

$$\text{Derivation of } \cos\left(\frac{A}{2}\right) = \sqrt{\frac{\cos A + 1}{2}}$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A + A) = \cos A \cos A - \sin A \sin A$$

Double \angle
Substitution

$$\begin{aligned} \cos(2A) &= \cos^2 A - \sin^2 A \\ &= \cos^2 A - (1 - \cos^2 A) \\ &= 2\cos^2 A - 1 \end{aligned}$$

Multiplication
Sub. Trig Id.
Distrib. Prop

$$\text{Let } A = 2\frac{A}{2}$$

Rewriting

$$\cos\left(2\frac{A}{2}\right) = 2\cos^2\left(\frac{A}{2}\right) - 1$$

Substitution

$$\cos A = 2\cos^2\left(\frac{A}{2}\right) - 1$$

Mult. Inv.

$$\cos A + 1 = 2\cos^2\left(\frac{A}{2}\right)$$

Add Prop. =

$$\frac{\cos A + 1}{2} = \cos^2\frac{A}{2}$$

Div. Prop. =

$$\sqrt{\frac{\cos A + 1}{2}} = \cos\frac{A}{2}$$

SQ. ROOT