

Using Composition to Identify Inverses

If f and g are functions with domains D_f and D_g , respectively, and for each all $x \in D_f$

$$g(f(x)) = x$$

and for each $x \in D_g$

$$f(g(x)) = x$$

then f and g are inverse functions

Example

$$f(x) = 2x + 15 \quad \text{and} \quad f^{-1}(x) = \frac{x-15}{2}$$

$$f(x) = 2x + 15$$

$$f(f^{-1}(x)) = 2\left(\frac{x-15}{2}\right) + 15$$

$$f(f^{-1}(x)) = x - 15 + 15$$

$$f(f^{-1}(x)) = x$$

Example If $f(x) = 6x - 2$ and $g(x) = \frac{x-2}{6}$, are $f(x)$ and $g(x)$ inverses?

Determine if $f(g(x)) = g(f(x)) = x$, then they are inverses.

$$\begin{aligned} f(g(x)) &= 6(g(x)) - 2 \\ &= 6\left(\frac{x-2}{6}\right) - 2 \\ &= x - 2 - 2 \\ &= x - 4 \end{aligned}$$

f and g are NOT inverses because $f(g(x)) \neq x$. I can stop here.