Derivation of $\sin\left(\frac{A}{2}\right) = \sqrt{\dfrac{1-\cos A}{2}}$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$	Double $\angle$
$\cos(A + A) = \cos A \cos A - \sin A \sin A$	Substitution

$\cos(2A) = \cos^2 A - \sin^2 A$

Multiplication

$$= (1 - \sin^2 A) - \sin^2 A \qquad \textbf{Sub. Trig Id.}$$
$$= 1 - 2\sin^2 A \qquad \textbf{Dist. Prop}$$

$\cos(2A) - 1 = -2\sin^2 A \qquad$ **Sub Prop =**

Let $A = 2\,\dfrac{A}{2}$

Rewriting

$$\cos\left(2\,\frac{A}{2}\right) - 1 = -2\sin^2\left(\frac{A}{2}\right) \qquad \textbf{Substitution}$$

$$\cos A - 1 = -2\sin^2\left(\frac{A}{2}\right) \qquad \textbf{Mult. Inv.}$$

$$\frac{\cos A - 1}{-2} = \sin^2\left(\frac{A}{2}\right) \qquad \textbf{Div.Prop. =}$$

$$\frac{1 - \cos A}{2} = \sin^2\left(\frac{A}{2}\right) \qquad \textbf{Div. Neg \#}$$

$$\sqrt{\frac{1 - \cos A}{2}} = \sin\frac{A}{2} \qquad \textbf{SQ. ROOT}$$