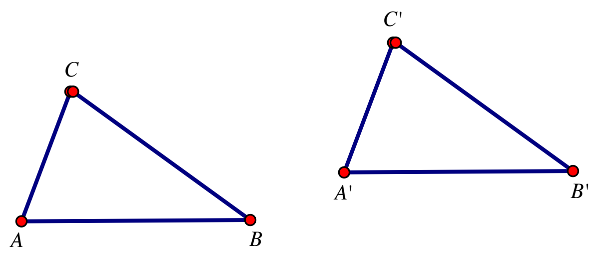
# Ch. 00 Transformations

## Sec. 1 Mappings & Congruence Mappings

Moving a figure around a plane is called mapping. In the figure below, ΔABC was moved (mapped) to a new position in the plane and the new triangle formed, ΔA'B'C' is called the image of ΔABC.




*ΔABC is mapped into ΔA'B'C'*

Mathematically we write, ΔABC ———> ΔA'B'C'

**A transformation is a mapping such that;**

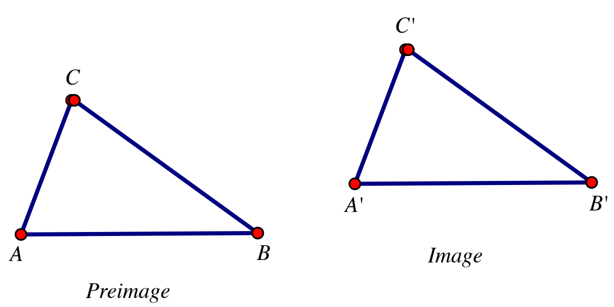
1. **each point in the plane has exactly one image**
2. **for each point in the plane there is exactly one preimage**

We have seen transformations in algebra when we moved graphs around the coordinate plane.

We want to look at different kinds of transformations that maintain their same size and shape, they are called isometries or congruence mappings. By definition, an isometry is a transformation that preserves distance and angle measure.

So, looking at the figure above, the distance between AA', BB', and CC' are all the same and all the angles, respectively are congruent – have the same angle measure.

Again, looking at the figure below, the original figure is called the preimage and the final shape and position of the figure is called the image.




*ΔABC ---> ΔA'B'C'*

**Sec. 2 Isometries**

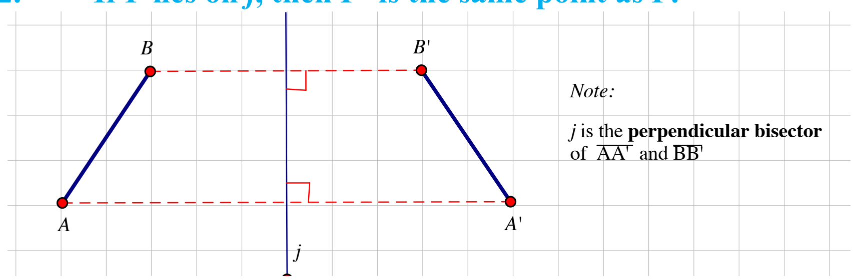
In this section, we will look at three transformations that are isometries; a reflection, translation and a rotation.

**Reflection**

We all have experienced a reflection, look in the mirror. Pretty simple, right? We will formalize that definition this way.

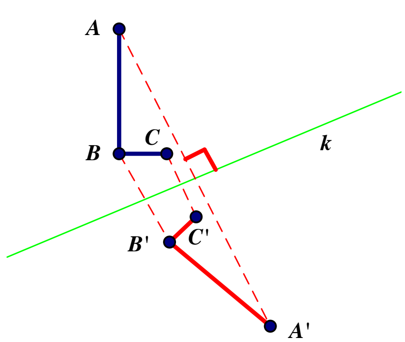
**A reflection in some line *j* maps every point P into a point P', such that:**

1. **If P does not lie on *j*, then *j* is the perpendicular bisector of $\overline{PP'}$**
2. **If P lies on *j*, then P' is the same point as P.**




That reflection was rather obvious because we reflected the points across a vertical line. Not so obvious when reflecting across "any" line.

Using the definition, let's reflect the "L" across the line ***k*** below.




By definition, line ***k*** is the **perpendicular bisector** of AA', BB' and CC'.

That means when we draw our lines they must be perpendicular.

And the distance from point A to line ***k*** = the distance from point A' to line ***k.***

While the definition of a reflection will allow us to either draw or construct reflections in any line, we are going to look at specific reflections in the x-axis, y-axis and the line y = x by examining their coordinates.

And of course, since we are doing math, we introduce mathematical notation. The notation we will use to indicate a reflection is the small letter "r". Some books use a capital "M" for a reflection so it not confused with a rotation. An "M" for mirroring.

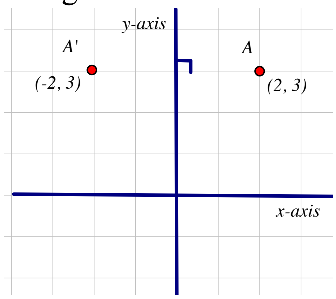
We can look at some simple reflections being mapped over vertical and horizontal lines and see some patterns that develop that allow us to give immediate answers to reflection problems. For instance, looking at reflections across the y-axis, we can see in an ordered pair, the sign of x changes. That suggests the following rule.

$$\mathbf{r}_{\text{y-axis}}\ (\mathbf{x},\mathbf{y}) \longrightarrow (-\mathbf{x}, \mathbf{y})$$

This is read as *"reflection in the y-axis of (x, y) is mapped into (–x, y)."*

**Example 1** Find r $_{\text{y-axis}}$ (2, 3)

Using the definition above, we change the sign of the x-coordinate, (–2, 3)




Notice the y-axis would be the perpendicular bisector of line segment AA'

So, putting it simply, to reflect points across the y-axis, we just change the sign of the x coordinate.

**Example 2** Find r $_{\text{y-axis}}$ (–4, 5)

Changing the sign of the x-coordinate, we have (+4, 5)

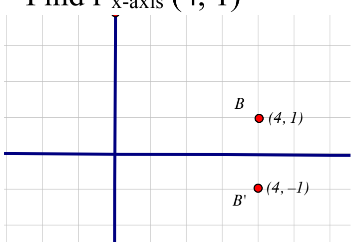
If you were to plot those points on the coordinate axes, you would see the y axis would be the perpendicular bisector of the line segment connecting those points.

Let's look at what happens if we want to reflect points across the x-axis. To reflect across the x-axis, what value will need to be changed? If you said the y value, you got it. Let's write the rule.

$$\mathbf{r}_{\text{x-axis}}\ (\mathbf{x}, \mathbf{y}) \longrightarrow (\mathbf{x}, -\mathbf{y})$$

Seems pretty simple, right. If you just try to memorize that without visualizing that reflection, you might confuse it with a reflection across the y-axis – so don't! Picture a point being reflected across the x-axis and you will know the y coordinate is the one that changes.

**Example 3** Find r $_{\text{x-axis}}$ (4, 1)



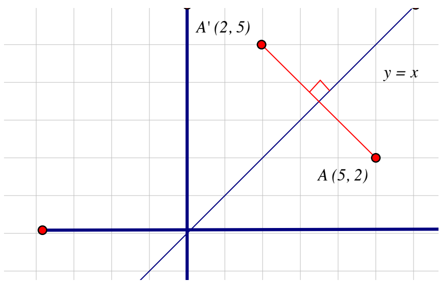

Knowing the rule, or visualizing, we just change the sign of the y-coordinate to (4, –1)

**Example 4** Find the $r_{x\text{-axis}}$ (–7, –2)

Using the rule or reasoning, we change the sign of the y-coordinate; (–7, +2)

Our next reflection is across the line, y = x. To find that reflection, we graph y = x on the coordinate axes and pick some point (x, y). Then, by the definition of a reflection, we draw a line through the point (x, y) that is perpendicular to the line y = x. Now the distance from the point (x, y) to the line must be equal to the distance from the line y = x to the new point.

If we drew a few of these, you would notice something very interesting occurring.



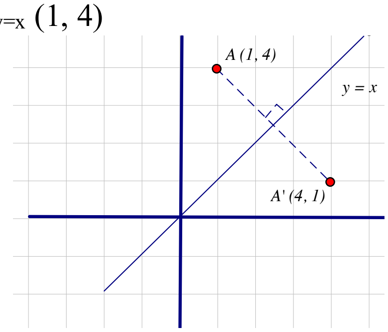

Looking at the graph, if the preimage was the point (5, 2), its image on the other side of the line y = x is (2, 5). *The x and y coordinates are interchanged!*

Those types of observations would lead us to a new rule for a reflection of a point across the line y = x.

$$\mathbf{r_{y=x}\,(x,\,y) \longrightarrow (y,\,x)}$$

That is, we simply **interchange** the x and y coordinates.

**Example 5** Find $r_{y=x}$ (1, 4)




Interchanging the coordinates, we have (4, 1). Piece of cake, right?

**Example 6** Find $r_{y=x}$ (2, –4)

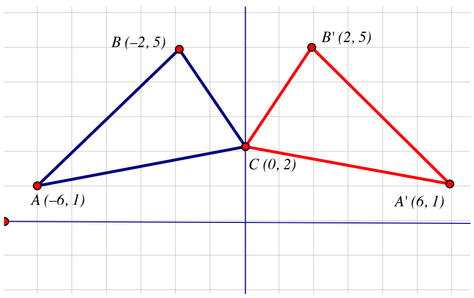
Using the rule that came from our observations, interchange the x and y coordinates, we have (–4, 2)

In all the above examples, we used the “rules” to reflect just one point across the axes or the line y = x. If we had to reflect polygons or other figures, we would need to map more points. For instance, a triangle is defined by 3 points, we would need to map all three to get the image. A quadrilateral would require four points to be reflected, and so on.

So, the good news is that if you can do one reflection, it's not more difficult to reflect four, the problem only takes longer.

Let's look at an example.

Example 7 Find $r_{\text{y-axis}}$ ΔABC




Notice, to find the reflection across the y-axis, we used the same rule 2 times. That is, we changed the sign of the x-coordinate for points A and B. Since C was on the y-axis, it stayed the same.

The good news is, like in all math, I just can't make these problems more difficult, only longer. And, if you can find these images, you can reverse the rule if given the image to find the original points.

For instance;

**Example 8** If A'(2, –3) is the image of a point A reflected across the y-axis, find the coordinates of A.

The rule to reflect a point across the y-axis is $r_{\text{y-axis}}$ A (x, y) → A'(–x, y)

We have A(x, y) and A'(2, –3), we can see we changed the sign on the x-coordinate, so let's change it back, that results in A(–2, –3).

Always in math, definitions are important. Using the definition of a reflection in a line, we can answer questions like the following:

**Example 9** If point B is 3.5 inches from the line of reflection, then BB' =

Remember, the line of reflection is the perpendicular bisector of the $\overline{BB'}$. That means that line $\overline{BB'}$ is cut in half. So if the distance to the line of reflection is 3.5, then the other side is 3.5, so BB" = 7

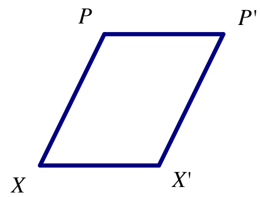
## Translations

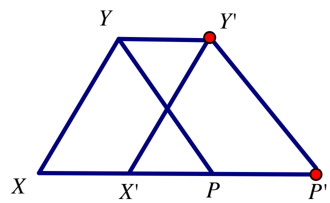
Picture yourself going down a slide, you start at the top and slide down to the bottom. That's a translation.

While that "slide" description works in elementary school, we need a more formal definition for secondary students.

**A translation that maps X into X' maps every point P into P' such that:**

1. **If P does not lie on $\overleftrightarrow{XX'}$ , then PXX'P is a parallelogram.**
2. **If P does lie on $\overleftrightarrow{XX'}$, then there is a segment $YY'$ such that both XYY'X' and PYY'P' are parallelograms.**



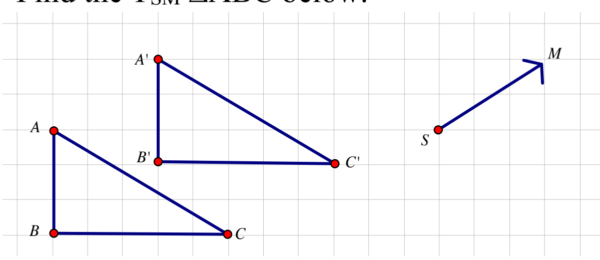




Now, that's impressive. But the great news is the translations are the easiest of all the transformations.

To do a translation, we move the points around the coordinate plane by looking how the translation is defined either graphically or algebraically.

The notation we use for a translation is that is moved graphically is T $_{(SM)}$ (x, y). That is read the point (x,y) is mapped under the translation SM. We look at SM and use the slope to move the ordered pair.

**Example 1** Find the $T_{SM}$ ΔABC below.




Using the slope of $\overline{SM}$, every point must go over 3 and up 2 to find the image.

We can also do perform a translation given points and how those points will be translated by being given the movement along the x and y axes.

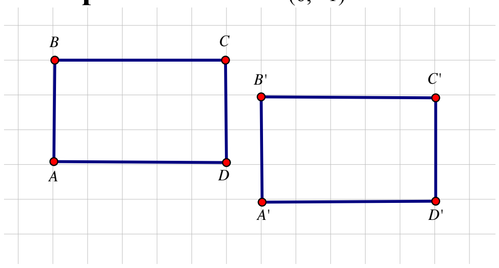
If we were not given a drawing, we might see this notation for a translation. The "a" moves the points P over "a" and the "b" moves the points P up "b' units.

$$\mathbf{T_{(a,b)}\ (x, y) \longrightarrow (x + a, y + b)}$$

**Example 2** Find the $T_{(3,-4)}(5, 2)$

Under this translation, the point (5, 2) will move over 3 and down 4 to find the image. $(5+3, 2+-4) = (8, -2)$

**Example 3** Find $T_{(6,-1)}$ □ABCD




Notice all the points, A, B, C, and D were moved 6 spaces to the right and 1 space down.

These are just too easy. Translations, like reflections are isometries, which mean they maintain their size and shape, they are congruence mappings.

If we played with these new transformations, we start to see patterns form. For instance, if I reflect an picture across two parallel lines, the end result is the same as a translation of theorigian figure. What does that composition mean, it means I reflect the picture over one line, then reflect it again over the other line that is parallel.

**The composition of two reflections over parallel lines has the same effect as a translation twice the distance between the parallel lines.**

**Example 4** Convert the double reflection over parallel lines to a single translations if $r_{y=4} \circ r_{y=1}(P) = T_{a,b}\, P(-5, 2)$.

So, the first thing we do is find the image of the two reflections. *Compositions are done from right to left!* So we reflect (−5, 2) over y = 1 first. Then we take that image and reflect it across y = 4 to find the composition.

Reflecting (–5, 2) over y = 1 results in the image at (–5, 0) – draw the picture. Now reflect (–5, 0) over the line y = 4, results in (–5, 8). Now, since we know the coordinates of the final image, let's plug that into our translation formula to find a and b.

$$T_{a,b}\, P(-5, 2) = (-5, 8)$$

How many spaces to the right were moved? None, so a = 1, how'd we get from 2 to 8, we moved up 6, so b = 6. Too easy, right?

$$\text{So } T_{0,6}\, P(-5, 2) = (-5, 8)$$

And finally, notice the distance between the parallel lines was 4 −1 = 3, doubled will give us the distance between the picture and image → 6.

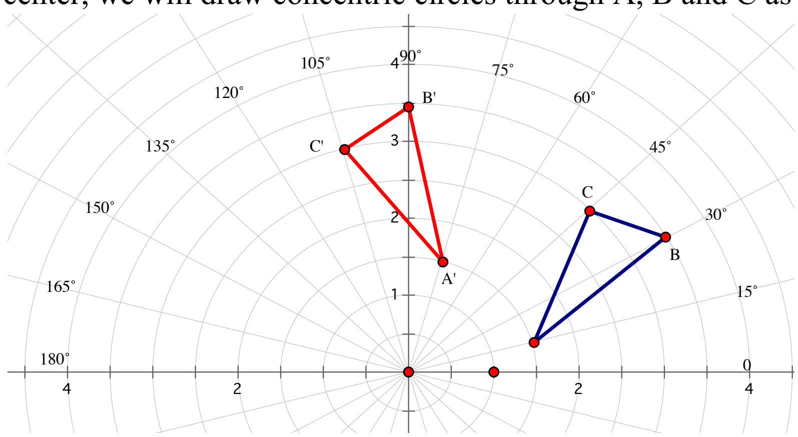
Let's look at another transformation that is an isometry, a rotation.

**Rotation**

If you have ever been on a merry-go-round, then you have experienced first-hand a rotation. Let's take an informal look at those merry-go-rounds and assume a person looks like a capital T. If T stayed in the same position as the merry-go-round went around (rotated), T would always be the same distance from the center of the merry-go-round. A complete rotation would be circular. As the merry-go-round went along, if T did not move, what he would see would be different from what he saw from his original position. If T had a friend, call him P, that was next to him, his distance from the center would not change and would not be the same as T's distance from the center.

We can use that information to find rotations. What we will do is look at a figure $\Delta ABC$ being rotated 60˚ around some point O, called the center. To help identify points on $\Delta ABC$, I will label them A, B and C.

Using O as the center, we will draw concentric circles through A, B and C as shown below.




As we indicated earlier, A, B, and C will stay on their own circles. Now, what would that figure look like if we rotated the $\Delta ABC$ around the center at 60˚? Looking at the polar graph, each radius represents a change in 15˚so we can count to get to 60˚ rotation, that means each point would move 60˚ and stay on it's on circle.

All the points A,B and C that made up $\Delta ABC$ were all moved (rotated) 60˚ and stayed on their own circle, the radii of each stayed the same.

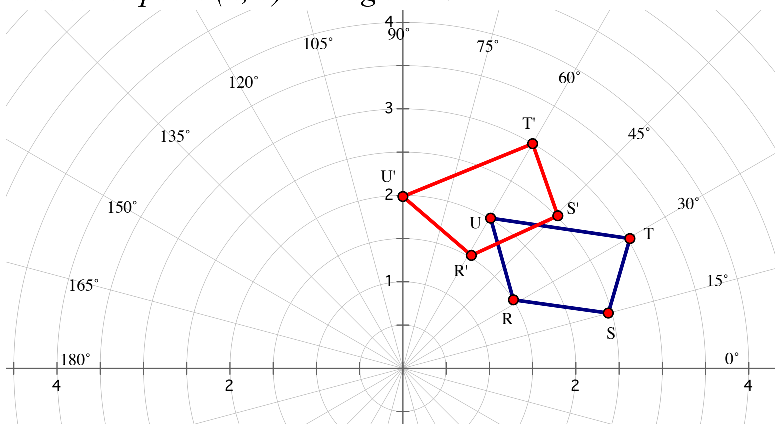
Let's look at a more formal definition of a rotation.

**A rotation about a point O through $\alpha$˚ maps every point P into P' such that:**

1. **If P is different from O, then OP' = OP and $m\angle P'OP = \alpha$˚**
2. **If P is the point O, then P' is the same as P**

I could do the same thing for any angle measurement, $\boldsymbol{\alpha}^\circ$. All I would need to know is a center point and the degree measure. From there, I would draw concentric circles, and use my protractor to find the degree measure required. To do that, I would connect each point defining my figure to the center, then measure the degrees that number of times.

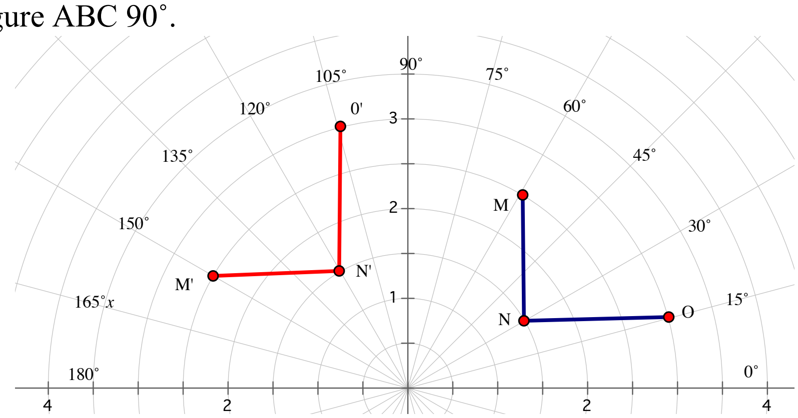
Now, the mathematical notation used to describe a rotation is $R_{(a,b)\ 30^\circ}(x, y)$. That is read *a rotation of (x, y) about the point (a, b) through 30˚.*




Notice each point was rotated on its own circle 30˚.

If we played with these rotations long enough, specifically with rotations of 90˚, we could see some patterns develop that would allow us to actually find coordinates of 90˚ rotations by inspection.

Let's rotate figure ABC 90˚.




And the special cases are:

$R_{(0,0)\ 90^\circ}(x, y) \longrightarrow (-y, x)$
$R_{(0,0)\ 180^\circ}(x, y) \longrightarrow (-x, -y)$
$R_{(0,\ 0)\ 270^\circ}(x, y) \longrightarrow (y, -x)$

These rules come from our circle coordinates studied in trig.

Those are mappings you need to know

**Example 1** Find the $R_{(0,0)\ 90^\circ}(5, 2)$

Using the formula, the coordinates are interchanged with a sign change
(–2, 5)

**Example 2** Find the $R_{(0,0)\ 180^\circ}$ (3, –2)

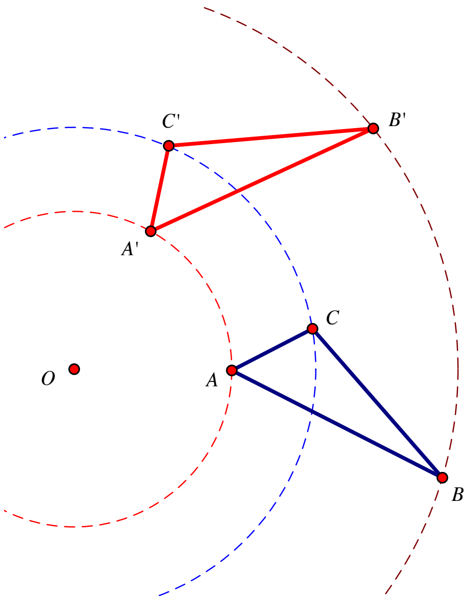
Using the formula, both x and y change signs: (–3, +2)

**Example 3** Find the $R_{(0,0)\ 270^\circ}$ (4, –6)

Using the formula, (–6, –4)

**Example 4** Find the $R_{O\ 60^\circ}$ ΔABC

In the previous graphs, we used polar graphs that made moving the points pretty simple. If I did not have a polar graph, I would have to construct my own circles and use a protractor to rotate the points. Here's how you'd do that.




We have studied no rule for 60˚, so we can draw the rotations using the following steps.

1. For A, draw radius OA, construct a circle.
2. Use a protractor, using OA as the initial ray and measure 60˚, plot point A' on the circle.
3. For B, draw radius OB, construct a circle.
4. Use a protractor, using OB as the initial ray and measure 60˚, plot point B' on the circle
5. For C, draw radius OC, construct a circle.
6. Use a protractor, using OC as the initial ray and measure 60˚, plot point C'
7. Connect A', B' and C' for the image.

So we have rules, shortcuts to find rotations of 90, 180 and 270˚. All others we'd find by constructing like in the last example.

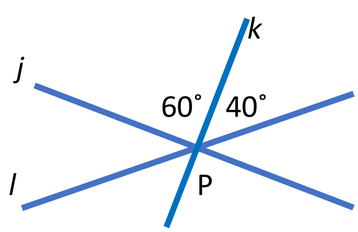
The transformations: reflections, rotations and translations are all isometries – they keep their same size and shape.

We could also explore and find other relationships. It turns out **a composition of a reflection over intersecting lines is the same as a rotation that is twice the measure of the angle formed by the intersecting lines.**

**Example 5** Use the figure on the right to find the measure of the angle of rotation if $r_j \circ r_k$ (A) = $R_P$ __(A).

The $m\angle$ between the intersecting lines is 60˚

$\therefore r_j \circ r_k (A) = R_{P\ 120}(A)$



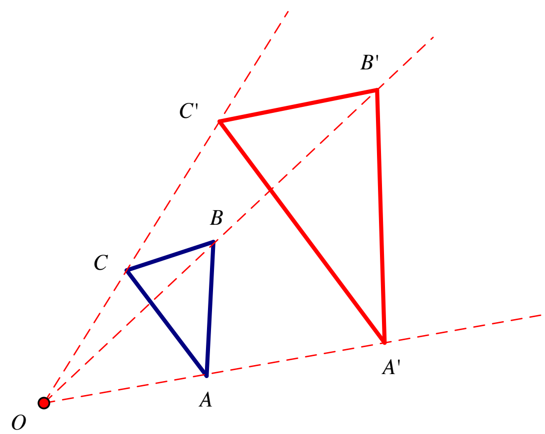
**Sec. 3 Dilations, a transformation that is not an isometry**

A dilation is a transformation that is not an isometry, it is not a congruence mapping. A dilation is a transformation that is related to similarity. A good example of a dilation is with the use of a projector. The further the projector is to a wall, the larger the picture. The closer the projector is to the wall, the smaller the picture.

**A dilation with center O and scale factor k (k > 0) is a mapping such that:**

1. **If P is different form O, then P' lies on $\overrightarrow{OP}$ and OP' = k (OP)**
2. **If P is the point O, then P' is the same point as P**

Beginning with ΔABC and point O, let A' lie on $\overrightarrow{OA}$ so that OA' = 2(OA).
Let B' lie on $\overrightarrow{OB}$ so that OB' = 2(OB), and let C' lie on $\overrightarrow{OC}$ so that OC' = 2(PC)




If k > 1, the dilation is called an expansion. If 0 < k < 1, the dilation is a contraction.

Writing a dilation mathematically, we have

$$\mathbf{D_{(0,0)\,k}\ (x, y) \longrightarrow (kx, ky).}$$

That is read, a dilation of the point (x, y) with center (0, 0) and scale factor k is mapped into the point (kx, ky)

**Example 1** Find the $D_{(0,\,0)\,4}$ (2, 7)

Using the above mapping, we multiply both coordinate by the scale factor 4. Therefore, we have (8, 28)

**Example 2** Find the $D_{(0,\,0)\,½}$ (6, 14)

Using the mapping, we multiply both coordinates by the scale factor.

Therefore, we have (3, 7)

If the dilation is not about the origin, (0, 0), then the mapping is described by:

$$\mathbf{D}_{(a,\,b)\,k}\,(\mathbf{x},\,\mathbf{y}) \longrightarrow (\mathbf{a} + \mathbf{k}(\mathbf{x}-\mathbf{a}),\ \mathbf{b} + \mathbf{k}(\mathbf{y}-\mathbf{b}))$$

That is read *the dilation of a point (x, y) with center (a, b) and scale factor k is mapped into the point (a + k(x–a), b + k(y–b)).*

In remembering this mapping, notice the x's and a's are x-coordinates, the y's and b's are the y-coordinates – keep them together when writing the ordered pair.

**Example 3** Find the $D_{(2,3)\,5}$ (4, 6)

Using the above mapping and substituting,

we have $(2 + 5(4 - 2), 3 + 5(6 - 3))$

$(2 + 5(2), 3 + 5(3))$

$(2 + 10, 3 + 15)$

$(12, 18)$

**Example 4** Find the $D_{(1,\,-2)\ \frac{1}{2}}$ (9, 10)

Using the mapping, $(1 + \frac{1}{2}(9 - 1), -2 + \frac{1}{2}(10 - -2))$

$(1 + \frac{1}{2}(8), -2 + \frac{1}{2}(12))$

$(1 + 4, -2 + 6)$

$(5, 4)$

If you were given the coordinates of the image, preimage and scale factor, you could find the center by just substituting those values back into the mapping formula.

**Example 5** Find the center of the dilation if the preimage was (10, 6), the image was located at (34, 22) and the scale factor was 5.

$(a + k(x - a), b + k(y - b)) \rightarrow a + k(x-a) = \text{image x coordinate}$

$a + 5(10 - a) = 34$

$a + 50 - 5a = 34$

$-4a + 50 = 34$

$-4a = -16$

$a = 4$

$b + k(y - b) = 22$ ; y-coordinate

$b + 5(6 - b) = 22$

$b + 30 - 5b = 22$

$-4b = -8$

$b = 2$

The center of dilation (a, b) is (4, 2)

Let's use the same mapping formula to find k, given the center, preimage, and image.

**Example 6** Find the scale factor k given the center is located at (2, 4), the preimage is (10, 5), and the image is located at (26, 7).

Use the mapping formula and substitute values for either the x or y coordinate, you don't need to do both.

| | |
|---|---|
| $a + k(x - a) = x$ coordinate | $b + k(y - b) = y$ coordinate |
| $2 + k(10 - 2) = 26$ | $4 + k(5 - 4) = 7$ |
| $2 + 8k = 26$ | $4 + k = 7$ |
| $8k = 24$ | $k = 3$ |
| $k = 3$ | |

The scale factor is 3.

**Review of transformation mappings**

$r_{x\text{-axis}}\ (x, y) \longrightarrow (x, -y)$
$r_{y\text{-axis}}\ (x, y) \longrightarrow (-x, y)$
$r_{y = x}\ (x, y) \longrightarrow (y, x)$
$r_{y = -x}\ (x, y) \longrightarrow (-y, -x)$
$r_{origin}\ (x, y) \longrightarrow ((-x, -y)$

$T_{(a, b)}\ (x, y) \longrightarrow (x + a, y + b)$

$R_{(0,0)\ 90^\circ}\ (x, y) \longrightarrow (-y, x)$
$R_{(0,0)\ 180^\circ}\ (x, y) \longrightarrow (-x, -y)$
$R_{(0,0)\ 270^\circ}\ (x, y) \longrightarrow (y, -x)$

$D_{(a,b)\ k}\ (x, y) \longrightarrow (a + k(x-a), b + k(y-b)))$

Keep in mind the rules show us how to map one point, when we have triangles or quadrilaterals, we will have to map three or four points to find the mapping.

## Sec. 4 Review Compositions of Mappings

Up to this point, we looked at single transformations and then performed a second transformation – calling those compositions. If you can do those transformations, then doing compositions of transformations (mappings) is just adding a step as we have seen. That is, you do the first transformation, then use that result to perform the second transformation with the new (x, y).

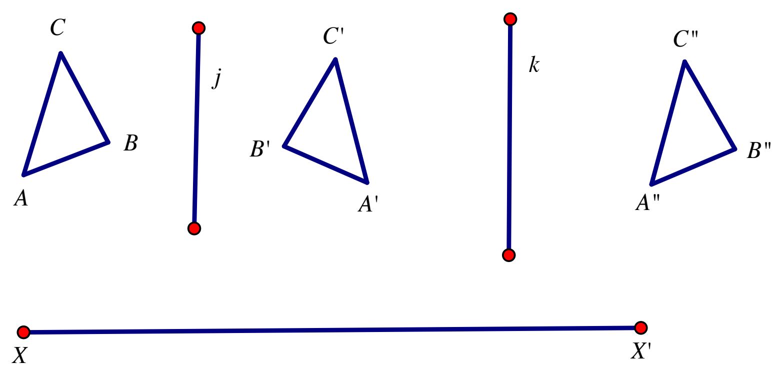
So, in essence, this section contains nothing new. It just requires you to do two or more transformations within one problem.

The notation for a composition of transformations looks like and works like composition of functions. There are two notations for compositions of both functions and transformations can be utilized. One notation is an open circle. A notation such as $T_{1,5}\,(x,y) \circ\ r_{\,y=x}\,(x,y)$ is read as *"a translation of (x, y) ——> (x+1, y+5) after a reflection in the line y = x."* or the translation $T_{(1,5)}$ of $r_{\,y=x.}$

You may also see this notation $T_{1,5}(x,y)\ (r_{\,y=x})$ which is read as *after* or *of* – either way. Using either notation, the process MUST be done from **right to left.** Just like in composition of functions in algebra.

And, like in all of math, we begin to see patterns form that we generalized. Those generalizations allow us to do our work quicker.

An example of a generalization is – Every translation is equal to the composition of two reflections.




$T_{XX'}\ \Delta ABC$ ——> $\Delta A''B''C''$ or

$r_{\,x=j}\ \Delta ABC$ ——> $\Delta A'B'C'$, $r_{\,x=k}\ \Delta A'B'C'$ ——> $\Delta A''B''C''$

As can be seen above, the translation using XX' results in ΔA"B"C" being in the same position and as the ΔABC and moved XX'.

We can also see the two reflections results in ΔA"B"C" being in the same position and has moved XX'. In other words, in either case, we get the same image.

Another popular generalization is every rotation is equal to the composition of two reflections.

The point, composition of mappings is pretty straight-forward as long as you know, have memorized, the mapping formulas and you remember to go from **right to left** when doing the composition mappings**.** While memorization is good, being able to visualize those also helps your memory, and the ability to recall the formulas.

And, with anything, the way you get better is by practice, practice, practice.

**Example 1** $\Delta$JKL has vertices J(6, –1), K(10, –2) and L(5, –3). Find the coordinates of its image after a translation of (0,4) and a reflection in the y-axis.

The notation for this example would be $\mathbf{r}_{y=x} \circ \mathbf{T}_{(0,4)}\,(\mathbf{x}, \mathbf{y})$ – **where J, K and L would be the ordered pairs (x, y).**

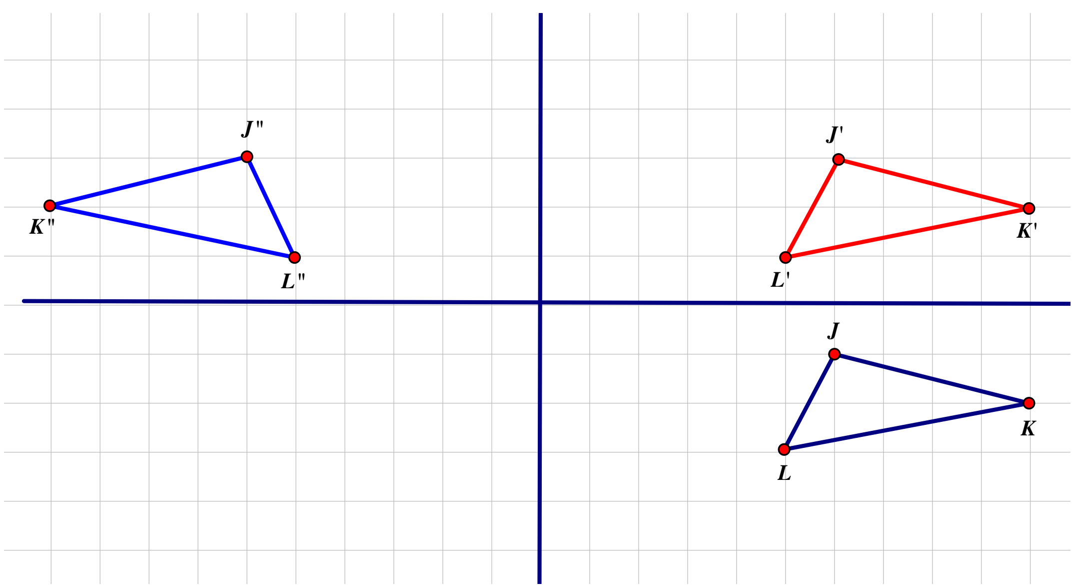
You could draw this composition to have a visual, but since a graph was not asked for, I'm just going to apply the rules we learned. In this case, just adding 4 to the y-coordinate.

$T_{(0,4)}$ J(6, –1) —> J' (6, 3)
$T_{(0,4)}$ K(10, –2) —>K' (10, 2)
$T_{(0,4)}$ L(5, –3) —>L' ( 5, 1)

Now I take my $\Delta$J'K'L' and do a reflection in the y-axis to find $\Delta$A"B"C". Remember, to do a reflection in the y-axis, we just change the sign of the x-coordinate.

$T_{(0,4)}$ J(6, –1) —> J' (6, 3) ———> J" (–6, 3)
$T_{(0,4)}$ K(10, –2) —>K' (10, 2)———> K" (–10, 2)
$T_{(0,4)}$ L(5, –3) —>L' (5, 1)———> L" (–5, 1)

Here is what this would look like if you did the problem graphically..



The good news with transformations, as long as we understand them and know the shortcuts, we can't make these more difficult – just longer.

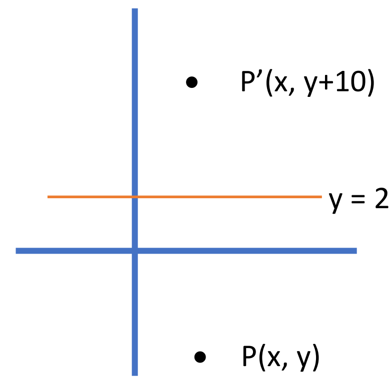
**Example 2** If you want to translate a shape up 10 units, you could reflect it over the line y = 2, if you do, find the value of y.

Let's draw the picture.

Since the translation is 10 units, half (5) has to be above the line of reflection, half has to be below.

So 5 units up from the line y = 2 results In y = 7.

So, y = 7




*If you did two or three of these, and looked at the question, you would see a pattern that would allow you to do the problem in your head.*

Moving 10 units up, half above the line of reflection, half below – just add 5 to the line of reflection and get 7.

**Example 3** If you want to translate a shape 6 units to the right, you could reflect it over the line x = 8, then x =

Without drawing the picture, if we are moving the shape 6 units to the right, half will be on each side of the line of reflection that's 3 units. Adding 3 to line of reflection, 8 + 3 = 11, so (11, y).

That's just too easy.

Let's convert some compositions to a single transformation and see how that's done.

**Example 4** Convert the double refection over parallel lines to a single transformation.

$r_{x=-1} \circ r_{x=7}(P) = T_{x,y}\, P(10, 7)$

So, I'm looking for the x and y in the translation.

The first thing I have to do is determine the results of the composition of the reflections. Once done, we set that equal to the translation.

As with all compositions, we work from right to left. So, let's find $r_{x=7}P(10, 7)$ 10 is 3 units to the right of the line of reflection x = 7, so P' has to be 3 units to the left of the line of reflection, resulting in (4, 7).

Now let's reflect that over the line x = –1 to complete the composition. Since 4 is 5 units to the right of x = –1, I need to go 5 units to the left of –1, which puts us at (–6, 7).

If it helps, go ahead and draw these on the coordinate axes.

Now, we finally get to answer the question being asked, to find the single translation that results in

$$T_{x,y}\, P(10, 7) = (-6, 7)$$

So, what do I add to 10 to get –6 and what do I add to 7 to get to 7?

The answers are –16 and 0 respectively. Therefore my single transformation that maps the composition of reflections into (–6, 7) is: $T_{-16,\,0}(10, 7)$.

*Remember, the more math you know, the easier it gets! You might recall that a reflection composition of reflections is a translation with the distance being the twice difference in the parallel lines.*

*So, instead of doing all the work, since we were reflecting over parallel lines, the translation is twice the – 1 – 7 = =–8, multiply by 2 = –16. And that's exactly what we got above; –16.*

That sure made that problem a lot easier – not to mention a lot less work.