## Theorems

Theorem In the same or congruent circles, if two central angles are congruent, their arcs are congruent.

Theorem In the same or congruent circles, if two minor arcs are congruent, the central angles are congruent.

If I played long enough with arcs and chords, I would find that congruent arcs have congruent chords and congruent chords have congruent arcs. Those would be easily proven using the congruence theorems for triangles.

Theorem In the same or congruent circles, congruent chords have congruent arcs.

Theorem In the same or congruent circles, congruent arcs have congruent chords.

Theorem A diameter that is perpendicular to a chord bisects the chord and its two arcs.

To prove this theorem, we draw the picture, draw lines so triangles are formed, prove the triangles are congruent by HL Congruence Postulate, the rest falls into place nicely.

Theorem: A diameter that is $\perp$ to a chord bisects the chord and its $\mathbf{2}$ arcs

| Given: |  |
| :--- | :--- |
|  | $\overline{\mathrm{AB}} \perp \overline{\mathrm{XY}}$ |
| Prove: | $\overline{\mathrm{XK}} \cong \overline{\mathrm{YK}}$ |
|  | $\overparen{X B}=\overparen{Y B}$ |



|  | Statements | Reasons |
| :--- | :--- | :--- |
| 1. | Draw $\overline{\mathrm{OX}}$ and $\overline{\mathrm{OY}}$ | Construction |
| 2. | $\overline{\mathrm{AB}} \perp \overline{\mathrm{XY}}$ | Give |
| 3. | $\overline{\mathrm{OK}} \cong \overline{\mathrm{OK}}$ | Reflexive |
| 4. | $\overline{\mathrm{OY}} \cong \overline{\mathrm{OX}}$ | Radii of circle are $\cong$ |
| 5. | $\Delta \mathrm{OKX} \cong \Delta \mathrm{OKY}$ | HL |
| 6. $\overline{\mathrm{XK}} \cong \overline{\mathrm{YK}}$ | cpctc |  |
|  | $\boxed{\angle 1 \cong \angle 2}$ |  |
| 7. | $\overparen{X B} \cong \overparen{Y B}$ | 2 central $\angle$ 's are $\cong$, the arcs |

