

Recursive Functions

Now we see a linear function can describe an arithmetic sequence by substituting values of x beginning with one and finding the corresponding values of y .

Example 1 Find the arithmetic sequence defined by $f(x) = 2x + 3$

| | | | | | |
|---|---|---|---|----|----|
| x | 1 | 2 | 3 | 4 | 5 |
| y | 5 | 7 | 9 | 11 | 13 |

The resulting sequence is 5, 7, 9, 11, 13, ...

Notice the slope is 2 – the common difference of each term of the sequence is 2.

To find the third term of the sequence, we can see from the chart it is 9. Or, we could find the value of f at 3, written mathematically as $f(3) = 9$ from the rule $f(x) = 2x + 3$.

Look at the chart, look at the sequence, and look how f is defined. Now look at the next three statements.

Note the 3rd term is found by adding 2 to the 2nd term. In other words

$$f(3) = f(2) + 2$$

Note the 4th term is found by adding 2 to the 3rd term. In other words

$$f(4) = f(3) + 2$$

Note the 5th term is found by adding 2 to the 4th term. In other words

$$f(5) = f(4) + 2$$

To find the function recursively, I'm merely adding the common **difference** (**which is the slope**) to the preceding term in the sequence which is described in functional form.

Now, to ensure we understand the notation, let's look at $f(5) = f(4) + 2$.

Another way to write that is

$$f(5) = f(5 - 1) + 2$$

Mathematically, we say that $f(n) = f(n - 1) + d$, where d is the common difference or slope.