

$$b^{\log_b x} = x$$

$$10^{\log a} = a$$

$$10^{\log b} = b$$

$$10^{\log ab} = ab$$

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|----|--|---------------------|
| 1. | $10^{\log ab} = ab$                                | Given               |
| 2. | $= (10^{\log a})(10^{\log b})$                     | Substitution        |
| 3. | $= 10^{\log a + \log b}$                           | Mult Rule Exp.      |
| 4. | $\therefore \rightarrow \log ab = \log a + \log b$ | Transitive Property |

$$\log ab = \log a + \log b$$

Therefore, we can say, *to find the logarithm of a product of positive numbers, you add the logarithms of the numbers.*

That follows our rules of exponents, **when you multiply numbers with the same base, you add the exponents.**

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$$10^{\log a} = a$$

$$10^{\log b} = b$$

$$10^{\log a/b} = a/b$$

Again, using the three equalities that are a direct application of  $b^{\log_b x} = x$ , let's look at what we can develop for division.

1.  $10^{\log a/b} = a/b$                       Given

2.  $= \frac{10^{\log a}}{10^{\log b}}$                       Substitution

3.  $= 10^{\log a - \log b}$                       Div Rule Exp.

4.  $10^{\log a/b} = 10^{\log a - \log b}$                       - Transitive Prop.

$$\log a/b = \log a - \log b \quad - \text{Exp Equation}$$

Knowing that  $a = 10^{\log a}$ . If each side is raised to the power of n, we have

1.  $a = 10^{\log a}$  Given
2.  $a^n = (10^{\log a})^n$  Exponent Power Rule
3.  $= 10^{n \log a}$  Exp. Raise Power to Power
4.  $(10^{\log a})^n = 10^{n \log a}$  Substitution
5.  $\log a^n = n \log a$  Definition

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Sometimes it is helpful to change the base of a logarithm such as  $\log_b n$  to a logarithm in another base.

$$\text{Let } x = \log_b n$$

$$b^x = n \quad - \text{Def of log}$$

$$\log_a b^x = \log_a n \quad - \text{log of both sides}$$

$$x \log_a b = \log_a n \quad - \text{Power rule – logs}$$

$$x = \frac{\log_a n}{\log_a b} \quad - \text{Div Prop. Equality}$$

$$\log_b n = \frac{\log_a n}{\log_a b} \quad - \text{Substitution}$$

So, we can see to change the base of a logarithm, we have

$$\log_b n = \frac{\log_a n}{\log_a b}$$

