Quadratics – Problem Solving

1. Find 2 consecutive positive integers whose product equals 6. (Use a quadratic equations)

   1st number – x
   2nd number – $x + 1$

   $x(x + 1) = 6$ or $x^2 + x = 6$;

   $x^2 + x - 6 = 0$
   $(x + 3)(x - 2) = 0$

   so $x = -3$ or $x = +2$

   The answer has to be positive, the numbers are 2, and 3

2. The length of a rectangle is three times its width. The area of the rectangle is 48, find the length and width.

   W- x
   L – $3x$
   LW = Area, $3x(x)= 48$

   $3x^2 = 48$
   $3x^2 - 48 = 0$
   $3(x^2 - 16) = 0$
   $3(x + 4)(x - 4) = 0$

   The length has to be positive, so the width is 4 and the length is 12

3. The length of a rectangle is 4 feet longer than its width. If the area of the rectangle is 45 square feet, find the dimensions.

   W – x
   L – $x + 4$
   LW = Area; $x(x + 4) = 45$

   $x^2 + 4x = 45$
   $x^2 + 4x - 45 = 0$
   $(x + 9)(x - 5) = 0$

   The answer has to be positive (length), width is 5 and length is 9

4. An object is thrown upward with an initial velocity of 128 feet per second. If the height above ground after t seconds is given by the formula $h = 20 + 128t - 16t^2$, when will it reach it's maximum height and what is the maximum height of the object?

The max or min on quadratic equations is given by $-b/2a$ (vertex) in the equation $y = ax^2 + bx + c$

In this case, $b = 128$ and $a = -16$, substitute those numbers into $-b/2a$
$-128/-32 = +4$. So the max height occurs at 4 seconds. Substitute 4 into the height equation; $h = 20 + 128t - 16t^2$
$= 20 + 128(4) - 16(4)^2 = 256$ feet

5. An object is thrown upward with an initial velocity of 128 feet per second from a height of 20 feet above ground. If the height above ground after t seconds is given by the formula $h = 20 + 128t - 16t^2$, when will it reach its maximum height and what is the maximum height of the object?

Same as problem 4 except the ball is thrown from a height of 20 feet. In the last problem it reached a max height of 256 feet, just add the 20, final answer is 276 feet.

6. In a 110 volt circuit having a resistance of 11 ohms, the power W in watts when a current I is flowing is given by $W = 110I - 11I^2$. Determine the maximum power that can be delivered in this circuit.

Using $-b/2a$ to find when the max I occurs. $b = 110$, $a = -11$
$-b/2a = -110/-22 = 5$
Substitute $I = 5$ in the for power equation
$W = 110(5) - 11(5)^2 = 275$ watts

7. A building developer estimates the monthly profit p in dollars from a building s stories high is given by $p = -2s^2 + 88s$. To maximize his profit, how many stories should he build?

Using $-b/2a$, to find the max stories
$-b/2a = -88/-4 = 22$, to max his profit, he should build 22 stories.
The problem did not ask for his max profit, if it did, you would substitute 22 in the profit equation given

8. A shuttle operator charges a fare of \$10 to the airport and carries 300 people per day. The owner of the shuttle service estimates for every dollar increase in fare, he will lose 15 passengers. Find the most profitable fare for him to charge.

Cost x people = fare
10 x 300 = \$3000

For every increase of \$1, he loses 15 people
$(10 + x)(300 - 15x) = \text{fare}$
$3000 + 150x - 15x^2 = \text{fare}$
Using $-b/2a$, $b = 150$, $a = -15$, we have $-150/-30 = 5$
He should increase the fare by \$5 for max profit
$(10 + 5)(300 - 15(5))$
$15 (225) = \$3375$