Compound Interest Derivation

$$
\mathbf{A}_{\mathbf{1}}=\mathbf{P}+\mathbf{r} \mathbf{P} \quad \text { or } \quad \mathbf{A}_{1}=\mathbf{P}(1+\mathbf{r})
$$

The second interest period would take the funds ( $\mathrm{A}_{1}$ ) and add interest to it.
Substituting $\mathbf{P}(\mathbf{1}+\mathbf{r})$ for $\mathbf{A}_{\mathbf{1}}$ Factoring out $\mathbf{P}(\mathbf{1}+\mathbf{r})$

$$
\begin{gathered}
\mathbf{A}_{2}=\mathbf{A}_{1}+\mathbf{r} \mathbf{A}_{1} \\
\mathbf{A}_{2}=\mathbf{P}(\mathbf{1}+\mathbf{r})+\mathbf{r} \mathbf{P}(\mathbf{1}+\mathbf{r}) \\
\mathbf{A}_{2}=\mathbf{P}(\mathbf{1}+\mathbf{r})[\mathbf{1}+\mathbf{r}] \\
\mathbf{A}_{2}=\mathbf{P}(1+\mathbf{r})^{2}
\end{gathered}
$$

The third interest period would take the total funds $\left(\mathrm{A}_{2}\right)$ and add interest to it.

$$
{ }_{2} \mathbf{A}_{3}=\mathbf{A}_{2}+\mathbf{r} \mathbf{A}_{2} \text { Substituting } \mathbf{P}(\mathbf{1}+\mathbf{r}) \text { for } \mathbf{A}_{2}
$$

2
Factoring out $\mathbf{P}(\mathbf{1}+\mathbf{r})$
$A_{3}=\mathbf{P}(\mathbf{1}+\mathbf{r})+\mathbf{r P}(\mathbf{1}+\mathbf{r}) \mathbf{A}_{\mathbf{3}}=\mathbf{P}(\mathbf{1}+\mathbf{r})^{\mathbf{2}}[\mathbf{1}+\mathbf{r}]$

$$
\mathrm{A}_{3}=\mathbf{P}(1+\mathbf{r})^{3}
$$

The formulas for $\mathbf{A}_{1}, \mathbf{A}_{\mathbf{2}}$, and $\mathbf{A}_{\mathbf{3}}$ suggest a generalization for the $\mathbf{k}^{\text {th }}$ period.

$$
A_{k}=P(1+r)^{k}
$$

