Ch. 5 PERCENTS

Percents can be defined in terms of a ratio or in terms of a fraction. Percent as a fraction – a percent is a special fraction whose denominator is 100. Percent as a ratio – a comparison between two quantities with the total being 100. The number in front of the percent symbol (%) is the numerator. The denominator is not written, but understood to be 100.

Examples $6\% = \frac{6}{100}$ $14\% = \frac{14}{100}$ $87\% = \frac{87}{100}$

Because a percent is a special fraction, then, just like with decimals, all the rules for percents come from the rules for fractions. That should make you feel pretty good. It's not like we are learning brand new stuff you're not familiar with.

Let's take a quick look. To add or subtract percents, you add the numerators and bring down the denominator just like in fractions.

Adding & Subtracting Percents

Example: $34\% + 15\% = 49\%$

Notice I added the numbers in front of the percent symbol, the numerators, then I brought down the common denominator, the percent symbol.

Oh, yes, this is really, really, really good stuff. Don't you wish that - just sometimes - you could make math difficult. As long as you see the patterns develop and you know your definitions and algorithms, math is just plain easy.

Multiplying Percents

If I wanted to multiply percents, again I would go back to my rules for multiplying fractions. To multiply fractions, you multiplied the numerators, then the denominators. To multiply percents, you do the same thing. Multiply the numerators, then the denominators.

Examples 5% x 12%

Multiplying the numerators, $5 \times 12 = 60$ Remember, the denominators are not written. They are defined to be 100. Therefore, we multiply $100 \; x \; 100,$ that equals 10,000.

$$5\% \text{ x } 12\% \rightarrow \frac{5}{100} \text{ x } \frac{12}{100} = \frac{60}{10,000}$$

Which could be expressed as a decimal; .006

Converting Percents to Fractions and Decimals

To convert a percent to a fraction, we just use the definition. The number in front of the percent symbol is the numerator, the denominator is 100, then simplify.

Example Convert 53% to a fraction

$53/100$ Too easy, right?

What if someone asked you to convert percents to decimals, would you do it the same way? Of course.

Example Convert 53% to a decimal

$\frac{53}{100}$, but that's a fraction.

How do you divide by 100? Move the decimal point 2 places to the left. So, $53\% = .53$.

If we did enough of these, we'd soon realize to convert a percent to a decimal, you move the decimal point 2 places to the left.

Example Convert 3% to a decimal.

Moving the decimal point 2 places to the left, we have .03.

Knowing that you convert a percent to a decimal by moving the decimal point 2 places to the left, how would you convert a decimal to a percent? That's right; you'd do just the opposite, move the decimal 2 places to the right and put the percent symbol at the end.

Example Convert .34 to a percent.

Move the decimal point 2 places to the right and put a percent symbol at the end. The answer is 34%.

That's just too easy.

Now, why are we moving the decimal point 2 places? Because the denominator for a percent is 100, two zeros, and we learned shortcuts for multiplying and dividing by powers of 10.

$$.34 = \frac{34}{100} \rightarrow 34\%$$

When you are first learning these problems and trying to apply shortcuts, remember we call them rules, sometimes we get them confused. So, here's a hint that might help you remember.
To convert **TO** a decimal, the loop on the "**d**" in decimal opens to the left, so move the decimal point to the left 2 places.

To convert **TO** a percent, the loop on the "**p**" in percent opens to the right, so move the decimal point to the right 2 places.

Again, those two hints came from patterns we recognized.

Example Convert 63% to a decimal.

The loop on the "d" opens left, move the decimal point 2 places in that direction. The answer is .63.

That's the shortcut, the reason why that works is because 63% means $63/100$.

Simplifying $63/100$ in decimal form is .63

Example Convert .427 to a percent.

The loop on the "p" opens to the right, move the decimal point 2 places in that direction. The answer is 42.7%.

That's the shortcut that allows you to compute the answer quickly. But, shortcuts are soon forgotten, it's important that you understand why the shortcut works.

Let's see what that would look like if we did not use the shortcut.

$$.427 = \frac{427}{1000}$$

To convert that to a percent, I have to rewrite that fraction with a denominator of 100.

$$\frac{427}{1000} = \frac{42.7}{100} \rightarrow 42.7\%$$

Once nice thing about mathematics is the rules don't change. Problems might look a little different, but they are often done the same way. The first example we discussed was converting 6% to a fraction. We said the number in front of the percent symbol was the numerator, the denominator was 100.

$$6\% = \frac{6}{100}$$ Simplifying, the answer would be $\frac{3}{50}$

What if I asked you to convert ¼ % to a fraction, could you do it? Of course you could. You would do exactly what you did to convert 6% to a fraction. The numerator is the number in front of the percent symbol, the denominator is 100.
By converting $\frac{1}{4}\%$ to a fraction by the definition of percent, we have

$$\frac{1}{4}\% = \frac{1/4}{100}$$

Simplifying that complex fraction, I'd invert and multiply, then simplify.

$$\frac{1/4}{100} \rightarrow \frac{1}{4} \div 100$$

$$= \frac{1}{4} \text{x} \frac{1}{100}$$

$$= \frac{1}{400}$$

Notice, the problems looked different, but we used the same strategy, put the numerator over 100 and simplified. Piece of cake! If you simplified a number of fractional percents, you'd probably see a nice pattern develop that would allow you to simplify them in your head.

Let's try a few.

Convert to fractions.

1.	83%	2.	9%	3.	520%	4.	30%
5.	45%	6.	$\frac{2}{3}\%$	7.	.4%	8.	3.5%

Convert to decimals

9.	65%	10.	7%	11.	324%	12.	.43%
13.	$\frac{1}{2}\%$	14.	8.3%	15.	$\frac{2}{5}\%$	16.	$8\frac{1}{4}\%$

Convert to percents

17.	$1/2$	18.	.23	19.	$\frac{3}{4}$	20.	8.6

Percent Proportion

For many of us, a percent is nothing more than a way of interpreting information. We have worked with percents since grade school as a statistic. In reality, all we are doing is looking at information in terms of a ratio, then rewriting the ratio so the denominator is 100.

For instance, let's say you got 8 correct out of 10 problems on your quiz. To determine your grade, your teacher would typically want to know how well you would have performed if there were 100 questions.

In other words, they would set up a proportion like this.

$$\frac{\#\,correct}{total} = \frac{?}{100}$$

Filling in the numbers, I have $\frac{8}{10} = \frac{\_\_}{100} \rightarrow \frac{8}{10} = \frac{80}{100}$

Getting 8 out of 10, I'd expect to get 80 out of 100

Notice the right side is a fraction whose denominator is 100. Just as we defined a percent.

Example Let's say you made 23 out of 25 free throws playing basketball. I might wonder how many shots I would expect to make at that rate if I tried 100 shots.

Again, I have a ratio

$$\frac{\text{attempts}}{\text{total}} = \frac{\_\_}{100}$$

$$\frac{23}{25} = \frac{\_\_}{100} \rightarrow \frac{23}{25} = \frac{92}{100}$$

Now I could solve that by making equivalent fractions or by cross-multiplying. Either way, the missing numerator is 92. I would expect to make 92 free throws out of 100 tries.

These problems are just like the ratio and proportion problems we have done before. The only difference is the denominator on the right side is 100 because we are working with percents.

A proportion that always has the denominator of the right side as 100 is called the **Percent Proportion.**

Percent Proportion

$$\frac{\textbf{\textit{part}}}{\textbf{\textit{total}}} = \frac{\%}{\mathbf{100}}$$

Remembering that we describe the ratios the same way on each side of a proportion, we might think this should read.

$$\frac{\text{part}}{\text{total}} = \frac{\text{part}}{\text{total}}$$

Well, the percent ratio actually does compare parts to total on both sides. For a percent, the total is always 100 and the percent is always the part you got.

The point I want to make is we have consistency with the math we have already learned. Now, the real good news. We can use the percent proportion to solve just about any problem involving percents. So, memorize it!

$$\frac{\boldsymbol{part}}{\boldsymbol{total}} = \frac{\%}{\mathbf{100}}$$

Speaking mathematically, the 100 always goes on the bottom right side. That's a constant. The only things that can change is the part, total or percent. You get that information by reading the problem and placing the numbers in the correct spot, then solve.

There are only 3 different problems, we can look for a part, a total or a percent. Let's go for it.

Example 1 Bob got 17 correct on his history exam that had 20 questions. What percent grade did he receive?

$\frac{\text{part}}{\text{total}} = \frac{\%}{100}$ filling in the numbers, $\frac{17}{20} = \frac{\quad}{100}$

Solving, either by equivalent fractions or by cross-multiplying, we find he made an 85%. In this problem we found a percent.

Example 2 A company bought a used typewriter for $350, which was 80% of the original cost. What was the original cost?

Now does the $350 represent the total or part? $\frac{350}{n} = \frac{80}{100}$

Cross multiplying, we have $80n = 350 \text{ x } 100$. Solving, $80n = 35{,}000$, $n = 437.5$

The original cost of the typewriter is $437.50. In this problem we found the total.

Example 3 If a real estate broker receives 4% commission on an $80,000 sale, how much would he receive?

Is the $80,000 representing the part or total?

$$\frac{n}{80,000} = \frac{4}{100}$$

$$100n = 4 \text{ x } 80,000$$

Solving, $100n = 320,000$

$$n = 3,200$$

He would receive $3,200 in commission. Here, we found the part.

While the first three examples were all percent problems and we used the percent proportion to solve them, in each case we were looking for something different. That's the beauty of the percent proportion.

In this next example, everything we learned stays the same, but there is a slight variation in how the problem is written. To do this problem, you must understand how proportion problems are set up.

Another percent proportion that is commonly used is the "is-of" proportion. It almost looks the same as the Percent Proportion we have already discussed, but the words are different.

$$\frac{is}{of} = \frac{\%}{100}$$

Now remembering the Percent Proportion is:

$$\frac{\boldsymbol{part}}{\boldsymbol{total}} = \frac{\%}{\mathbf{100}}$$

A way of remembering where to place the numbers is to substitute the word "part" where you see a percentage.

Example 4 75 is 20% of what number.

Substituting the word "part" for the percentage, we have 75 is part of what number.

That suggest that 75 is part and goes in the "part" position.

$$\frac{\mathbf{75}}{?} = \frac{\mathbf{20}}{\mathbf{100}}$$

Example 5 What is 25% of 80.

Substituting the word "part" for the percentage, What is part of 80.

That suggests that 80 is the total or the whole.

$$\frac{?}{80} = \frac{25}{100}$$

Example 6* Dad purchased a radio that was marked down 20% for $68.00. What was the original cost of the radio!

Now I need you to stay with me. Setting up the proportion, does $68 represent the part or total?

Filling in the proportion,

$$\frac{\text{paid}}{\text{total}} = \frac{\%}{100}$$

This is very, very important, the $68 represents the part you paid, what does the 20% represent? That's the part you got off.

We can**not** have a proportion with **paid is to total** as **amount off is to total**. The ratio on the left should match the ratio on the right. If Dad received 20% off, we have to have the same ratio on both sides. That is paid to total as paid to total. If he got 20% off, what percent did he pay? 80%

Now, filling in the numbers, we have $\frac{68}{n} = \frac{80}{100}$

Solving, we have

$$80n = 6{,}800$$
$$n = 85$$

Percent as Formulas

By solving the Percent Proportion for different components of the proportion, we can derive formulas that can make our work easier.

$\frac{\text{part}}{\text{total}} = \frac{\%}{100}$; by cross multiplying, we have **part(100) = total(percent)**

Solving for the part, we have: $\boldsymbol{part = \frac{percent\ x\ total}{100}}$

Solving for the percent, we have: $\boldsymbol{percent = \frac{part\ x\ 100}{total}}$

or solving for the total, we have: $\boldsymbol{total = \frac{part\ x\ 100}{percent}}$

Of these formulas, many use the part formula. Just remember, all these formulas come from the percent proportion.

The most common question people are asked when working with percents is looking for the part: By formula, that is percent times the total. What is 20% of a number. We are looking for part of 20. The word "of" often translates to multiplication on mathematics. So, if asked, what is 20% of 50. I multiply .20 x 50 = 10. If asked, what is 20% off an item selling for $50, I am taking 20% of the number, so I multiply. The answer, again, is $10.

***Example** 7* Theo sold a home for $350,000.00 and will make a 6% commission, how much will he earn from that sale?

I could set up the percent proportion and solve or I could determine what part of the $350,000.00 does he earn and multiply .06(350,000) = $21000.00.

Theo would earn $21,000.

Use the Part Formula

1. Find 25% of 300
2. Find 40% of 250
3. Find 80% of 600
4. Find 15% of 90
5. Find 35% of 380
6. Find 12% 0f 72

Percent of Increase/Decrease

We were able to solve 3 different type problems using the Percent Proportion. We solved for the part, total, and percent by using what we learned in ratios and proportions earlier.

Another popular type of problem is percent of increase or decrease. While they can be solved using the Percent Proportion, some like to look at them as distinct problem types.

The good news is everything we have learned still applies. We are just going to look at the problems a little differently. Up to this point, using our knowledge of ratios and proportions, the ratio on the right side of the equal sign was the same as the ratio on the left side.

That is $$\frac{part}{total} = \frac{part}{total} \rightarrow \frac{part}{total} = \frac{\%}{100}$$

Now we will make a minor adjustment. Rather than having the part as the numerator, we will change that to the ***amount changed***. The denominator will be the ***original amount***.

$$\frac{\textbf{\textit{amount changed}}}{\textbf{\textit{original amount}}} = \frac{\%}{\mathbf{100}}$$

So, If Bob earned $10 per hour last year and this year he earns $12 per hour, what percent increase did he receive?

Using the variation of the Percent Proportion, the original amount he earned was $10 per hour, he received a $2 per hour increase. Filling in the proportion, we have

$$\frac{2}{10} = \frac{n}{100}$$

Cross multiplying, $10n = 200$

$n = 20$, he received a 20 percent increase (raise)

Now, just for fun, but wanting to make a point. Let's take a look at the situation we just described. Bob got his 20% raise and now makes $12 per hour. What if his boss tells him at the end of the year that his business is not going well and wants to reduce his pay by 20%, how much will Bob make per hour if he accepts that condition?

Most people, without much thought, might be inclined to say $10 right? Let's do the math.

In this case, his original amount is $12 per hour, and we want to find out how much he will earn if he takes a 20% pay cut.

Let's fill in the numbers and see how this works out.

$$\frac{n}{12} = \frac{20}{100}$$

Cross multiplying, we have 100n = 240

So n = \$2.40

That results in Bob taking a \$2.40 pay cut or his new hourly rate will be \$9.60 per hour.

Two points I want to make, 1) always do the math. Many would have been lulled into thinking that if they got a 20% increase that resulted in them going from 10 to 12, then taking a 20% decrease would take them from 12 to 10 – that's wrong. Point 2) The new original amount changed from \$10 to \$12. That changes the math.

Percent Error

The percent error, very much like the percent of increase/decrease, is a ratio that compares the inaccuracy of an estimate (amount of error) to the actual amount.

$$\frac{amount\ of\ error}{actual\ amount} = \frac{percent}{100}$$

Example 8 Suppose you are asked to guess how many pieces of candy are in a carton. You guessed there were 100 pieces of candy. When actually counted, there were only 80 pieces in the carton. What was your percent error?

We were off by 20 pieces, the amount of the error. The actual number in the box was 80. Setting up our proportion, we have

$$\frac{amount\ of\ error}{actual\ amount} = \frac{percent}{100}, \qquad \frac{20}{80} = \frac{n}{100}$$

$$\frac{1}{4} = \frac{25}{100}$$

There is a 25% error

Simple Interest

I = Prt

Simple interest is the amount of money paid or earned for the use of money.

That amount is determined by the formula; I = Prt. I represents the interest, P is the principal which is the amount of money being used, r represents the rate, and t stands for time in years.

Simple interest is interest earned over a specific time period. Compound interest, which is actually how most money is earned or paid, is broken up during that time period and added to the original amount.

We will work only with simple interest in this chapter. To learn more about compound interest, go the algebra section and look under exponentials.

Example 9 Alicia put $500 into her savings account which pays 3% simple interest per year, how much will she earn in 4 years.

Using I = Prt, P = $500 r = 3% t = 4

I = (500)(.03)(4)

I = $60.

She will have earned $60 and will now have $560 in her savings account.

Example 10 Bob borrowed $600 at a rate of 8% for 6 months, how much interest will he have to pay for use of the money and how much will he have to pay back altogether.

Using the formula - I = Prt; P = 600 r = 8% t = ½

Remember, t is given in terms of years, 6 months is ½ year

I = 600(.08)(1/2) = $24

Bob will have to pay $24 in interest and repay a total of $624

Solving Percents using Algebra

There are always different ways to solve problems. If you know how to solve equations, we can solve percent problems algebraically.

Example 1 12 is what percent of 30.

Since I'm looking for a percent, I will call that x. The word "is" is translated into math as "=". "Of" indicates a multiplication problem,

Writing an equation $12 = x\,(30)$

$$\frac{12}{30} = x; \qquad \textbf{that simplifies to 4/10}$$

$$\mathbf{40\% = x}$$

Example 2 Ted received a 10% raise and his new hourly rate is $13.20. What was his hourly rate of pay before he got the raise?

Looking for his hourly rate, we will call that *x*.

Writing an equation, his new rate is his old rate plus his percent increase, and that equals $13.20.

$$x + .10x = 13.20$$

$$1.10x = 13.20$$

$$x = \frac{13.20}{1.10}$$

$x = 12.00$ His original pay was $12.00

1. On last week's math test Carol had 21 correct out of 25 problems. What percent grade did she earn?

2. Bob received an 85% on his history exam. If there were 20 questions, how many did he get correct?

3. Juan receives a 5% commission on his sales. If he received $30, how much merchandise did he sell?

4. Ted got an 84% on his science test, if there were 50 questions, how many did he get wrong?

5. If you receive 30% off on a pair of slacks that cost $25, how much would you change should you receive if you gave the clerk a $20 bill?

6. Jessie earns $250 per week. If 18% is deducted for federal tax and 8% for social security, what is his net income?

7. A radio costs $20, Harold buys it for $16, what percent off the original price did he receive?

8. The Pep Club was decreased from 15 members to 12, what percent decrease was there in the club?

9. Find the 15% tip for a restaurant bill of $42.

10. A store drops the price of a radio 24% to a sales price of $36.48. What was the original price of the radio?

11. After a person receives a 20% raise, his salary is $9,600. What was his old salary?

12. James spends 12 hours per week studying. He spends 3 of those hours studying math. What percent of his study time is spent on math?

13. The Johnson's purchased their home for $160,000. Two years later, the house had increased in value by 15%. How much is the house worth after two years?

14. The membership at a certain church was 450 members five years ago. Today, the membership is 525 members. What percent increase is this?

15. In a recent survey, people were asked if they had a digital camera at home. Six people had digital cameras, this was 30% of the total number of people surveyed. How many people were surveyed?

16. Last week the hotel occupancy rate was 75%. If 600 rooms were rented, how many rooms were still available?

17. Bonnie used 4 gallons of paint-to-paint 80% of her bedroom. How many gallons of paint will it take to paint the entire room?

18. On a restaurant bill of $58.00, Susan want to leave a 20% tip. How much was her total bill?

19. On a test with 30 questions, Bill answered 80% correctly. How many answers did he get wrong?

20. Tom bought a radio on sale for 20% off for $68. What was the original cost of the radio?

21. Bob guessed there was $400 in the bank, the actual amount in the bank was $500, what was his percent error?

22. Ted said he would bowl a score of 150 in the next game, his score was 180, what was his percent error/

23. If Maria borrowed $700 for 3 years at 6% interest, how much would she have to pay back to repay the loan?

24. Jose loaned his friend Juan $800 for 9 months at 4% interest, how much will Jose earn from lending hos money?

PERCENT PROBLEMS

1. Find the 7.5% sales tax on the purchase of a sports coat costing $150.

2. Find the 6% down payment on the purchase of a house costing $120,000.

3. 54 is 12% of a certain number. Find the number.

4. A stockbroker sold $350,000 worth of stocks and bonds during the month of October. If her commission rate is 2%, how much money did she earn in commissions?

5. What is the tax rate in Nevada, if a tax of $19.50 is charged on a television set costing $300?

6. Mr. Jones spent $4,800 for a used car. This amount was 20% of his annual income. What was his annual income?

7. What is the sales tax on a dress selling for $70, if the tax rate is 6%. What is the total cost of the dress?

8. Ms. Smith gets a 14% raise in salary, if her original salary is $28,000 per year what is her new salary?

9. A store drops the price of a certain type of radios 24% to a sale price of $36.48. What was the original price of the radios?

10. Anne borrows $1,200 to fix up her home. If the money is borrowed at 7% simple interest, how much does Anne have to pay back at the end of one year?

11. Find the 15% tip for a restaurant bill of $42.

12. The sale price of a used car is $1,837. Find the list price if the discount rate is 16.5%.

13. A pro shop discounted all merchandise in the store by 40%. What is the discount and the sale price on a tennis racket regularly priced at $135.50?

14. Jane was paid a commission of $287 on sales of $5,740. What was her commission rate?

15. The tax rate in Indiana is 5%. If the sales tax charged on the purchase of a coat is $13.75, how much does the coat sell for?

Percents A

Name
Date

1.*** Define “percent” in terms of a fraction.

2.*** Define “simple interest”.

3.*** Write the Percent Proportion.

4.*** Write the Percent of Change Proportion.

5.*** Write the Percent of Error Proportion.

6.*** Write the Simple Interest Formula and label the parts.

7.** Convert 17% to a fraction.

8.** Convert 9% to a decimal.

9.** Convert $\frac{3}{5}$% to a fraction.

10.** Convert 0.05% to a decimal.

11.** Convert 18/25 to a percent.

12.** Convert 0.4 to a percent.

13.** Dave reported that 7 out of 35 members of the club paid their dues. What percent paid their dues?

14.** Bob received a 75% on a test that had 24 questions. How many did he have correct?

15.** Twenty percent of the sophomore earned straight A's. If 136 students received awards for that accomplishment, how many students are in the sophomore class?

16.** A house that sells for $92,000 requires a 20% down payment. What is the amount of the down payment?

17.** Last year Ernie earned $25, 000. His new job pays $28,000, what percent raise did he receive?

18.** A store drops the price of a certain type of radio 25% to a sale price of $68. What was the original price of the radio?

19.** If Juan borrowed $1500 for 2 years at 8% interest, how much would he have to pay back to satisfy the loan?

20. Guillermo placed $900 in the bank for 4 months earning 5% interest, how much money will he earn after 4 months?

21.** John estimates the weight of a gallon of water is 5 pounds (lbs). The actual weight of a gallon of water is 7 pounds. Find the percent error.

22.* SBAC A car is purchased for $18,000. If each year the car depreciates by 10% of it's value the preceding year, what will be its value at the end of three years?

23.* SBAC The price of a suit that sold for $100 was reduced by 25%. By what percent must the price be increased to bring the price back to $100?

24.*SBAC A store owner buys a compact stereo for $300. She wants to price it so she can offer a 20% discount off the posted price and make a profit of 25% of the price she paid. What will the posted price of the stereo be?

25.*** Provide parent/guardian information; phone, cell phone, email, etc. (CHP)