Chapter 5

Indirect Proofs

There are times when trying to prove a theorem directly is either very difficult or impossible. When that occurs, we rely on our logic, our everyday experiences, to solve a problem. One such method is known as an *indirect proof* or a *Proof by Contraction*.

The logic of an indirect proof is based on the contra-positive of a conditional. You might remember a conditional such as $A \rightarrow B$. We said earlier that the converse of that conditional is $B \rightarrow A$. We also mentioned that if the original conditional was true, that did not mean the converse was true.

However, we learned a conditional and its contra-positive are logically equivalent statements because they have the same truth-values. That is, if the conditional is true, then the contra-positive is also true. The contra-positive of the conditional $A \rightarrow B$ is $B' \rightarrow A'$.

It is very important to know – the contra-positive assumes the **negation** of the conclusion.

To begin an indirect proof using the contra-positive, you suppose the negation of what you would like to prove is true. Then you reason logically from that assumption until you encounter a contradiction of a known fact – something given to you to as true. You then point out that everything followed from that assumption and the only place you could have gone wrong was the assumption in your opening statement is false and that it follows that the desired conclusion must be true. The deductive reasoning leads to a contradiction where two statements cannot both be true.

Example 1 Given: $\angle A$ is not congruent to $\angle B$

Prove: $\angle A$ and $\angle B$ are **not** both right angles

Assuming the **negation** of the conclusion, we have $\angle A$ and $\angle B$ are right angles.

By theorem, we know all right angles are congruent, that is

$\angle A \cong \angle B$ #

That contradicts the given fact that $\angle A$ is **not** congruent to $\angle B$. That means our assumption must be false and it follows that our desired conclusion is true. That is, since $\angle A$ and $\angle B$ are both right angles is false, then it follows that $\angle A$ and $\angle B$ are not both right angles.

Many people use the # to show where the contradiction appears in an indirect proof.

The success of an indirect proof depends upon finding a contradiction of a known fact. The known fact may be part of the hypothesis (given) of the statement to be proved, a postulate or theorem. Indirect proofs are also called **"Proofs by Contradiction."**

Indirect proofs are especially useful when you want to prove there is exactly one of something. You assume there is more then one, then find a contradiction.

While indirect proofs are often done in paragraph form, we can write them in a T-Proof. Here are your steps:

1. Assume the negation of what you wish to prove
2. Write down what is given to you as true
3. Reason logically until you encounter a contradiction of a known fact
4. Point out assumption in your first step must be false and it follows that the desired conclusion is true.

Let's redo that previous example in T-Proof format. You have a choice, proving theorems indirectly in paragraph form or in a T-Proof. Choose whichever makes you more comfortable.

Example 2 Given: $\angle A$ is not congruent to $\angle B$

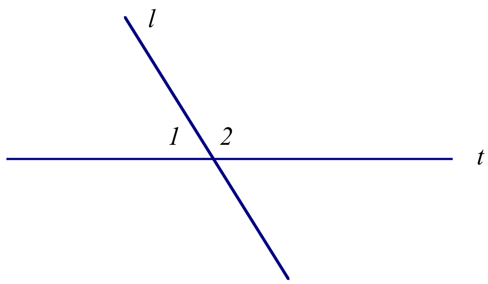
Prove: $\angle A$ and $\angle B$ are not both right angles

	STATEMENTS	REASONS
1.	Assume $\angle A$ and $\angle B$ are RT $\angle$s	Assumption
2.	$\angle A$ not $\cong$ $\angle B$	Given
3.	$\angle A \cong \angle B$	All RT $\angle$s are $\cong$
4.	# $\therefore \rightarrow \angle A$ and $\angle B$ are not both RT $\angle$s	Assumption - false

In step 3, we see that the 2 angles were congruent because they were both right angles from our assumption. But that contradicts what we said in step 2 that was given as true. That results in our contradiction.

So, indirect proofs are used when it is difficult to prove something directly. They are often used when trying to prove there is *one and only one* of something, like a midpoint or an angle bisector.

Example 3 Prove indirectly.



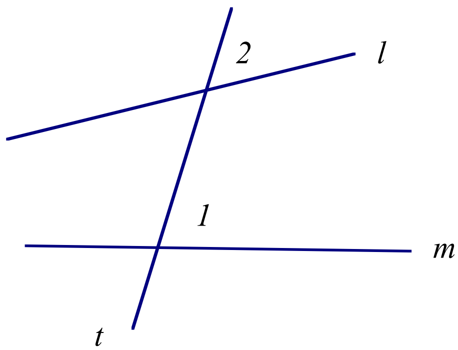
Given: t not perpendicular to l

Prove: $\angle 1$ not $\cong$ $\angle 2$

To begin, we assume the negation of the conclusion.

	STATEMENTS	**REASONS**
1.	Assume	Assumption
2.	t not perpendicular to l	Given
3.	$\angle 1 \wedge \angle 2$ linear pair	Ext. sides lie in a line
4.	t is perpendicular to l	2 lines form $\cong$ adj s$\angle$, lines are perpendicular
5.	# $\therefore \rightarrow \angle 1$ not $\cong$ $\angle 2$	Assumption must be false

Example 4 Prove by contradiction



Given: l and m cut by t
$\angle 1$ not $\cong \angle 2$

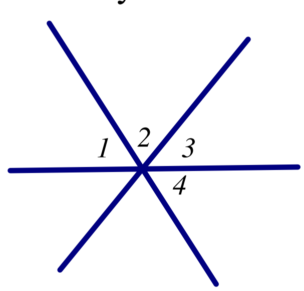
Prove : l not $\parallel$ to m

	STATEMENTS	REASONS
1.	Assume $l \parallel m$	Assumption
2.	$\angle 1$ not $\cong \angle 2$	Given
3.	$\angle 1 \cong \angle 2$	2 $\parallel$lines cut by t, corr $\angle$s $\cong$
4.	# $\therefore \rightarrow l$ not $\parallel$ to m	Assumption must be false

Try this one on your own using the procedure from Page 2

1)

Prove indirectly



Given: Plane figure
$m\angle 2 \neq m\angle 3$

Prove: $m\angle 1 + m\angle 2 \neq m\angle 3 + m\angle 4$