CHAPTER 1 Linear Equations & Inequalities

Sec 1. Solving Linear Equations

Kids began solving simple equations when they worked missing addend problems in first and second grades. They were given problems such as $4 + \Delta = 6$ and had to find the value that goes into the Δ by guessing and substituting numbers to find a number that worked.

As students learned to evaluate arithmetic expressions in mathematics, they were taught the **Order of Operations**. The Order of Operations, an agreement that allows us all to evaluate arithmetic expressions the same way, also serves as the foundation for solving linear equations.

An agreement such as the Order of Operations is common place in our daily lives. For instance, we typically agree that married folks wear their wedding rings on their left hand. When you watch sports, the home team is always listed last. My guess another agreement we find useful is most people will drive on the right side of the road – unless of course they want to meet new friends. So, as we have those types of agreements, in mathematics, we have an agreement called the Order of Operations, which like the agreements we make in life, will lead to consistency and not cause confusion.

To evaluate an expression such as; $4 + 2 \times 5$, we'd use the Order of Operations.

Order of Operations

1. Grouping
2. Exponentials
3. Multiply/Divide
4. Addition/Subtraction

From left to right

That agreement indicates that all operations within a parenthesis will be done first, followed by any exponentials. After that, any multiplication of division will be done, then any addition or subtraction. Each line done from left to right.

In the problem given above, notice there were no signs of grouping, nor were there any exponentials. So going to step #3, we do any multiplication or division – from left to right.

$$4 + 2 \times 5 \quad = \quad 4 + 10$$

Finally, we do any addition or subtract. $= 14$

Using that agreement, we find that 4 + 2 x 5 = 14.

Without this agreement, some students may have just done the computation from left to right resulting in a wrong answer of 30.

Example 1 Evaluate: 4 + 30 ÷ 5 x 2

Again in this problem, there are no signs of grouping and there are no exponentials. So we begin with #3. Multiplication/Division from left to right - whichever comes first. In this case, it's the division. Doing that, we have

$$4 + 30 \div 5 \times 2$$
$$4 + 6 \times 2$$
$$4 + 12$$

Therefore, 4 + 30 ÷ 5 x 2 = 16

A common mistake for students is to perform multiplication before division. The agreement is multiplication and division are equal in rank (on the same line because division is defined as a multiplication of the inverse) and you do whichever comes first – from left to right.

Let's go back to our original problem, 4 + 2 x 5 = 14. I could rewrite that as 2 x 5 + 4 = 14 using the commutative property. In elementary school, students might be asked to find the missing number for 2 "x" Δ+ 4 = 14 by guessing. Later in algebra, that problem would look like 2x + 4 = 14 and rather than finding the missing number by guessing or trial & error, students would be asked to solve the equation for x in a more systematic way.

Now, if I asked students to solve 2x + 4 = 14 they would have to find the value of x that makes that open sentence true. To find the value of x, they would have to undo the expression on the left to isolate the x.

Before we do that, let's look at a gift wrapping analogy. If I were to give you a present, I would put it in a box, put the cover on the box, put paper on the box, tape it, and place a ribbon on it. For you to get the present out of the box, you would take the ribbon off, take the tape off, remove the paper, take the top off, and take the present out. In other words, you would do exactly opposite of what I did to give you the present.

We do the same to solve an equation. The way we arrived at 4 + 2 x 5 = 14 was using the Order of Operations. For us to solve an equation, undo an algebraic expression, we need to use the Order of Operations in reverse using the inverse (opposite) operation to undo the expression.

Strategy for Solving Linear Equations

Rewrite an equation in ax + b = c format using the Properties of Real Numbers, then use the Order of Operations in reverse using the inverse operations to isolate the variable.

a. Identify what is physically different from ax + b = c

b. Get rid of it

To solve 2x + 4 = 14, we need to get rid of any addition or subtraction. Since we have an addition, we'll do the opposite and subtract 4 from both sides of the equation. That leaves us with 2x = 10.

Continuing to use the Order of Operation in reverse, we need to get rid of any multiplication or division. We have a multiplication by 2, so to get rid of the multiplication we divide, the answer, the solution, the value of the variable that makes the open sentence true is 5.

In general, we were solving equations written in the form, ***ax + b = c***

Example 2: Solve for x; 4x – 2 = 10

Choose which side you'd like to have the variable on. If you choose the left side, then we isolate the variable by using the Order of Operations in reverse, using the OPPOSITE OPERATION. That means we have to get rid of any addition or subtraction first. How do you get rid of a minus 2?

$$4x - 2 = 10 \quad \text{Given}$$

$$4x - 2 + 2 = 10 + 2 \quad \text{Addition Property of equality}$$

$$4x = 12 \quad \text{Combine Like terms}$$

Now, how do we get rid of a multiplication by 4? That's right, divide by 4. Therefore,

$$x = 3 \quad \text{Division Property of equality}$$

An important problem solving/learning strategy is to take a problem that you don't know how to do and transform that into a problem that you know how to solve. Right now, we know how to solve problems such as $\boldsymbol{ax + b = c}$. I cannot make solving equations more difficult, but what I can do is make them longer problem.

Equations like $4x + 3 = 27$ and $5x - 2 = 18$ are in $\boldsymbol{ax + b = c}$ format so we use the Order of Operations in reverse using the inverse operation to isolate the x.

Example 3 Solve for x; $7x + 3 = 31$

$7x + 3 = 31$	Given
$7x + 3 - 3 = 31 - 3$	Subtract Prop Equality
$7x = 28$	Add Inverse/ Combine terms
$x = 4$	Div Prop Equality

The nice thing about math is I can never make the problems more difficult, I can only make them longer. By making longer problems look like shorter problems we have already solved, math becomes a real blast.

When more is added to an equation, such as parentheses or fractions, students should get rid of those by one of the Properties of Real Numbers, then rewrite the equation in $\boldsymbol{ax + b = c}$ format. In that way, all linear equations can be solved using the Order of Operations in reverse using the inverse operation.

Sec. 2 Solving Equations **not** in $ax + b = c$ format

When solving equations that are **not** in the $ax + b = c$ format, the trick is to physically identify what is different and get rid of it by using the Properties of Real Numbers.

Example 1 Solve for x, $5x - 2 = 2x + 19$

Now this example looks a little different from the equations we have solved this far. The question, the important question, is how is this problem physically different from the examples we have done?

$$5x - 2 = 2x + 19$$

Hopefully you noticed the there was a variable on the right side of the equation, 2x.

Using the strategy of getting rid of what I don't recognize, how do we get rid of the 2x? That's right, subtract 2x from both sides of the equation using the Subtraction Property of Equality

$5x - 2 = 2x + 19$	Given
$5x - 2 - 2x = 2x + 19 - 2x$	Subtract Prop Equality
$3x - 2 = 19$	Add Inverse

That results in $3x - 2 = 19$. This is now in $ax + b = c$ format.

Now, since it is in $ax + b = c$ format, I can use the Order of Operations in reverse using the inverse operation to solve for x.

Using the Order of Operations in reverse, I will get rid of the –2 by adding 2 to both sides of the equation.

$3x - 2 + 2 = 19 + 2$	Add Prop Equality
$3x = 21$	Add Inverse/CLT
$x = 7$	Div. Prop Equality

Let's look at another example.

Example 2 Solve for x $5(2x + 3) - 4 = 19$

That problem again looks different from the equations in the ***ax + b = c*** format. The question you have to ask is; what is physically different in this problem?

The answer is the new problem has parentheses. That means to make these problems look alike, we need to get rid of those parentheses. We'll do that by using the Distributive Property.

$5(2x + 3) - 4 = 19$	Given
$10x + 15 - 4 = 19$	Distributive Property
$10x - 11 = 19$	Combine Like Terms

Now the equation is in ***ax + b = c*** format

$10x = 30$	Add Prop Equality
$x = 3$	Div Prop Equality

Example 5: **Solve for x;** $5(x + 2) - 3 = 3(x - 1) - 2x$

This problem is clearly longer, but it is not harder. What's different is we have two sets of parentheses.

Getting rid of the parentheses by the Distributive Property and combining like terms, we have

$5x + 10 - 3 = 3x - 3 - 2x$	Distributive Property
$5x + 7 = x - 3$	Combine Like Terms

To get this into ***ax + b = c*** format, I will need to subtract x from both sides using the Subtraction property of Equality.

$5x + 7 - x = x - 3 - x$	Subtract Prop Equality
$4x + 7 = -3$	CLT

Now we have transformed the original equation to a problem we know how to solve - ***ax + b = c***. Continuing by subtracting 7 from both sides.

$4x + 7 = -3 - 7$	Subtract Prop Equality
$4x = -10$	Add Inv/ CLT
$\frac{4x}{4} = \frac{-10}{4}$	Division Property of equality
$x = \frac{-5}{2}$	Simplify

The general strategy being used to solve linear equations is to rewrite (transform) them into the ***ax + b = c*** format by using the Properties of Real Numbers.

Once we have transformed the equations into equivalent equations in the ***ax + b = c*** format, we then use the **Order of Operations** in reverse using the inverse operation to isolate the variable.

Solve, find the solution set, find the value of the variable that makes the open sentence true.

1. $4x - 2 = 18$	$5x + 3 = 43$	$3x - 3 = 6$
2. $5x - 2 = 3x + 10$	$7x + 3 = 3x - 17$	$6x + 8 = 2x + 4$
3. $4(2x + 3) = 36$	$2(3x + 5) = 40$	$5(2x - 3) + 5 = 20$
4. $-2(5x + 2) = 6$	$--3(2x - 4) = 36$	$2x + 3(2x + 5) = 39$
5. $4x + 3 = 5x - 6$	$2x - 3 = 5x + 9$	$3x + 4 = 7x + 4$
6. $2x + 3x - 4 = 26$	$7x - 2x - 5 = x + 7$	$2(x - 3) - 3 = 3x + 2$

Sec. 3 Solving Linear Equations with Rational Coefficients

You might recall the overall strategy we used last time was to isolate the variable by using the **Order of Operations** in reverse using the inverse operation to make the equation look like $\boldsymbol{ax + b = c}$. The good news is that does not change.

Having fractional coefficients does not make the problems any more difficult. The question is, can we make equations with fractional coefficients look like the problems we have already solved?

If I multiplied both sides of an equation containing rational coefficients by a common denominator, that will get rid of the fractions. Let's try that.

Example 1: Solve $\frac{x}{2} + \frac{x}{3} = 10$

We have an equation with fractions – we were not taught how to solve these. However, by using the strategy of making this problem look like problems we have solved before, we could end up solving this pretty easily. To do that, we have to ask, what is physically different about this problem and problem in the $\boldsymbol{ax + b = c}$ format? You're right, it has fractions. How do you get rid of fractions?

Using magic, better known as the Multiplication Property of Equality. I'll make the denominators 2 and 3 disappear by multiplying both sides of the equation by the common denominator - 6

$6 \{\frac{x}{2} + \frac{x}{3}\} = 6\ (10)$ Multiplication Property of equality

Simplifying, I have

$3x + 2x = 60$ Distributive Property

$5x = 60$ Combine Like Terms

$x = 12$ Division Property of Equality

Notice, once we multiplied both sides by the common denominator, we no longer had fractions and we solved the resulting equation the same way we did last time. Piece of cake! Makes you want to do a longer problem, doesn't it?

Example 2: $\frac{x+3}{4} - \frac{4x-5}{5} = -1$

As you can see, these problems with rational coefficients are no big deal. How do I get rid of the denominator? Good, multiply both sides by the common denominator - 20.

$\frac{x+3}{4} - \frac{4x-5}{5} = -1$	Given
$20[\frac{(x+3)}{4} - \frac{4x-5}{5}] = 20(-1)$	Mult. Prop of Equality
$5(x + 3) - 4(4x - 5) = -20$	Simplifying
$5x + 15 - 16x + 20 = -20$	Distributive Prop
$-11x + 35 = -20$	Combine Like Terms

Notice the problem is now in $ax + b = c$ format.

$-11x = -55$	Subtraction Prop of Equality
$x = +5$	Div prop of equality

The reason this problem seemed longer than the other problems we have solved is because we had to not only multiply both sides by the common denominator to get rid of the fractions, we then had to use the distributive property to get rid of the parentheses, then combine terms. After all that was accomplished, the resulting equation looked like the ones we solved before in the **$ax + b = c$** format.

How can you be sure if you got the right answer? A solution should satisfy the original equation. Substitute +5 into that equation and check.

Try this one on your own, the answer is 7. $\frac{2x-2}{4} + \frac{2x+1}{3} = x + 1$

Exercises: Solve and check as directed by the teacher. Assume R = {all rational numbers}

1. $\frac{x}{2} - \frac{x}{3} = 2$

2. $\frac{x}{3} - \frac{x}{4} = 3$

3. $\frac{x}{2} + \frac{x}{3} = 11$

4. $\frac{y}{4} + \frac{1}{2} = -3$

5. $\frac{x}{4} - \frac{x}{5} = 4$

6. $3 + \frac{x}{4} = x + 6$

7. $\frac{x}{4} - 5 = 4 - \frac{x}{5}$

8. $\frac{x}{3} - \frac{x}{15} = 14 - \frac{x}{5}$

9. $\frac{3}{4} - \frac{1}{2} = x$

10. $x - \frac{3x}{2} = 7\frac{1}{2}$

Sec. 4 Solving Inequalities

We can solve linear inequalities the same way we solve linear equations.

We rewrite the inequalities in the $ax + b > c$ format, then use the Order of Operations in reverse using the opposite operation. Linear inequalities look like linear equations with the exception they have a greater than (>) or less than (<) sign rather than just an equal sign.

Let's put an equality and inequality side by side and solve both.

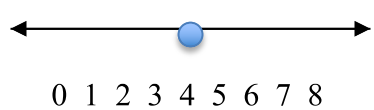
Example 1 Linear Equation $3x - 2 = 10$

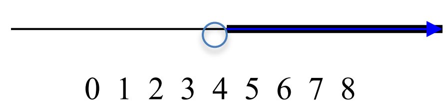
Linear Inequality $3x - 2 > 10$

Solving them both, we have

$3x - 2 = 10$	$3x - 2 > 10$
$3x = 12$	$3x > 12$
$x = 4$	$x > 4$

Graphing

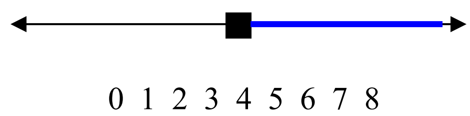




Notice the graph on the left only has the point representing 4 dotted. That translates to $x = 4$. The graph on the right has an open dot on 4, it is not shaded because 4 is not included as part of the solution. Also notice that all there is a solid line to the right of the open dot representing all the numbers greater than 4 are part of the solution set.

If the inequality contained an equal sign, $3x - 2 \geq 10$, then everything would be done the same in terms of solving the inequality, except the answer and graph would look a little different because it would include 4 as part of the solution set. To show 4 was included, we would shade it.

The solution is $x \geq 4$.



I can't make these problems more difficult, I can only make them longer as I did with linear equations.

Multiplying or dividing by a negative number is a concern.

What would happen if I had an inequality such as $6 > 5$ and multiplied both sides by a positive number, would the inequality sign stay the same? In other words would $10(6)$ still be greater than $10(5)$? The answer is yes. What if I multiplied by a negative number, by (-2)? In other words, is $(-2)6 > (-2)5$?

Multiplying, would the result $-12 > -10$ be true? The answer is no. Notice that -12 would be to the left of -10 on the number line. Numbers to the left are smaller. That leads us to the following:

When we multiply or divide by a negative number, to make the statement true, the order of the inequality must be reversed.

$$-12 < -10$$

<u>Axioms of Inequality</u>

Additive Axioms	**if $a < b$ and $c > 0$, then $a + c < b + c$**
	if $a > b$ and $c > 0$, then $a + c > b + c$
Multiply Axioms	**if $a < b$ and $c > 0$, then $a\,c < b\,c$**
	if $a > b$ and $c > 0$, then $a\,c > b\,c$
	if $a > b$ and $\underline{c < 0}$, then $a\,c < b\,c$

When you multiply or divide an inequality by a negative number, you reverse the sign of the inequality.

Example 2 Find the solution set. $-3x - 2 > 10$

$-3x - 2 > 10$	Given
$-3x - 2 + 2 > 10 + 2$	Add Prop Inequality
$-3x > 12$	Add Inverse
$x < -4$	Mult Ineq by Neg.

Notice that when we divided by a negative number, we reversed the order of the inequality.

We could have done the same problem without multiplying or dividing by a negative number by keeping the variable positive. That would result in not having to reverse the inequality. Let's look at the same problem.

Example 3 Find the solution set. $-3x - 2 > 10$

Rather than having the variables on the left side and have the coefficient negative, I could put the variables on the right side (add 3x to both sides) that would result in a positive coefficient.

$-3x - 2 > 10$	Given
$-2 > 3x + 10$	Add Prop Inequality
$-2 - 10 > 3x + 10 - 10$	Subtract Ax Ineq
$-12 > 3x$	Add Inv/ CLT
$-4 > x$	Div Ax. Of Ineq

In math, we read the variable first, so the correct way to say $-4 > x$ is *x is less than negative four.*

In both cases, we have the same answer. In the second example, since I kept my variables positive, I did not have to reverse the order of the inequality.

Bottom line, you solve linear inequalities the same way you solve linear equations – use the Order of Operations in reverse using the opposite operation after they are in ***ax – b = c*** format.

Remember, if you don't keep the coefficients of the variable positive, then you will likely have to multiply or divide by a negative number that will require you to change the order of the inequality.

Sec. 4 Double or Compound Inequalities

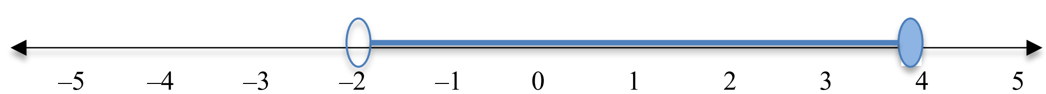
A double or compound inequality consists of two inequalities connected by an "and" or "or".

Example 1 $x > -2$ **and** $x \leq 4$

In this example, we are looking for numbers that meet two conditions, numbers greater than two **and** at the same time, numbers less than or equal to 4.

That would be all numbers greater than –2 and less than 4, including 4 which would be described as numbers between –2 and 4. We would write that ~ $-2 < x < 4$. If I wanted to include 4, I could say the numbers between –2 and 4 including 4. To show 4 was included, we would write $-2 < x \leq 4$.

As we did with simple inequalities, to graph that we would place an open circle around 2 and a shaded circle around 4 to show it was included in the solution, then graph all the pointes between –2 and 4.

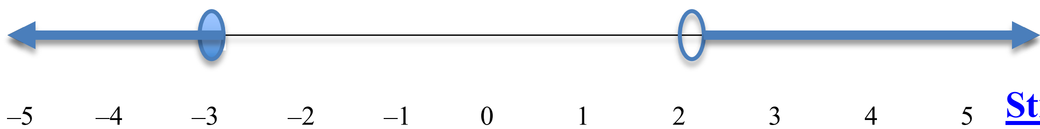


When reading a compound inequality connected by "and", it is often written as a double inequality. So rather than write those inequalities as $x > -2$ and $x \leq 4$, we would write that as $-2 < x \leq 4$. The conventional way of reading that is to read the variable first, then read the inequalities using "and". So, that would be read, x is less than or equal to 4 **AND** greater then –2.

Example 2 $x \leq -3$ **or** $x > 2$

In this example using an "**or**" statement. In an "or" statement either one or both of the statements can be true. So x could be

–3 or a number less than negative three or x could be greater than 2. If either of those statements are true, then those numbers satisfy the compound inequality.



Strategy for Solving Double Inequalities

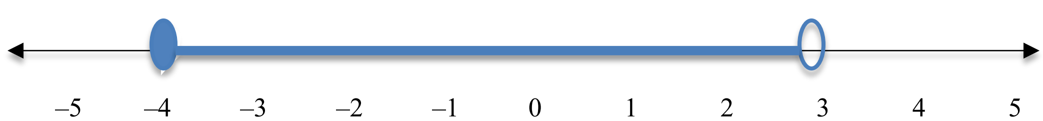
To solve a double inequality, you solve the two inequalities independently, then use the "and" or "or" statement to determine the solution set. In other words, solve the middle to the right of the inequality, then solve from the middle to the left.

Example 3 Solve the inequality and graph; $1 \leq x + 5 < 8$

I recognize the notation as an "and" statement, so both simple inequalities have to be true. Writing them from middle to left, then middle to right, we have

	$1 \leq x + 5$	and	$x + 5 < 8$
Solving each;	$-4 \leq x$	and	$x < 3$

Values of the variable that are greater than –4 and less than 3 are all the numbers between – 4 and 3 including –4. That is $-4 \leq x < 3$



Example 4 Solve the inequality; $-3 < 2x + 1 \leq 9$

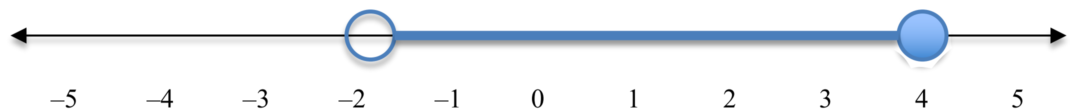
Solving the two simple inequalities joined by an "and", we have

$-3 < 2x + 1$ and $2x + 1 \leq 9$

$-4 < 2x \qquad\qquad 2x \leq 8$

$-2 < x \qquad\qquad x \leq 4$

The solution are all the numbers greater than negative two and less than positive 4, including 4. That is, $-2 < x \leq 4$

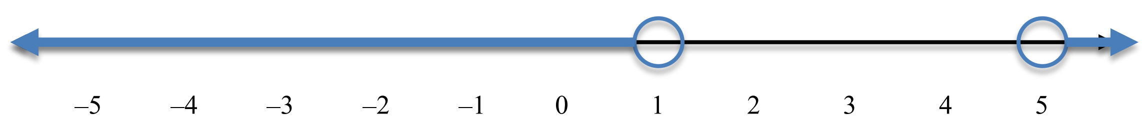


Example 5 Solve the inequality, $2x - 1 > 9$ or $3x + 2 < 5$

Solve the two simple inequalities connected by "or", we have

$2x - 1 > 9$	or	$3x + 2 < 5$
$2x > 10$		$3x < 3$
$x > 5$		$x < 1$

We want numbers that are either less than one or greater than five. Since either one of the simple inequalities has to be true to make the entire compound statement true, the solution set is the set of all possible numbers that make either of those true. So, the solution set is $x > 5$ OR $x < 1$



So, to solve inequalities you have to know the difference between an "and" and an "or" statement. In an "and" statement, both conditions must be met. In an "or" statement only one of the conditions must be met.

Sec. 5 Solving Equations containing Absolute Value

Mathematics uses many symbols such as: +, – , x, and ÷. Another symbol we use is the ABSOLUTE VALUE sign.

Absolute Value is used when we want to concern ourselves with positive numbers. Finding a distance is a good example of using absolute values.

The absolute value of a number is defined to be a positive number. For example, the $|5|$, read the absolute value of 5, is POSITIVE 5.
The $|-6| = +6$. In arithmetic, that's pretty simple, we always take the positive value when finding the absolute value.

In algebra, we are looking for all possible values of the variable that make an open sentence true. So, we will define the $|x|$ to ensure we get those values. The argument inside the absolute values signs could be positive or negative, but when you take the absolute value, the answer is positive.

Mathematically, we write

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

That definition of absolute values suggests we will always have two answers when solving equations containing absolute value. One answer when the expression in the absolute values signs is positive, a second answer when the expression in the absolute value signs is negative.

When solving equations containing absolute values, we are looking for ALL values of the variable that will make the open sentence (equation) true - just as we have done in the past.

Example 1: Solve for x, $|x| = 8$

I am looking for values of x so the $|x|$ is 8. The number inside the absolute values signs could be positive or negative. Clearly x could be 8 because the $|8|$ is 8. But x could also be negative, x could equal –8 because the $|-8|$ is 8.

We have two values of the variable that make the open sentence true. So x could be 8 or –8.

Let's make the problem a little longer.

Example 2: Solve for x, $|x - 1| = 10$

Let's reason this out a bit. The $(x - 1)$ within the absolute value signs represents a number. That number, $(x - 1)$ could be 10 or –10 because the $|10|$ is 10 as is $|-10|$ is 10 also.

What that means algebraically is

$x - 1 = 10$, if $x - 1$ is positive or $-(x - 1) = 10$, if $x - 1$ is negative

If I solve those two equations, I will find the values of the variable that will make the open sentence true. One value will represent when $(x - 1)$ is positive, the other when it is negative. Let's solve it.

$$
\begin{array}{rcr}
x - 1 = 10 & \text{or} & -(x - 1) = 10 \\
x = 11 & & x - 1 = -10 \\
& & x = -9
\end{array}
$$

If we substitute these values into the original equation, we see they both work.

If we did enough of these problems, we'd realize there would always be two answers - solutions. One solution will occur when the expression inside the absolute value sign is positive (greater than zero), the other solution will occur when that expression is negative.

Solving Equations with Absolute Value

1. Isolate the absolute value
2. Set the positive and negative of the expression inside the absolute value signs equal to the number on the outside
3. Solve the resulting equations in the $ax + b = c$ format

Example 2: Solve $|2x - 3| = 13$

Since the expression $2x - 3$ could be positive or negative we write

$$2x - 3 = 13 \qquad \text{or} \qquad -(2x - 3) = 13$$

Solving each equation, we have

POSITIVE		NEGATIVE
$2x - 3 = 13$	or	$-(2x - 3) = -13$
$2x = 16$		$2x - 3 = -13$
$x = 8$		$2x = -10$
		$x = -5$

Let's substitute those in to $|2x - 3|$ to check. If $x = 8$, then $2x - 3$ is $+13$ and the $|+13|$ is 13. That's true.

If $x = -5$, then $2x - 3$ is -13 and the $|-13|$ is 13. Both solutions work.

There are two solutions, $x = 8$ or $x = -5$.
Mathematically, we write the solution set in brackets $\{8, -5\}$.

Absolute value inequalities are solved in much the same way

Example 3 Solve: $|x| < 8$

In this problem, I am looking for all the values of the variable that make the open sentence true. In other words, for what values of x is the $|x| < 8$.

If I plugged in numbers, I would see 7, 6, 5, 4, 3, 2, 1, 0. –1, – 2, –3, _–4, –5, –6, and –7 all work. Notice – 9 does not work.

To write that mathematically, I want all the values of x that meet these two conditions, numbers that are less than 8 and greater than – 8

$$x < 8 \ \underline{\text{and}}\ x > -8$$

$$-8 < x < 8$$

Example 4 Solve: $|\mathbf{x}| > 8$

How is this problem different than the last example? The inequality has been reversed. So what numbers work? Clearly numbers greater than 8 work. Any other numbers work? Well. –9, – 10, etc work or numbers less than a – 8 work.

So our solution is $x < -8$ **or** $x > 8$. That's written $x < -8 \cup x > 8$

Example 5 Solve: $|5x - 3| > 27$

In this problem, we are looking for a value of x that makes the absolute value of $(5x - 3) > 27$

That means we are looking for values of x so that $(5x - 3)$ is less than 27 or when $(5x - 3)$ is greater than 27.

Translated that means that

$$\begin{array}{lcl} 5x - 3 > 27 & \text{OR} & -(5x - 3) > 27 \\ 5x > 30 & & 5x - 3 < -27 \\ x > 6 & & 5x < -24 \\ & & x < -24/5 \end{array}$$

So, the solution set, values of the variable that make the open sentence true are

$$x < -24/5 \ \ \mathbf{U} \ \ x > 6$$

Example 6 Solve: $|2x - 5| < 9$

Translated that means that

$$2x - 5 < 9 \quad \text{AND} \quad -(2x - 5) < 9$$

$$2x < 14 \qquad\qquad 2x - 5 > -9$$

$$x < 7 \qquad\qquad 2x > -4$$

$$x > -2$$

$$-2 < x < 7$$

It might be helpful to keep in mind when the absolute value is less then, you have a conjunctive (and) statement, when it's greater than, you have a disjunctive (or) statement.

*Remember, when you take an absolute value of a number, that is, take the number out of the absolute value brackets, the number has to be positive – by definition - greater than zero!

Example 7 Solve $|2x + 3| = -5$

The solution is the empty set, Ø, there is no value of the variable that will make this open sentence true. The number on the outside must be positive.

Solve the following equations containing absolute value

1. $|y - 5| = 8$
2. $|x + 2| = 10$
3. $|2x - 1| = 9$
4. $|3x + 4| = 10$
5. $|x + 8| - 2 = 11$
6. $|z - 5| + 4 = 16$
7. $2|x - 2| + 5 = 17$
8. $3|2x + 1| - 7 = 21$
9. $|x + 3| > 4$
10. $|2x - 1| \leq$