Evaluating & Supervising Math Booklet

Recommendation Booklet
Solving Linear Equations & Inequalities

Sample
Suggestions & Recommendations
for initial instruction
Designed for Formal & Informal Evaluations
By School Administrators
that
Address Math Content Strategies

Addressing the needs of struggling students in math

Nevada has experienced a documented math teacher shortage since 1985. We are hiring less and less qualified teachers to fill vacant positions. Those less qualified are being supervised and evaluated by school administrators who know less math than the teachers they are supervising and rely on "instructional" strategies in their evaluations. To actually improve instruction that directly address math achievement, these booklets and associated workshops focus on "math content" strategies that directly impact what teachers teach, how they teach it, and assessments that support the understanding of mathematics.

The contents of this booklet contain specific math content strategies that could be used in the supervision and evaluation of math teachers. Those math content strategies include linkages to previously learned material or outside experiences that provide opportunities to review, reinforce or address student deficiencies as they introduce new material. Those linkages also allow teachers to introduce new concepts & skills in more familiar language which makes students more comfortable learning math.

Also provided in this booklet are simple, straight-forward examples that could be suggested to be used by teachers when first introducing a topic. These examples do not distract or bog students down in needless arithmetic – they keep the focus on the new learning. These problems are chosen specifically to build improve student confidence by building success on success. Those examples are followed by practice problems for students using numbers that don't distract students.

There are examples of practice problems that administrators could suggest to be used when scaffolding – again using simple straight, forward examples.

In addition, there are also strategies, procedures, and/or formulas that might be recommended to teachers so students have a written guide, with examples, as they process what they are learning and put that knowledge into words – language acquisition.

And finally, a sample test is included that is specifically designed to set students up for success.

Linear Equations & Inequalities – Evaluation & Supervision

LINEAR EQUATIONS – 1 VARIABLE

RECOMMENDATION

A suggestion would be to review the Order of Operations, then using examples provided below that require division before multiplication to emphasize multiplication and division have the same rank and must be done in order from left to right. Explain, not everyone did the operations in the same order & the importance of the having an agreement – the Order of Operations – so everyone does the problem the same way and arrives at the same answer.

$2 + 4 \times 5 = \begin{matrix} 30 \\ < \\ 22 \end{matrix}$ 2 reasonable answers – only one can be correct

Order of Operations – just an agreement like driving on the right side of the road so we all get the same answer.

Parentheses
Exponentials
Multiply/Divide } From Left
Add/Subtract } to Right

RECOMMENDATION

Provide a few examples in which division comes first to address common misconceptions.

EXAMPLES

Simplify the following: (emphasizing division can come first!)

$2 + 20 \div 2 \times 5 + 1$
$4 + 24 \div 6 \times 2 - 3$
$20 - 12 \div 2 \times 3 + 2$

RECOMMENDATION

Ask students the following question; if it costs $10.00 to enter the amusement park and $5.00 per ride, how many rides can go on if you have $90.00?

After students provide the answer using mental math, show them how they actually used algebra to solve the problem. $10 + $5r = $90

REOMMENDATION

Consider quickly introducing simple equations used in elementary school such as using $4 + \Delta = 6$ where they had to find the value that goes into the Δ by guessing and substituting numbers to find a number that worked. Then scaffold to a problem such as $2 \times \Delta + 4 = 14$.

EXAMPLES

Fill in the missing number – Guess & Check

$5 \times \Delta + 3 = 13$

$10 \times \Delta + 2 = 52$

Using the elementary example, $2 \times \Delta + 4 = 14$, show them how that looks in algebra. $2x + 4 = 14$. The only change being vocabulary & notation.

RECOMMENDATION

Go over the Gift Wrapping Analogy and show how that is used to develop a systematic approach to solving equations in algebra.

Based on the Gift Wrapping Analogy and the Order of Operations, provide students with a systematic strategy for solving all linear equations and have them write it in their notes.

More systematic approach for equations already in $ax + b = c$ format

Order of Operations

Parentheses
Exponentials
Multiply/Divide
Add/Subtract

RECOMMENDATION

Have the students write the following strategy in their notes

Strategy for Solving Linear Equations

Rewrite an equation in $ax + b = c$ format using the Properties of Real Numbers, then use the Order of Operations in reverse using the inverse operations to isolate the variable.

a. Identify what is physically different from $ax + b = c$

b. Get rid of it

EXAMPLES

$5x + 3 = 13$
$10x + 2 = 52$
$3x - 7 = 5$
$x/4 - 3 = 2$

PRACTICE PROBLEMS

$4x - 3 = 17$
$10x + 4 = 54$
$5x + 6 = 36$
$x/2 - 3 = 7$

Important Formulas, procedures or strategies, such as the Order of Operations should be written and left on the board for reference

When introducing equations, use numbers that don't distract students from the concept or skill being taught or bog them down in arithmetic.

RECOMMENDATION

After the students are comfortable solving equations using the before mentioned strategy, then have them begin to justify each step.

On high stakes tests, students must justify their answers. So after the students are comfortable with solving simple equations, do a couple of more examples by providing justifications making sure the equations solved are horizontally.

$7x + 3 = 31$	Given
$7x + 3 - 3 = 31 - 3$	Subtract Prop Equality
$7x + 0 = 28$	Add Inverse/Combine terms
$7x = 28$	Property of Zero
$x = 4$	Div. Prop Equality

Continually reference the strategy developed when doing problems.

SCAFFOLDING - Variables on Both Sides

RECOMMENDATION

Explain that problems cannot be made more difficult – only longer. Scaffold up by providing a couple of examples to be worked out for them using the *Get Rid of It*

strategy. Examples such as $5x - 2 = 2x + 19$ and $7x + 3 = 3x + 23$. Have students identify how these equations look physically different, by underlining or circling, than the equations solved previously – comparing and contrasting.

Use the strategy for solving linear equations, step by step, continually referring to that strategy as you are doing each step by speaking out loud as you proceed.

EXAMPLES

Do equations with variables on both sides following strategy written on board;

$8x - 2 = 3x + 28$
$10x + 4 = 7x + 25$
$7x - 2 = 3x + 30$

PRACTICE PROBLEMS

Solve:

$5x + 3 = 2x + 18$
$7x - 2 = 4x + 19$
$8x + 6 = 3x + 21$

RECOMMENDATION

As you continue to scaffold, continue to reference the strategy for solving linear equations, that is rewrite the equations into $ax + b = c$ format using the *Get Rid of It* strategy. Taking something you don't recognize and changing that into a pattern you do recognize. Comparing and contrasting the equations. Introduce equations containing parentheses by asking what is physically different, then have the students follow the *Get Rid of It* strategy.

SCAFFOLD AGAIN– Distributive Property

EXAMPLES

Solve for x

$4(3x - 2) - 2x = 22$
$3(2x + 1) - 4 = 11$
$5(2x + 3) - 4 = 21$

PRACTICE PROBLEMS

$5(2x - 3) + 5 = 20$
$2(x - 3) - 3 = 3x + 2$
$5(2x + 3) - 2(x - 4) = 2x - 1$

RECOMMENDATION

After the students are comfortable with solving equations with variables on both sides of the equation and the Distributive Property, have them then do a couple of longer problems.

EXAMPLES

$2x + 3(2x + 5) = 39$
$2(x – 3) – 3 = 3x + 2$
$5(2x + 3) – 4(x –2) = 3x + 26$

PRACTICE PROBLEMS

$3(2x – 3) + 4x = 5x + 16$
$4(3x – 2) – 2x = 22$
$5(2x + 3) – 2(x – 4) = 2x – 1$

RECOMMENDATION

After the students are comfortable and successfully solving equations, have them redo a few problems with justifications for each step

EXAMPLE

$6x + 8$	$= 2x + 4$	Given
$6x + 8 – 2x$	$= 2x + 4 – 2x$	Sub. Prop of Equality (SPE)
$4x + 8$	$= 4$	Add. Inv. / CLT
$4x + 8 – 8$	$= 4 – 8$	SPE
$4x + 0$	$= – 4$	Add. Inv. / CLT
$4x$	$= –4$	Prop of Zero or Add Identity
x	$= – 1$	DPE

PRACTICE PROBLEMS

$2x + 3(2x + 5) = 39$
$2(x – 3) – 3 = 3x + 2$
$5(2x + 3) – 4(x –2) = 3x + 26$

INEQUALITIES

RECOMMENDATION

To clarify solving inequalities when multiplying or dividing by a negative number, give students a few examples using a number line stressing numbers to the right on a number line are greater than numbers to the left.

If $a > b$, then $a + c > b + c$.
If $5 > 2$, then $5(3) > 2(3)$ ——≥ if $a > b$, then $ac > bc$
However, when multiplying /dividing by a negative number If $5 > 2$,
then $5(–3)$? $2(–3)$

if $c < 0$, and $a > b$, then $ac < bc$

When we multiply or divide by a negative number, to make the statement true, the order of the inequality must be reversed.

RECOMMENDATION

Write that finding as a rule. When multiplying or dividing by a negative number, reverse the order of the inequality.

RECOMMENDATION

Use same numbers in inequalities that were used in equalities so students know the strategy does not change.

EXAMPLES

Inequalities – (same procedure as equalities)

$5x + 3 < 2x + 18$
$7x – 2 \geq 4x + 19$
$8x + 6 \leq 3x + 21$

RECOMMENDATION

After students recognize that inequalities are essentially solved the same way as equalities, then do two examples where the order of the inequality changes because of either multiplication or division by a negative number

EXAMPLES

Inequalities with negative number

$–2x + 3 < 13$
$7x – 2 \geq 10x + 19$

PRACTICE PROBLEMS

$–4x + 2 > 30$
$8x – 2 \leq 10x + 6$
$10 – 4(x + 3) \geq 2x + 4$

RECOMMENDATION

Have the students write the following procedure in their notes

1. Rewrite the inequality in ax + b > c format
2. Isolate the variable using inverse operation using the Order of Operations in reverse
3. If you multiply or divide the inequality by a negative number, reverse the order of the inequality.

PRACTICE PROBLEMS

$-4x + 3 \geq 23$

$-2x + 3 < 8x + 33$

$4(2x - 3) + 5 > 5(3x + 7)$

COMPOUND (DOUBLE) INEQUALITIES

RECOMMENDATION

When reading reading a double inequality, read the variable expression first, then read the inequalities using "and". So, $-2 < x \leq 4$ would be read, x is less than or equal to 4 **AND** greater then –2.

Emphasize "and" and "or" statements with double inequalities and how an "and" statement occurs when the graphs overlap.

Stress solving inequalities and double inequalities follows our same strategy, rewrite the double inequality as two separate inequalities

RECOMMENDATION

Require students to write the following strategy in their notes.

Solving Double Inequalities

To solve a double inequality, you solve the two inequalities independently, then use the "and" or "or" statement to determine the solution set. In other words, solve the middle to the right of the inequality, then solve from the middle to the left.

Procedure:

1. Determine if the compound inequality is connected by "and" or "or".

2. Solve each inequality
3. If compound inequality is connected by an "and", then the solution must satisfy each statement. The solution is where the graphs of each overlap.

3a. If the compound inequality is connected by an "or" statement, then the solution can satisfy either or both statements. The solution is the graph of both solutions.

EXAMPLES

Solve for x;

$-11 < 2x + 1 < 7$

$-13 \leq 2x + 3 < 5$

$x + 1 < 3$ or $2x - 1 \geq 7$

PRACTICE PROBLEMS

$12 < 2x \leq 16$

$-3 \leq 2x + 1 < 9$

$2x - 1 > 9$ or $3x + 2 < 5$

ABSOLUTE VALUE EQUATIONS

Introduce absolute value as students learned form elementary school, then emphasize in algebra, we are looking for all the values of a variable that makes an open sentence true and therefore must have a more precise definition. We define absolute value as a piecewise function.

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

To increase student understanding, make the arguments simple, |x|, and scaffold to make longer problems. Then use the definition to the two possibilities – when the argument is positive and when the argument is negative.

Use the following procedure to solve equations containing absolute value using the get rid of it

RECOMMENDATION

Solving Equations with Absolute Value

1. Isolate the absolute value
2. Set the positive and negative of the expression inside the absolute value signs equal to the number on the outside creating 2 equations
3. Solve the resulting equations in the $ax + b = c$ format

EXAMPLES

$|x| = 8$

$|x - 1| = 12$

$|2x + 1| = 13$

PRACTICE PROBLEMS

$|2x + 3| = 13$

$|\ 2x - 1\ | = 9$

$|3(\ x - 2)| = 12$

$2|2x - 1| - 4 = 8$

Caution – if the definition of absolute value is not stressed and the first step is skipped, then students will experience difficulty with when solving inequalities.

ABSOLUTE VALUE INEQUALITIES

RECOMMENDATION

To determine if an absolute value inequality is an "and" or an "or" statement, ask students to identify what numbers would work for $|x| > 3$ and $|x| \leq 2$. Then summarize by indicating when the absolute value is less than, it's an "and" statement, when its greater then, it's an "or" statement.

Do the same absolute value problems from above but use inequalities.

EXAMPLES

$|x| > 8$

$|x - 1| \geq 12$

$|2x + 1| < 13$

PRACTICE PROBLEMS

$|2x + 3| < 13$

$| 2x - 1 | \geq 9$

$|3(x - 2)| < 12$

$2|2x - 1| - 4 \leq 8$

TESTING

RECOMMENDATION

Construct a test specifically designed to increase student performance using our test template with 3-star, 2-star and 1-star questions. The 3-star questions have no computation or manipulation and are reviewed daily during the QCPR and are on the test. The 2-star questions are problems based on the 3-star that are checked for proficiency everyday right after the QCPR. And the 1-star questions are ACT/SAT, conceptional type questions. A copy of the practice test should be constructed and posted on the math department's website so students and parents have access.

|| Practice Test Name_____

Date _____

1. ***Using mathematical notation, define the Distributive Property.

2. ***Using mathematical notation, define Absolute Value.

3. ***Write the following in word form. $3 < 2x + 1 \leq 10$

4. ***Write the Order of Operations

5. ***Write the strategy for solving linear equations.

6. ***Write the strategy for solving equations containing absolute value.

7. ** $4x + 2 = 22$

8. ** $\frac{2x}{3} - 4 = 10$

9. ** $8x + 6 = 6x + 10$

10. ** $7x + 4 = 4x - 23$

11. ** $5(3x - 2) + 4 = 10x + 29$

12. ** $4(2x + 1) - 2(3x - 2) = x + 9$

13.** $6x - 3(x - 8) = 4(x - 7) + 6$

14. ** Solve and graph: $2x - 1 \leq 13$

15. ** Solve and graph: $-4x + 6 \leq 2x - 30$

16.** Solve and graph: $2x - 5 > 1$ or $3x + 2 \geq 14$

17.** Solve and graph: $-10 \leq 2x + 3 \leq 5$

18.** Solve: $|2x - 1| = 13$

19.** Solve & graph: $|2x - 1| \leq 13$

20.* Fill in the justifications:

$2x - 1 = 9$	Given
$2x - 1 + 1 = 9 + 1$	_____
$2x + 0 = 10$	_____
$2x = 10$	Identity for Add
$x = 5$	_____

21.*** Parental contact information: Provide phone number, home address, email or text. (CHP)