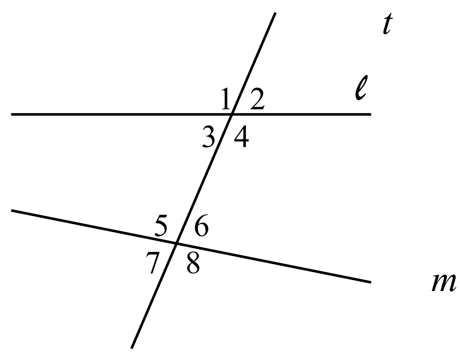
Chapter 4

Angles formed by 2 Lines being cut by a Transversal

Now we are going to name angle pairs that are formed by two lines being intersected by another line called a transversal.



If I asked you to look at the figure above and find two angles that are on the same side of the transversal, one an interior angle (between the lines), the other an exterior angle that were not adjacent, could you do it?

$\angle 2$ and $\angle 4$ are on the same side of the transversal, one interior, the other is exterior – whoops, they are adjacent. How about $\angle 2$ and $\angle 6$?

Those two angles fit those conditions. We call those angles **corresponding** angles.

Corresponding Angles – two angles on the same side of the transversal, one interior, one exterior that are not adjacent.

Can you name any other pairs of corresponding angles?

If you said $\angle 4$ and $\angle 8$, or $\angle 1$ and $\angle 5$, or $\angle 3$ and $\angle 7$, you'd be right.

Alternate Interior – two angles are on opposite sides of the transversal, both interior and not adjacent. $\angle 4$ and $\angle 5$ are a pair of alternate interior angles. Name another pair.

Based on the definition of alternate interior angles, how might you define **alternate exterior** angles?

Alternate Exterior – two angles on opposite sides of the transversal, both exterior and non adjacent.
How about same side interior angles? Can you describe them? One pair of same side interior angles is $\angle 3$ and $\angle 5$, can you name another pair?

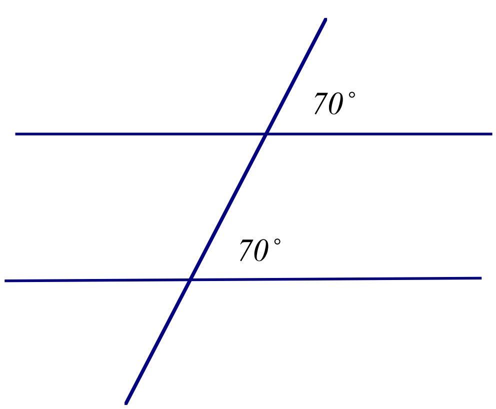
Same Side Interior - two angles that are on the same side of the transversal, non-adjacent, both interior angles. $\angle 3$ and $\angle 5$ are one pair and $\angle 4$ and $\angle 6$ is another pair of same side interior angles.

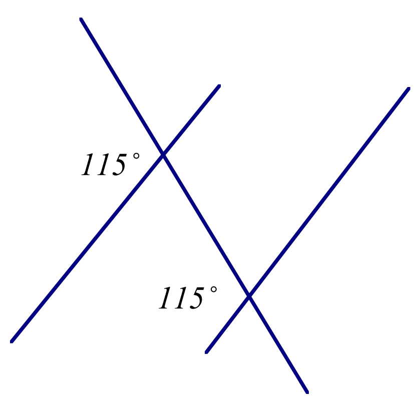
Parallel lines are lines that are in the same plane and have no points in common.

Skew lines are lines that are not parallel and do not intersect, they do not lie in the same plane.

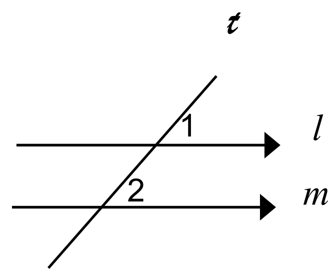
Angles Pairs formed by Parallel lines

Something interesting occurs if the two lines being cut by the transversal happen to be **parallel**. It turns out that every time I measure the corresponding angles, they turn out to be equal. You might use a protractor to measure the corresponding angles below. Since that seems to be true all the time and we can't prove it, we'll write it as an axiom – a statement we believe without proof.





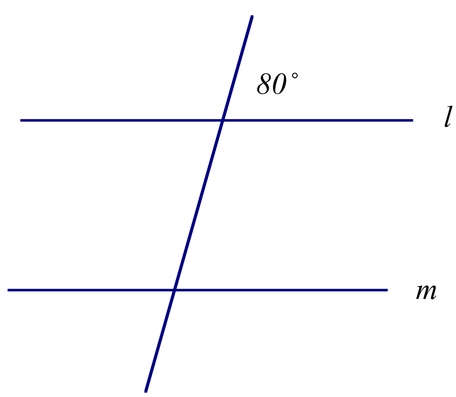
Postulate If two **parallel** lines are cut by a transversal, the corresponding angles are congruent.

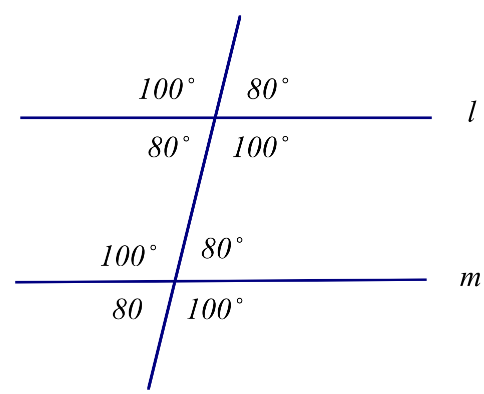


$\angle 1 \cong \angle 2$

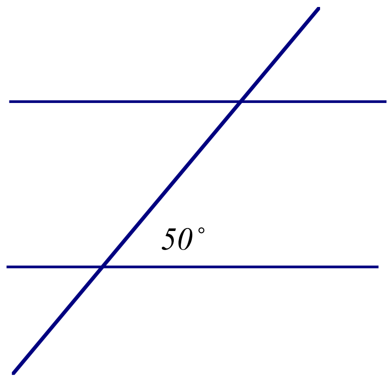
Now let's take this information and put it together and see what we can come up with. Knowing the corresponding angles are congruent, we can use our knowledge of vertical angles being congruent and our knowledge of angles whose exteriors sides lie in a line to find the measure of other angles.

If $l \parallel m$ and we are given the 80° angle in the left diagram, then we can fill in all the other 80° angles in the right diagram using corresponding angles are congruent and vertical angles are congruent. And two angles whose exteriors sides lie in a line are supplementary.

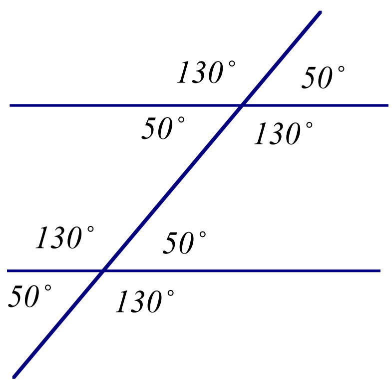




Example 1 Given the lines are parallel and the angle measures 50˚, find the measure of all the other angles.



Begin by filling in the corresponding angles, followed by the vertical angles and finally the angles formed by their exterior sides being in a line – they are supplementary.



NOTICE

The alternate interior angles and the alternate exterior angles have the same measure. Also notice the same side interior angles are supplementary in these examples.

In the last couple of examples we saw the relationships between different angles formed by parallel lines being cut by a transversal. Let's formalize that information.

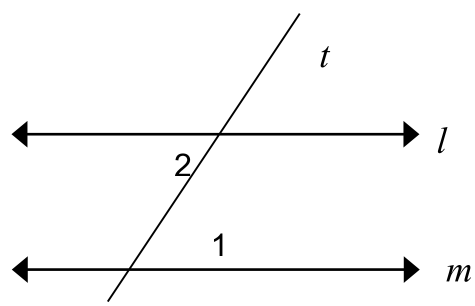
Proofs: Alternate Interior Angles

Let's see, we've already learned vertical angles are congruent and corresponding angles are congruent, if they are formed by parallel lines. Using this information we can go on to prove alternate interior angles are also congruent, if they are formed by parallel lines.

What we need to remember is drawing the picture will be extremely helpful to us in the body of the proof. Let's start.

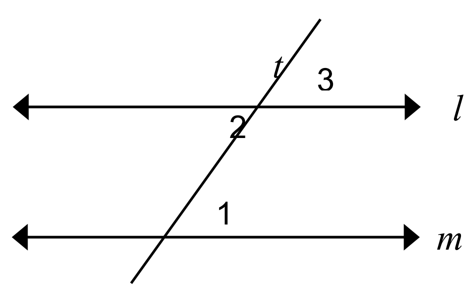
Theorem
If 2 parallel lines are cut by a transversal, the alternate interior angles are congruent.

By drawing the picture of parallel lines being cut by a transversal, we'll label the alternate interior angles.



The question is, how do we go about proving $\angle 1 \cong \angle 2$?

Now this is important. We need to list on the picture things we know about parallel lines. Well, we just learned that corresponding angles are congruent when they are formed by parallel lines.
Let's use that information and label an angle in our picture so we have a pair of corresponding angles. So, we will put $\angle 3$ into the problem.



Since the lines are parallel, $\angle 1$ and $\angle 3$ are congruent. Oh wow, $\angle 2$ and $\angle 3$ are vertical angles! We have studied a theorem that states all vertical angles are congruent.

That means $\angle 1 \cong \angle 3$ because they are corresponding angles and $\angle 2 \cong \angle 3$ are congruent because they are vertical angles, that means $\angle 1$ must be congruent to $\angle$ 3.

$$\angle 1 \cong \angle 3$$
$$\angle 2 \cong \angle 3$$

That would suggest that $\angle 1 \cong \angle 2$ by substitution.

Now we have to write that in two columns, the statements on the left side, the reasons to back up those statements on the right side.

Let's use the picture and what we labeled in the picture and start with what has been given to us, line l is parallel to m.

	Statements	Reasons
1.	$l \parallel m$ $\angle 1$ and $\angle 2$ are alt int$\angle$'s	Given
2.	$\angle 1$ and $\angle 3$ are corr. $\angle$s	Def of corr. $\angle$s
3.	$\angle 1 \cong \angle 3$	Two ll lines, cut by t, corr.$\angle$'s $\cong$
4.	$\angle 3 \cong \angle 2$	Vert$\angle$'s
5.	$\angle 1 \cong \angle 2$	Transitive Prop

Is there a trick to this? Not at all. Draw your picture, label what's given to you, then fill in more information based on your knowledge. Start your proof with what is given, the last step will always be your conclusion.

I could have done this proof differently, I could have added more steps to further clarification if needed. Just like a proof in the last chapter, I added more information than was given to me. We were given alternate interior angles. I added $\angle 3$ to the diagram so I could use the corresponding angles theorem. So, to get $\angle 3$ into the problem, I could have added the step, $\angle 3 \cong \angle 3$ by the Reflexive Property

Now, we have proved vertical angles are congruent, we accepted corresponding angles formed by parallel lines are congruent, and we just proved alternate interior angles are congruent. Could you prove alternate exterior angles are congruent? Try it. Write the theorem, draw the picture, label the alternate exterior angles, add more information to your picture based on the geometry you know, identify what has been given to you and what you have to prove.

Let's write those as theorems.

Theorem If two parallel lines are cut by a transversal, the alternate exterior angles are congruent.

Theorem If two parallel lines are cut by a transversal, the same side interior angles are supplementary.

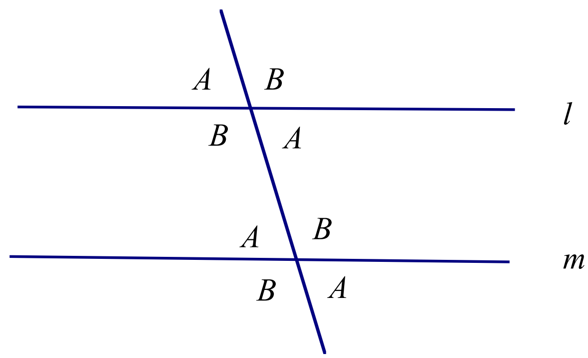
Theorem If a transversal is perpendicular to one of two parallel lines, it is perpendicular to the other line also.

Summarizing, we have:
If ***two parallel lines*** are cut by a transversal, then the
- corresponding $\angle$'s are $\cong$
- alt. int. $\angle$'s are $\cong$
- alt. ext. $\angle$'s are $\cong$.
- same side int. $\angle$'s are supplementary.

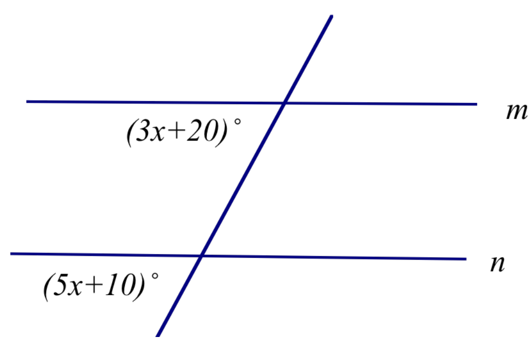
ABBA

Another way to remember that postulate and those three theorems is by remembering ABBA. All the angles marked A have the same measure, all the angles marked B have the same measure, and A+B = 180 if $l \parallel m$.

Look at the diagram below.



Example 2 Given that $m \parallel n$ and the information in the diagram, find the value of x.

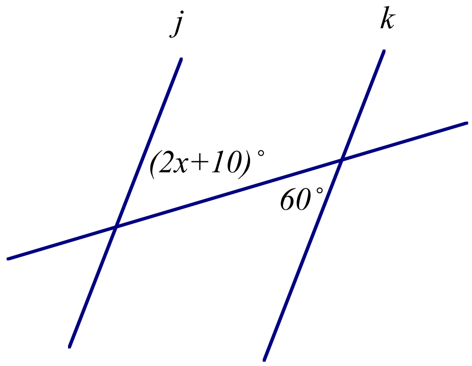


Since the two lines are parallel, we know the corresponding angles are congruent, so we know they have the same measure.

$$5x + 10 = 3x + 20$$
$$2x = 10$$
$$x = 5$$

So, to answer the question, x = 5. If I wanted to know the measure of each angle, I would substitute 5 into each expression. Each angle measures 35˚

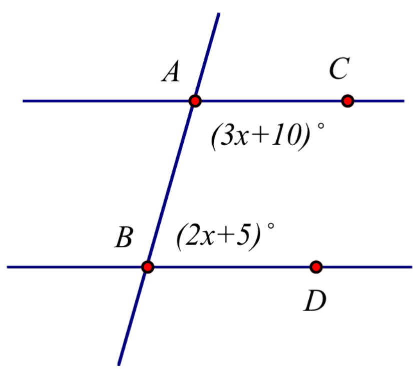
Example 3 Given j || k and the information in the diagram, find the value of x.



In this case, we have two parallel lines being cut by a transversal, we know the alternate interior angles are congruent. We can set those two angles equal and solve the resulting equation.

$$2x + 10 = 60$$
$$2x = 50$$
$$x = 25$$

Example 4 Given l || m and the information in the diagram, find the measure of $\angle CAB$ and $\angle DBA$.



Same Side Interior angles are supplementary

Since $\angle CAB$ and $\angle DBA$ are same side interior angles, we know their sum is 180°

$$(3x + 10) + (2x + 5) = 180$$
$$5x + 15 = 180$$
$$5x = 165$$
$$x = 33$$

Remember, the question was NOT to find the value of x, we were to find the measure of each angle, $\angle CAB$ and $\angle DBA$. Substituting 33 into those expressions, we have

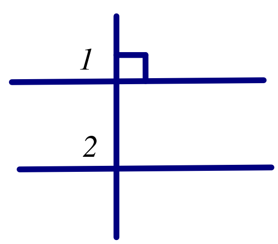
$$\angle CAB = 109^\circ \text{ and } \angle DBA = 71^\circ$$

Up to this point, these relationships were built on the axiom/postulate that if lines are parallel and cut by a transversal, corresponding angles are congruent. That axiom and the subsequent theorems (alt int $\angle$s, alt ext $\angle$s, same side int $\angle$s) are "important" for you to know.

Lets look at another theorem that follows directly from these theorems.

Theorem If a transversal is perpendicular to one of 2 parallel lines, it is perpendicular to the other line also.

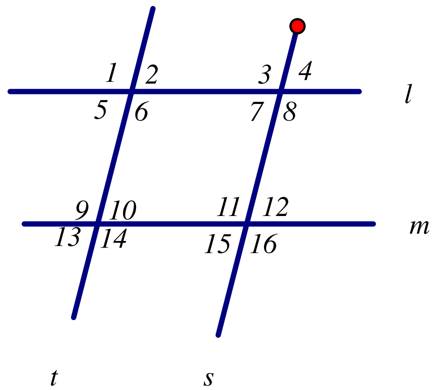
If you draw the diagram, you can quickly see this theorem that since the lines are parallel, the corresponding angles would be equal. Since perpendicular lines form right angles, a 90° angle would be formed. That results in the corresponding angle also being 90°, which means they are both right angles by definition of a right angle and thus the lines are perpendicular by the definition of perpendicular lines.



As always in math, I can not make problems more difficult, only longer.

Try these problems – Use ABBA

1. Given $\angle 2 = 70^\circ$, l || m and t || s find the measure of the following angles.



Proving that Lines are Parallel

In chapter 1, we stated the converse of a theorem is not necessarily true, but could be true. As it turns out, many of the theorems and postulates dealing with parallel lines, the converses are also true. We will start with a postulate. This is the converse of the postulate that read; if two parallel lines are cut by a transversal, the corresponding angles are congruent. Now what I will accept as true is if the corresponding angles are congruent, the lines must be parallel.

Postulate If to lines are cut by a transversal so that the corresponding angles are congruent, the lines are parallel.

The converse of a conditional is not always true, so this development is fortunate. As it turns out, the other three theorems we just studied about having parallel lines converses' are also true. That leads to the following three theorems.

Theorem If two lines are cut by a transversal so that the alternate interior angles are congruent, then the lines are parallel.

Theorem If two lines are cut by a transversal so that the alternate exterior angles are congruent, then the lines are parallel.

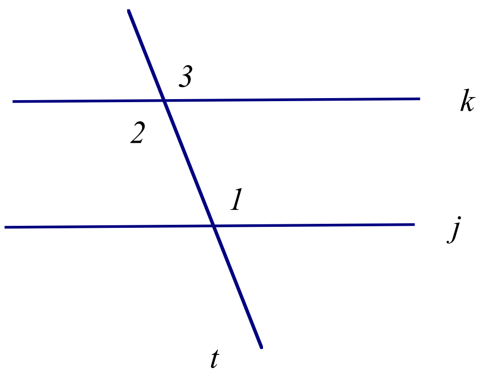
Theorem If two lines are cut by a transversal so that the same side interior angles are supplementary, then the lines are parallel.

Now I have four ways to show lines are parallel; corresponding $\angle$'s congruent, alternate interior $\angle$'s congruent, alternate exterior $\angle$'s congruent, same side interior $\angle$'s supplementary.

You should know those theorems because if we know lines are parallel, that will give us equations.

Let's look at a proof of the theorem concerning alternate interior angles. The other proofs will be very similar.

Example 1 **Prove the Theorem** If two lines are cut by a transversal so that the alternate interior angles are congruent, then the lines are parallel.



Given: $\angle 1 \cong \angle 2$
k and j cut by t

Prove: $j \parallel k$

	STATEMENTS	REASONS
1.	k and j cut by t $\angle 1 \cong \angle 2$	Given
2.	$\angle 2 \cong \angle 3$	Vertical angles $\cong$
3.	$\angle 1 \cong \angle 3$	Transitive Property
4.	$j \parallel k$	2 lines cut by t & corr. $\angle$'s are $\cong$

Remember, your proof does not have to look exactly like mine. For instance, after step 1, I could have put in another step $\angle 2$ and $\angle 3$ are vertical angles. The reason – the definition of vertical angles. The reason for step 3 could have been substitution. My point is a proof is just an argument (deductive reasoning) in which the conclusion follows from the argument.

In a nutshell, we have learned the postulates and theorems dealing with parallel lines are bi-conditional. That is, the converses are also true if the statements were true.

Summarizing and writing those as bi-conditionals, we have:

Two lines cut by a transversal are parallel if and only if the corresponding angles are congruent.

Two lines cut by a transversal are parallel if and only if the alternate interior angles are congruent.

Two lines cut by a transversal are parallel if and only if the alternate exterior angles are congruent.

Two lines cut by a transversal ate parallel if and only if the same side interior angles are supplementary.

Theorem **In a plane, if two lines are perpendicular to a third line, they are parallel to each other.**

Theorem **Through a point not on a line, exactly one parallel can be drawn to the line.**

Theorem **Through a point not on a line, exactly one perpendicular can be drawn to the line.**

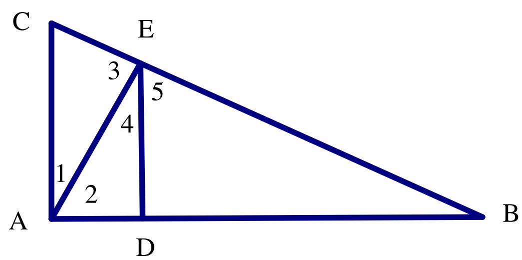
Since the last two theorems indicate exactly one parallel or perpendicular can be drawn, these theorems would be proved by contradiction – indirect proofs.

As we have mentioned before, we can't make these problems more difficult – only longer. There are times the problem might look more complicated than problems done earlier, but if we apply our knowledge, it just shakes out nicely.

To address problems that seem to have additional information, remember your definitions, postulates and theorems and mark those on your diagrams.

Let's look at an example that is showing more information that is needed.

Example 2 If $\overline{AC} \perp \overline{AB}$ *and* $\overline{DE} \perp \overline{AB}$, what angles could you prove congruent?



Your given some information in the problem, apply what that information means in terms of the geometry you have learned.

$\overline{AC} \perp \overline{AB}$ *and* $\overline{DE} \perp \overline{AB}$, you have 2 lines perpendicular to the same line, that means $\overline{AC} \parallel \overline{DE}$.

If $\overline{AC} \parallel \overline{DE}$, then we have corresponding $\angle$s congruent, alternate interior $\angle$s congruent, etc. $\angle 1$ and $\angle 4$ are alternate interior angles formed by || lines, therefore they are congruent.

So, $\angle 1 \cong \angle 4$