of letters, the other axis made up of numbers.

We are studying math, so we use just numbers. But to be able to label or find points in a plane, you must remember ordered pairs are called ordered for a reason, the coordinates are written in order x, followed by y – (x, y); horizontal followed by vertical. The points in the plane are labeled using capital letters.

Simply stated, (5, 2) is a different location (point) than (2, 5). Remember alphabetical order with this means the (x, y) corresponds to the (horizontal, vertical axes).

Plot (graph) the following points on the Rectangular Coordinate System.

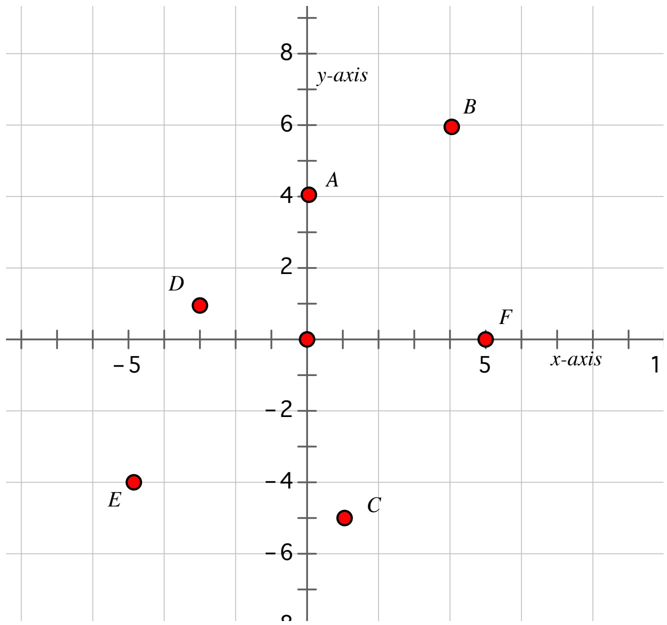
A(5, 2)   B(1, 5)   C(4, – 3)   D(–1, 4)   E(–2, –4)



What point is located in the second quadrant?
What points is in Quadrant III?
What point is located in Quadrant IV?

You should also be able to look at points on the Rectangular Coordinate System and label the ordered pairs associated with each point.

Label the ordered pairs associated with points A, B, C, D, E, and F



The ordered associated with each point are:

A(0, 4)
B(4, 6)
C(1, – 5)
D(–3, 1)
E(–5, –4)
F(5, 0)

What ordered pair represents the origin?
What quadrant is E located?

Integers & Name
Rectangular Coordinate System Date

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1. ***Define "integers"

2. ***How is a sign of value different than a sign of operation?

3. ***Write the six rules for operating with signed numbers.

4. ***Define |x|

5. ***Define origin

6. **Identify the operation of each of the following problems; add, subtract, multiply, or divide.

    a. (+8) + (–2) ________________

    b. (–5) – (+3) ________________

    c. (5)(–4) ________________

    d. – 6 – 7 ________________

7. **Evaluate

a. $|5| =$

b. $|-7| =$

c. $|-5 + 3| =$

8. **On January 8th, the temperature in Miami was 67° while the temperature in Boston was four below zero. What was the difference in temperatures?

9. **True or False

a. $-5 > -2$

10. **Use the number line model to illustrate $(+3) + (-4)$

11. **Bob had $250 in his savings account on July 1st. If he wrote checks for $110, $165, and $85 during that month and made 2 deposits, one for $100, the second for $75, what is his balance at the end of the month?

12 **Evaluate

a. -3^2

b. $(-5)^2$

Perform the indicated operations

13 ** $(-5) + (-6)$

14. ** $(+6) - (-2)$

15. ** $(-6)(-5)$

16. ** $(-3)(-2)(-5)$

17. ** $-3 - 2 - 5$

18. ** $+8 - 10 + 3 - 6$

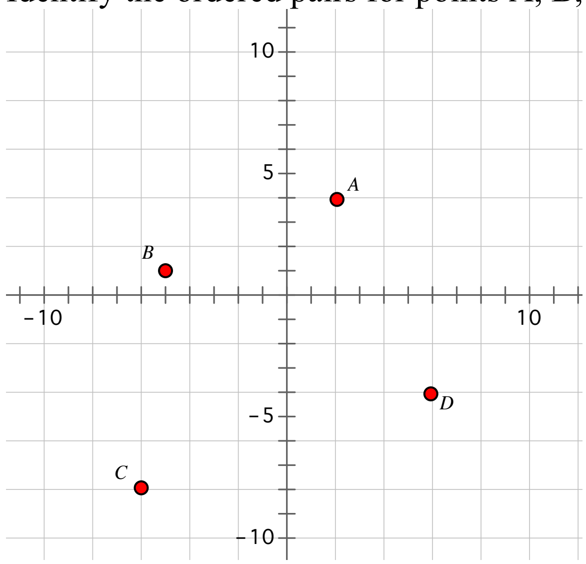
19. ** $6 + 5(-4) - 2(3)$

20. ** $(-5)^3$

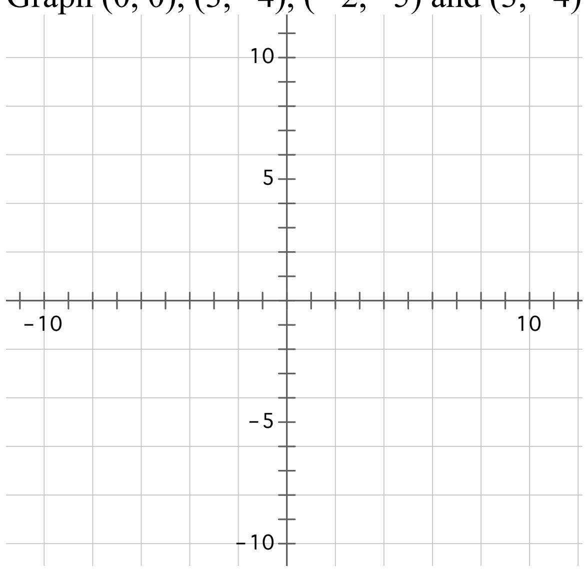
21. ** $-10 - 2(-3) + 6(-4)$

22. **Draw a block model to show that (+7) – (–3) = +10

23.** Identify the ordered pairs for points A, B, C, and D



24. ** Graph (0, 0), (3, –4), (– 2, –5) and (3, –4)



25. ***Provide Parent contact information, phone, cell or email. (CHP)